Dynamical contribution to isospin symmetry breaking and see saw mechanism in extended technicolor

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Recibido el 10 de junio de 1998; aceptado el 12 de enero de 1999

We study the possibility of breaking isospin in an extended technicolor model with left-right symmetric gauge group $SU(2)_L \otimes SU(2)_R \otimes SU(4)_C \otimes SO(N)_{ETC}$. The model contains two sectors of technicolor embedded in $SO(N)_{ETC}$. One of these sectors, called metacolor, gives rise to a Majorana type condensate which in turns induces large Majorana masses for right handed neutrinos through the ETC interactions. The other technicolor sector generates Dirac type condensates. A dynamical see saw mechanism is implemented in a natural way, and an isospin breaking is generated in the leptonic sector. This isospin breaking is transmitted to the ordinary quark sector through two loop diagrams involving extended technicolor and $SU(4)_C$ gauge fields. For the leptonic sector we can accommodate a realistic spectrum of masses. However for the quark sector the transmission of isospin breaking is not enough to generate the known isospin splitting between members of an isodoublet. This situation in the quark sector may be improved by combining it with some other ideas of isospin splitting.

Keywords: Isospin breaking; technicolor

Estudiamos la posibilidad de romper el isoespín en un modelo de tecnicolor extendido con grupo de norma izquierdo-derecho simétrico $SU(2)_L \otimes SU(2)_R \otimes SU(4)_C \otimes SO(N)_{ETC}$. El modelo contiene dos sectores de tecnicolor metidos dentro de $SO(N)_{ETC}$. Uno de estos sectores, llamado metacolor, da lugar a un condensado tipo Majorana, el cual induce masas tipo Majorana muy grandes para los neutrinos derechos a través de las interacciones de ETC. El otro sector de tecnicolor genera condensados tipo Dirac. En esta forma se implementa un mecanismo *see-saw* dinámico en forma natural y se genera un rompimiento de isoespín en el sector de leptones. Este rompimiento de isoespín se transmite al sector de quarks a través de diagramas a dos lazos por medio de los bosones de tecnicolor extendido y de $SU(4)_C$. Para el sector de leptones se puede acomodar un espectro realista de masas. Sin embargo, para los quarks la transmisión de rompimiento de isoespín no es suficiente para generar la diferencia de masas entre miembros de un isodoblete. Esta situación en el sector de quarks puede mejorarse si se combina con otras ideas de producir diferencia de masas.

Descriptores: isoespín; tecnicolor

PACS: 12.60.Nz; 12.15.Ff; 11.15.Ex

1. Introduction

Extended technicolor models [1, 2] were invented to deal with two of the main problems in present day theory of elementary particle physics, namely the question of the origin of the mass of elementary fermions and the breaking of the electroweak gauge symmetry. These models contain besides the ordinary fermion sector a new fermionic sector populated by the technifermions. Technicolor forces produce, at the scale where they become strong, a technicondensate which breaks the electroweak gauge symmetry. This technicondensate plays also a crucial role in the graphs which give rise to the mass of ordinary fermions. These graphs, at the one loop level contain a transition, mediated by an extended technicolor gauge field, from an ordinary fermion to a technifermion whose condensate plays the role of a seed of mass for ordinary fermions. The magnitude of the mass obtained from these graphs depends on the square of the inverse of the mass of the extended technicolor gauge fields in QCD like technicolor theories or depends on the inverse of the mass of the extended technicolor gauge field for walking [3] technicolor theories.

The hierarchy in the fermion spectrum, and in particular the splitting between isospin doublets, has its origin in the hierarchy and splitting of masses of the extended technicolor gauge fields. Here one can either stop, since the origin of the breaking of the extended technicolor gauge group to the technicolor one is unknown, or one can play with Higgses and vacuum expectation values to obtain the desired breaking of the extended technicolor gauge fields. In the later case one ends, effectively, putting by hand the splitting among masses of ordinary fermions. This feature is common to any theory where the electroweak group is the standard $SU(2)_L \otimes U(1)_Y$ one. If one wants to have a better understanding of the origin of isospin splitting one has to start considering a theory which does not break explicitly isospin. The electroweak sector has to be therefore left-right symmetric. But this is not enough, a seed for isospin splitting has to be introduced.

In this article we discuss a mechanism which contains as a seed for isospin splitting a condensate of new fermions called metafermions [4]. A preliminary version of these ideas was published in Ref. 5.

2. The origin of isospin splitting

Let us start considering a gauge theory based on the local gauge group

$$G = SU(2)_L \otimes SU(2)_R \otimes G_{\text{extc}}, \quad (1)$$

where G_{extc} contains the group of color, technicolor, and extended technicolor. The technicolor interaction should become strong at a certain scale where condensates are produced with the property that the electroweak gauge group is broken down to the electromagnetic one,

$$SU(2)_L \otimes SU(2)_R \otimes U(1)_{\text{B-L}} \to U(1)_Q. \quad (2)$$

This breaking has to proceed however in two steps:

$$SU(2)_L \otimes SU(2)_R \otimes U(1)_{\text{B-L}} \to SU(2)_L \otimes U(1)_Y \to U(1)_Q. \quad (3)$$

One needs therefore two type of condensates: an object, Δ_R, which respects $SU(2)_L$ and breaks $SU(2)_R \otimes U(1)_{\text{B-L}}$ down to $U(1)_Y$, as it would do a Higgs scalar transforming as a $(1, 3, -2)$ of the left-right symmetric group, and another object, ϕ, which breaks $SU(2)_L \otimes U(1)_Y$ down to $U(1)_Q$. In the usual technicolor theories the condensates

$$\langle \bar{Q}Q \rangle = \langle 0|\bar{Q}Q|0 \rangle, \quad (4)$$

where Q is a techniquark field, break $SU(2)_L$ and $SU(2)_R$ at the same scale; that is, they generate the symmetry breaking

$$SU(2)_L \otimes SU(2)_R \to SU(2)_{\text{L+R}}. \quad (5)$$

Since two types of condensates are needed, we assume that there are two different technicolor-type interactions. One, which we call metacolor, produces at a scale Λ_m a condensate of the form

$$\Delta_R = \langle N_{\text{R}}^{'T} C^{-1} N_R' \rangle, \quad (6)$$

where N' is an electrically neutral metafermion field and where through C, the charge conjugation matrix, a Lorentz invariant form is completed. The other technicolor-type interaction, for which we reserve the name technicolor, becomes strong at an energy scale Λ_t and produces condensates

$$\phi = \langle \bar{Q}Q \rangle. \quad (7)$$

N_R' transforms as a $SU(2)_L$ singlet with zero weak hypercharge; therefore $N_{\text{R}}^{'T}\ C^{-1}\ N_R'$ transforms trivially under $SU(2)_L \otimes U(1)_Y$.

The left-right symmetry of the electroweak group should be broken spontaneously. Therefore G_{extc} is left-right symmetric; that is either G_{extc} is vectorial or it may be written as $G_v \otimes G_L \otimes G_R$ where G_v is vectorial, G_L and G_R are isomorphic and if the left handed fermions transform as the representation N of G_L, the right handed ones transform as the representation N of G_R.

3. The model

We assume that

$$G_{\text{extc}} = SU(4)_C \otimes SO(N)_{\text{ETC}}, \quad (8)$$

where $SU(4)_C$ is the color group of the Pati Salam model [6] and $SO(N)_{\text{ETC}}$ is the group that contains technicolor and metacolor interactions as well as transitions among the three fermionic sectors (fermions, technifermions and metafermions). The fermionic sector consists of

$$\begin{aligned}
&quarks: & q_i^c &= \begin{pmatrix} u \\ d \end{pmatrix}_i^c, \\
&techniquarks: & Q_j^{cc'} &= \begin{pmatrix} U \\ D \end{pmatrix}_j^{cc'}, \\
&metaquarks: & Q_k^{cc''} &= \begin{pmatrix} U' \\ D' \end{pmatrix}_k^{cc''}, \\
&leptons: & \ell_i &= \begin{pmatrix} \nu \\ e \end{pmatrix}_i, \\
&technileptons: & L_j^{c'} &= \begin{pmatrix} N \\ E \end{pmatrix}_j^{c'}, \\
&metaleptons: & L_k^{c''} &= \begin{pmatrix} N' \\ E' \end{pmatrix}_k^{c''},
\end{aligned} \quad (9)$$

where

$$\begin{aligned} c &= 1,2,3, \\ c' &= 1,\cdots,n, \\ c'' &= 1,\cdots,m \end{aligned} \quad (10)$$

are the number of colors, technicolors, and metacolors respectively, while

$$\begin{aligned} i &= 1,\cdots,n_f, \\ j &= 1,\cdots,n_f', \\ k &= 1,\cdots,m_f', \end{aligned} \quad (11)$$

correspond to the number of families, technifamilies and metafamilies.

Quarks and leptons of a given family i form an irreducible 4-plet of $SU(4)_C$. The same is true for each family j of technifermions of a given technicolor c' and each family k of metafermions of a given metacolor c'',

$$f_i = \left[\begin{pmatrix} u \\ d \end{pmatrix}^1 \begin{pmatrix} u \\ d \end{pmatrix}^2 \begin{pmatrix} u \\ d \end{pmatrix}^3 \begin{pmatrix} \nu \\ e \end{pmatrix} \right]_i , \qquad (12)$$

$$F_j^{c'} = \left[\begin{pmatrix} U \\ D \end{pmatrix}^1 \begin{pmatrix} U \\ D \end{pmatrix}^2 \begin{pmatrix} U \\ D \end{pmatrix}^3 \begin{pmatrix} N \\ E \end{pmatrix} \right]_j^{c'} , \qquad (13)$$

$$M_k^{c''} = \left[\begin{pmatrix} U' \\ D' \end{pmatrix}^1 \begin{pmatrix} U' \\ D' \end{pmatrix}^2 \begin{pmatrix} U' \\ D' \end{pmatrix}^3 \begin{pmatrix} N' \\ E' \end{pmatrix} \right]_k^{c''} . \qquad (14)$$

Left handed and right handed parts of these fermions transform as irreducible N-plets of $\mathrm{SO}(N)_{\mathrm{ETC}}$,

$$\Psi_{\mathrm{L,R}} = \left(f_1, f_2, \cdots, f_{n_f}, F_1^1, \cdots, F_{n'_f}^n, M_1^1, \cdots, M_{m'_f}^m \right)_{\mathrm{L,R}}^T , \qquad (15)$$

where

$$N = n_f + nn'_f + mm'_f , \qquad (16)$$

For simplicity, let us deal in the following with a model of three ordinary families, one family of technifermions and one family of metafermions,

$$\Psi_{\mathrm{L,R}} = \left(f_1, f_2, f_3, F^1, \cdots, F^n, M^1, \cdots, M^m \right)_{\mathrm{L,R}}^T ,$$
$$N = 3 + n + m. \qquad (17)$$

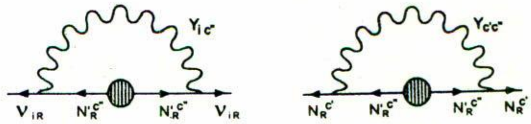
4. Symmetry breaking and fermion masses

$\mathrm{SU}(4)_C$ should be broken down to $\mathrm{SU}(3)_c \otimes \mathrm{U}(1)_{\mathrm{B-L}}$. This breaking should be triggered by the dynamics of $\mathrm{SO}(N)_{\mathrm{ETC}}$, that is by the technicolor and metacolor interactions that it contains.

We assume that the technicolor and metacolor interactions are given by the gauge groups $\mathrm{SU}(\ell)_{c'}$, $\ell = [n/2]$, $[\mathrm{SU}(\ell)_{c'} \subset \mathrm{SO}(n)]$ and $\mathrm{SO}(m)_{c''}$ respectively. Starting at some high energy scale $\mathrm{SO}(N)_{\mathrm{ETC}}$ is broken, by some still unspecified mechanism, gradually to [7] $\mathrm{SU}(\ell)_{c'} \otimes \mathrm{SO}(m)_{c''}$. In this process all the extended technicolor and extended metacolor gauge fields acquire mass. As we continue lowering the energy scale we arrive at the scale $\wedge_m$ where the $\mathrm{SO}(m)_{c''}$ metacolor interactions become strong and produce a metacondensate which should be $\mathrm{SO}(m)_{c''}$ singlet and which we assume to be electrically neutral. It is therefore of the form

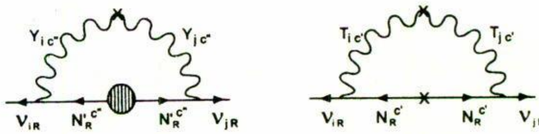
$$\Delta_R = \sum_{c''=1}^{m} \langle N'^T_{c''R} C^{-1} N'_{c''R} \rangle \qquad (18)$$

with a similar one with left handed metafermions. Δ_R breaks $\mathrm{SU}(2)_R$ and therefore it breaks isospin but it does not break $\mathrm{SU}(2)_L$. We suspect that, as in the case of QCD, there are no condensates which involve two different fields, like $\langle E'^T_{\mathrm{C''R}} C^{-1} N'_{\mathrm{C''R}} \rangle$ and that electromagnetic repulsion forbids the formation of QED breaking condensates of the form $\langle E'^T_{\mathrm{C''R}} C^{-1} E'_{\mathrm{C''R}} \rangle$. Furthermore, we assume, and this is our strongest hypothesis, that at this point the left-right symmetry of $\mathrm{SO}(N)_{\mathrm{ETC}}$ is broken in such a way that we may ignore the metacondensates with left handed metafermions. This amounts to assume that the scale of isospin breaking is higher than the scale of $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ breaking.



(a) (b)

FIGURE 1. Majorana masses induced by the metacondensate for a) neutrinos and b) technineutrinos. $Y_{ic''}$ are the gauge bosons which mediate the transitions between ordinary fermions and metafermions while $Y_{c'c''}$ are those which mediate the technifermion-metafermion transitions



(a) (b)

FIGURE 2. Additional off diagonal contributions to the mass of the right handed neutrinos arising from the mixing of a) extended metacolor and b) extended technicolor gauge fields.

Δ_R breaks dynamically not only $\mathrm{SU}(2)_R$ and isospin but also $\mathrm{SU}(4)_C$. At the scale $\wedge_m$ the gauge group G is therefore reduced to $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \otimes \mathrm{SU}(\ell)_{c'} \otimes \mathrm{SO}(m)_{c''}$ and the gauge bosons W_R and $X_c^{2/3}$ obtain masses of order $\wedge_m$. The leptoquarks $X_c^{2/3}$, $c = 1, 2, 3$ are those gauge bosons of $\mathrm{SU}(4)_C$ which make quark $\leftrightarrow$ lepton transitions and their mass is of the order of the scale of isospin breaking.

On the other hand the metacondensate Δ_R induces a Majorana mass for neutrinos and technineutrino through the diagrams depicted in Figs. 1a and 1b. In these figures $Y_{ic''}$ are the gauge bosons which mediate the transitions between ordinary fermions and metafermions while $Y_{c'c''}$ are those which mediate the technifermion-metafermion transitions. The induced diagonal mass terms are of the form

$$\sum_{i=e,\mu,\tau} m_{iR} \nu_{iR}^T C^{-1} \nu_{iR} + M_R \sum_{c'=1}^{n} N_{c'R}^T C^{-1} N_{c'R}. \qquad (19)$$

The mass matrix for right handed neutrinos receives an additional off diagonal contribution from diagrams involving $Y_{ic''}$-$Y_{jc''}$ mixing (Fig. 2a), where the source of mass is Δ_R and from diagrams where the source of mass is the Majorana mass of the technineutrino. This last type of diagrams

(Fig. 2b) involve fermion-technifermion transitions mediated by gauge fields $T_{ic'}$. The mixing between Y fields and between T fields depends on the details of the breaking of $\mathrm{SO}(N)_{\mathrm{ETC}}$ down to $\mathrm{SU}(\ell)_{c'} \otimes \mathrm{SO}(m)_{c''}$.

At the scale $\wedge_t \ll \wedge_m$ the technicolor forces become strong and form technicondensates

$$\langle \bar{Q}Q \rangle = \sum_{c'=1}^{\ell} \sum_{c=1}^{3} \langle \overline{Q}_{cc'} Q_{cc'} \rangle, \qquad Q = U, D, \tag{20}$$

$$\langle \bar{L}L \rangle = \sum_{c'=1}^{\ell} \langle \bar{L}_{c'} L_{c'} \rangle, \qquad L = N, E, \tag{21}$$

and using the result that in $\langle \bar{Q}Q \rangle$ there is a summation over the three colors,

$$\langle \bar{Q}Q \rangle = 3 \langle \overline{L}L \rangle. \tag{22}$$

These technicondensates contribute, through diagrams, (Fig. 3), involving extended technicolor gauge fields $T_{ic'}$ to the mass of ordinary fermions. These contributions are of the Dirac type and have the property that

$$M_u = M_d \equiv M_q, \qquad M_\nu^D = M_e \equiv M_l, \tag{23}$$

$$M_q = 3M_l, \tag{24}$$

where in an obvious notation M_u, M_d, M_ν^D, M_e, are the Dirac mass matrices for up quarks, down quarks, neutrinos and charged leptons respectively. These mass matrices come from the operators $\bar{\mathrm{U}}\mathrm{U}\bar{u}_i u_j$, $\bar{D}D\bar{d}_i d_j$, $\bar{N}N\bar{\nu}_i\nu_j$ and $\bar{E}E\bar{e}_i e_j$, where i and j are family indices. The degeneration in Eq. (23) is a consequence of the isospin symmetry of the condensates $\langle \bar{\mathrm{U}}\mathrm{U} \rangle = \langle \bar{D}D \rangle$, $\langle \bar{N}N \rangle = \langle \bar{E}E \rangle$ and the fact that the $T_{ic'}$ extended gauge bosons do not differentiate between the up and down sectors and neither between quarks and leptons of a given family.

In the leptonic sector there is no problem with the above degeneration in the up and down sectors, because for neutrinos we can implement a see saw mechanism from which one obtains

$$m_{\nu_i} \sim \frac{(m_{\nu_i}^D)^2}{m_{iR}}. \tag{25}$$

A constraint on $m_{\tau R}$ is set by the upper bound on the τ neutrino mass, $m_{\nu_\tau} \leq 100$ ev from cosmology: With $m_{\nu_\tau}^D \sim m_\tau \sim 1.5$ Gev, one obtains from Eq. 25, $m_{\tau R} \geq 3 \times 10^4$ Tev.

To break the isospin symmetry in the quark sector one needs to bring into play the $X_c^{2/3}$ gauge bosons of $\mathrm{SU}(4)_C$, as well as the heavy right handed neutrinos. These type of contributions are illustrated in Fig. 3. We can see that the contribution from these figures breaks the degeneration in the masses of the up and down quark sector, because the right handed technineutrino and neutrinos are much heavier than the technilepton "E" and the charged leptons e_j. However, the magnitude of this contribution to the isospin splitting in the quark sector is not of great significance, because it is an order of perturbation theory smaller than usual contribution from the extended technicolor one loop graphs. So, we have the relations

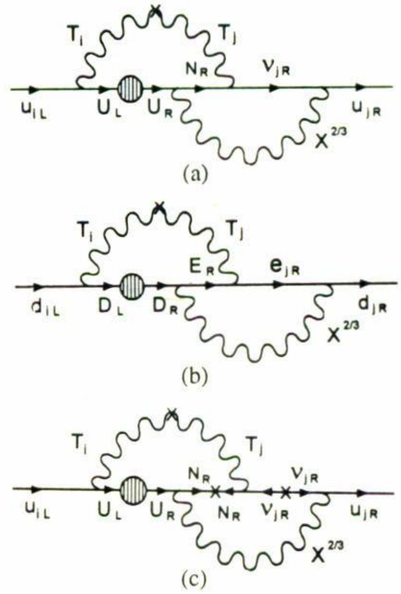


FIGURE 3. The isospin splitting in the leptonic sector is fed into the quark sector through the gauge fields $X_c^{\frac{2}{3}}$ of $\mathrm{SU}(4)_C$.

$$(\delta M_d)_{ij} \ll (\delta M_u)_{ij} \leq M_{qij}, \tag{26}$$

where $(\delta M_u)_{ij}$ is the contribution from Figs. 3a and 3c, $(\delta M_d)_{ij}$ the one from Fig. 3b and M_{qij} the degenerated contribution from the usual extended technicolor [Eq. (23)].

5. Numerical results

Now we proceed to do some numerical analysis to obtain the relations among fermion masses predicted by our model, and then we describe briefly how we can improve our results following some ideas already proposed by other authors.

Let us suppose here that masses and mixing of the $T_{ic'}$ gauge fields are generated in a SSB process of the ETC symmetry in such a way that the eigenvalues of M_ℓ give us the correct masses for the charged leptons. The very small masses for neutrinos can be controlled by chosing properly the heavy masses of the $Y_{ic''}$ gauge bosons in the see-saw mechanism. It is in this sense that we argue that it is possible to accommodate a realistic spectrum of lepton masses.

For the quark sector, from Eqs. (23), (24) and (26) we have:

d quarks: $M_d \approx 3M_\ell$ and therefore

$$m_b \approx 3m_\tau, \qquad m_s \approx 3m_\mu, \qquad m_d \approx 3m_e; \tag{27}$$

u quarks:

$$3m_\tau < m_t \leq 6m_\tau, \qquad 3m_\mu < m_c \leq 6m_\mu,$$
$$3m_e < m_u \leq 6m_e. \tag{28}$$

The mass relation between b and τ in Eq. (27) is the best prediction of the model. The other two relations in this equation are not the correct ones, although they are within the orders of magnitude of the known values. On the other hand, the t-b and c-s mass ratios resulting from Eqs. (27) and (28) turn out to be very small. Note from the two-loop contributions to quark mass matrices that $\delta M_d, \delta M_u$ and M_q in Eq. (26) have different matrix structure. This fact is important because then M^u and M^d, the final mass matrices, can not be diagonalized simultaneously and then in principle is possible to generate the CKM mixing.

We now try to incorporate some other ideas to increase the isospin splitting between the up and down sectors of quarks. In the context of "walking technicolor" a walking TC coupling allows high energy physics well above the weak scale to play an important role in the chiral symmetry breaking dynamics responsible for electroweak symmetry breaking. In particular effective four-fermion operators induced by the high energy physics may distinguish different flavors of technifermions and thereby split the corresponding condensates [8]. The question is whether such isospin breaking in the techniquark condensates is a source of isospin breaking for quark masses, while at the same time an approximate custodial SU(2) symmetry in the techniquark sector is preserved so that the ρ parameter is kept close to unity within the known experimental uncertainty. Authors of Ref. 8 argue that in this scheme it is reasonably to generate most or all of the c-s mass ratio, whereas the so large t-b mass ratio is rather difficult to accommodate within this framework.

Another scenario to get a splitting of the t and b quark masses is due to Holdom's [9] idea of considering a ETC gauge symmetry with at least two doublets of techniquarks (U_1, D_1) and (U_2, D_2), assuming that the ETC interactions preserve an approximate symmetry that forbids operators of the form $\bar{Q}_i Q_j \bar{q} q,\ i \neq j$, and also that condensates in the up sector are flavor diagonal, but the corresponding ones in the down sector are flavor violating. By defining new Dirac fermion fields one is able to define a custodial SU(2) symmetry that keeps the ρ parameter equal to unity, while the ETC interactions are such that t receives a mass while b remains massless. However, this contribution to the mass of t can not be larger than a few Gev's if FCNC are sufficiently suppressed even in the walking technicolor context.

6. Conclusions

To conclude, we have presented a plausible mechanism which may be the source of the splitting in mass inside isodoublets. The mechanism that we are suggesting works very well to accommodate the masses of the known leptons. Assuming this fact, the model predicts for the quark sector the relations giving by Eqs. (27) and (28). The masses of quarks are enhanced with respect to the masses of leptons because the techniquarks carry color, while technileptons does not.

The model predicts almost a degenerate spectrum of masses for the up and down sectors of quarks. The c-s mass ratio can be accomplished either by the isospin breaking in the techniquark condensates [8] or by the ideas suggested by Holdom [9]. However, to account for the great t-b mass ratio we need another sources of isospin breaking such as the models called "Topcolor" [10] and "Topcolor-assisted technicolor" [11].

Acknowledgments

This work was partially supported by CONACyT.

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