A comparison between spectral and fractal methods in electrotelluric time series

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By using electrotelluric time series, that is, data sets of the electric self-potential variations in a place close to the Middle American Tectonic Trench, we compare two methods of dynamical analysis. First, we calculate the spectral exponent α by means of fast Fourier transforms, and on the other hand we calculate the fractal dimension D of the time series by using the so-called Higuchi's algorithm. We find that the second method has remarkable advantages over the spectral analysis.

Keywords: Fractals; power spectrum; electrotelluric analysis

Comparamos dos métodos de análisis dinámico mediante series de tiempo electrotelúricas, es decir, conjuntos de datos de variaciones del autopotencial eléctrico de un lugar cercano a la Trinchera Mesoamericana. Primero calculamos el exponente espectral α mediante la transformada rápida de Fourier y por otro lado calculamos la dimensión fractal D de la serie de tiempo usando el algoritmo de Higuchi. Encontramos que el segundo método tiene notables ventajas sobre el análisis espectral.

Descriptores: Fractales; potencia espectral; análisis electrotelúrico

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1. Introduction

Nowadays, many natural phenomena are investigated through the collection of a discrete set of values of some characteristic variable in the course of time. This set of values is called a time series. For many physical systems, when this series is unfolded in the time axis, one finds a very irregular or "chaotic" aspect. Nevertheless, it is possible to look for valuable physical information contained in this data set by means of the techniques developed in the context of the theory of chaos and nonlinear dynamics [1–5]. One of the main signatures of the nonlinear behavior of one system is the great sensitivity of its temporal evolution on the initial conditions [1–5]. This property represents a great limitation on the predictability of the future evolution of the system. In spite of these restrictions the theory of nonlinear systems has much to do about the analysis of very irregular time series. Some of the most usual quantities for characterizing discrete time series are for example, the power spectrum, the fractal dimension and the Lyapunov exponent among others. The power spectrum (PS) of a fluctuating quantity is a measure of the power in each bandwidth or "intensity per Hertz" and it is calculated by squaring the normalized Fourier transform of the time series [6]. The power spectrum is very useful to characterize the kind of noise contained in a time series. This is accomplished by plotting in a log-log graph the PS versus the frequency. When this graph is a horizontal straight line, we have white noise and when the slope of the straight line is different from zero, we have a noise of color. These behaviors are summarized by means of a relationship of the following form (*i.e.* a power law):

$$S(f) \sim f^{-\alpha}, \tag{1}$$

where S(f) is the power spectrum of the time series (usually calculated by means of fast Fourier transform), f is the frequency and α is an exponent that defines the kind of noise involved, for example $\alpha = 0$ for white noise, which is uncorrelated and has a power spectrum that is independent of the frequency. Other relevant cases are $\alpha = 1$ for the so-called flicker or $1/f$ noise, which is moderately correlated and $\alpha = 2$ for Brownian noise which is strongly correlated. Another very important concept in time series analysis is the stationarity. A time series is stationary when it shows similar behavior throughout its duration. One definition of similar behavior is that the mean and standard deviation remain the same throughout the time series [1]. In general a white noise is stationary, and a fractional Brownian noise is non stationary [7]. If a time series is non stationary, then many analysis techniques as the spectral ones for example are of questionable application. Many time series have fractal properties [3], and in general they are of the kind of the statistically self-affine fractals, which are not isotropic, in contrast with self-similar fractals which are isotropic. Self-affine fractals in a 2-dimensional xy-space have the property that $f\left(rx, r^H y\right)$ is statistically similar to $f(x, y)$, where r is a scaling factor and H is known as the Hausdorff measure or the Hurst exponent [3, 4]. For self-affine fractals H and its fractal dimension D are linked by [3]

$$H = 2 - D. \tag{2}$$

This relation can be taken as the basic definition of the fractal dimension for a time series. For D in the interval $1 < D < 2$, it is required that $0 < H < 1$ which are the ranges for D and H for the so-called fractional Brownian noise [1, 8]. The exponent in expression (1) is also linked with H and D by means of the following relationships [3],

$$\alpha = 2H + 1 = 5 - 2D. \tag{3}$$

A very understandable demonstration of relations (2) and (3) can be found in chapter seven of Ref. 3.

Since five years ago, a research group [9, 10] has taken registers of the fluctuations of the electric self-potential of the ground (the so-called electrotelluric field) in several sites of Mexico. These registers are taken by means of electrotelluric stations whose details are in Ref. 9. Some stations are located along the coast of Guerrero state, near of Middle-American trench which is the border between the Cocos and the American tectonic plates. Another stations (control stations) were located at Cholula, Puebla and Mexico City more than 300 Km distant from the trench. The first is a very seismically active zone [11], while the second one has a moderate seismicity [12]. In a typical electrotelluric station thousand of data are taken each two or four seconds during periods in the scale of months and years. Those electrotelluric time series are analyzed by means of several techniques. Among these methods are the calculation of the power spectra and the fractal dimension. The PS are obtained by means of fast Fourier transform (FFT) and the fractal dimension by means of a method developed by Higuchi [13]. In this paper we compare the results of analyzing electrotelluric time series by the both mentioned methods and conclude that the Higuchi method has notorious advantages over the FFT spectral technique. We believe that this comparison has pedagogical consequences with regard to the care that must be taken when a time series analysis over field data is made.

2. Comparison between spectral and fractal analysis for electrotelluric time series

If a voltage time series $v(t)$ is specified over the interval of time T, the mean signal $\overline{v}\,(t)$ is given by

$$\overline{v}\,(T) = \frac{1}{T}\int_0^T v\,(t)\;dt. \tag{4}$$

The variance of the signal $V(t)$ is defined by

$$V(T) = \frac{1}{T}\int_0^T \left[v(t) - \overline{v}\right]^2 dt \tag{5}$$

and the standard deviation σ is the square root of $V(t)$.

The mean and the variance are the first two moments of the time series. If the time series is a self-affine fractal then [3, 8]

$$\sigma\,(T) \sim T^H. \tag{6}$$

As is well known, for Brownian noise (random walk) $\sigma \sim T^{\frac{1}{2}}$ [14] and thus $D = 3/2$ and $\alpha = 2$ [Eq. (3)]. For fractional Brownian noise the corresponding values are in the following intervals: $H \in (0, 1)$; $D \in (1, 2)$ and $\alpha \in (1, 3)$ [see Eq. (3)].

The time series $v\,(t)$ can be seen in the frequency domain in terms of the amplitude $A\,(f, T)$ which is the Fourier transform of $v\,(t)$

$$A\,(f, T) = \int_{-\infty}^{\infty} v(t)\; e^{2\pi i f t}\; dt. \tag{7}$$

The inverse Fourier transform is

$$v\,(t) = \int_{-\infty}^{\infty} A\,(f, T)\; e^{-2\pi i f t}\; df. \tag{8}$$

The quantity $|A\,(f, T)|^2$ is the contribution to the total energy of $v(t)$ from the components with frequencies in $[f, f + df]$. In real time series the samples are picked up in a finite interval $0 < t < T$, in such a way that the effect of finite time series shall be taken into account, the FFT method is the appropriate for this kind of analysis [15]. The power spectral density of $v\,(t)$ is defined by

$$S\,(f) = \lim_{T\to\infty} \frac{1}{T}\,|A\,(f, T)|^2\,. \tag{9}$$

The quantity $S\,(f)\,df$ is the power in the time series associated with the frequency in the interval $[f, f + df]$. If the time series is fractal then it satisfies relation (1) [3].

In Fig. 1, we show a typical segment of an electrotelluric time series registered at Ometepec-station (16.71 N, 98.45 W) near of Pacific coast in Guerrero state. As it can be seen the time series of self-potential differences between two electrodes E-W oriented is notoriously non-stationary. In contrast the corresponding N-S time series has some segments which are approximately stationary. If we calculate the PS [Eq. (9)] by means of the FFT (we use the Excel Microsoft package) of this data set we obtain in a log-log graph Figs. 2 and 3 for the E-W and N-S lines respectively.

The best least-squares fit for these graphs are $\alpha_{\rm NS} = 0.2693 \pm 0.224$ and $\alpha_{\rm EW} = 0.6132 \pm 0.28$, with correlation coefficients $R^2_{\rm NS} = 0.034$ and $R^2_{\rm EW} = 0.102$ respectively, which are very poor results, even for the N-S line which has quasistationary segments. In Figs. 2 and 3 the power spectra obtained by the FFT-method show noisy fluctuations superposed on the power law spectrum. Thus the unambiguous determination of the exponent α is difficult. Usually α is considered to be the index for representing the irregularity of a time serie [13], although the fractal dimen-

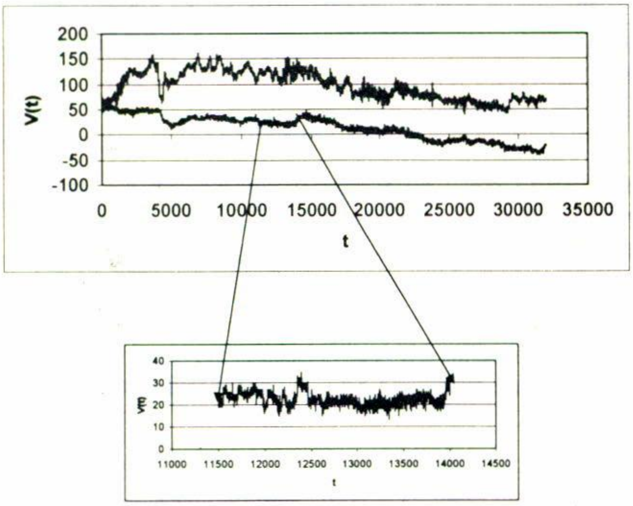

FIGURE 1. Time series of electric self-potential differences $v(t)$ between pairs of electrodes E-W (upper graph) and N-S (lower graph) oriented at Ometepec-station. The voltage files correspond to 36 hr of data registered each four seconds.

sion D can be also used as index of irregularity. In fact, in many cases the use of D is more appropriate than α for determining irregularity indices, as we will see below. The fractal technique developed by Higuchi [13] gives stable indices even for a small number of data.

Higuchi [13] considers a finite set of time series observations taken at a regular interval:

$$v(1), v(2), v(3), \cdots, v(n).$$

From the given time series, one first constructs a new time series, v_k^m, defined as follows

$$v_k^m : v(m), v(m+k), v(m+2k), \cdots, v\left(m + \frac{[N-m]}{k}\right)$$

with $m = 1, 2, \cdots, k$, and where [ ] denotes the Gauss notation, that is the bigger integer, and k and m are integers that indicate the initial time and the interval time respectively. For a time interval equal to k one gets k sets of new time series. For example, for $k = 4$ and $N = 100$, four new time series are obtained,

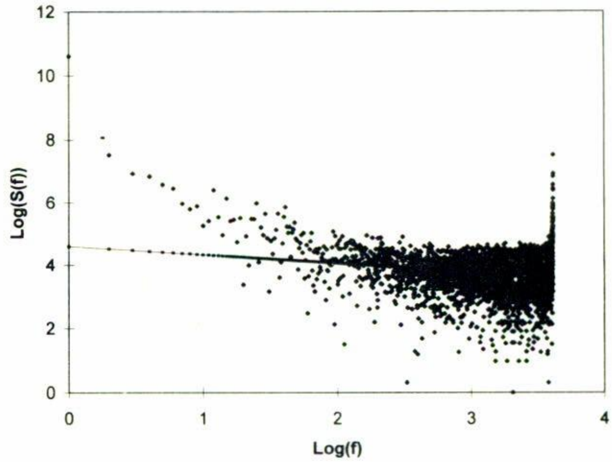

FIGURE 2. Log-log plot of the power spectrum against frequency for the N-S time series. The straight line is the best fit of the spectral data.

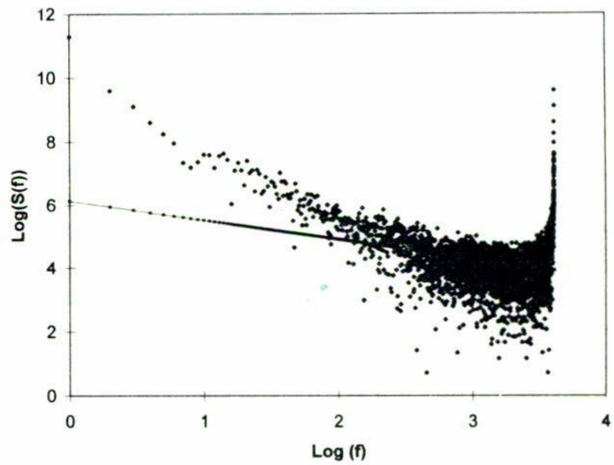

FIGURE 3. Log-log plot of the power spectrum against frequency for the E-W time series. The straight line is the best fit of the spectral data.

$$\begin{aligned} v_4^1 &: v(1), v(5), v(9), \cdots, v(97) \\ v_4^2 &: v(2), v(6), v(10), \cdots, v(98) \\ v_4^3 &: v(3), v(7), v(11), \cdots, v(99) \\ v_4^4 &: v(4), v(8), v(12), \cdots, v(100). \end{aligned} \tag{10}$$

Higuchi [13], defines the length of the curve associated to each time series, v_k^m as follows:

$$L_m(k) = \frac{\left[\sum_{i=1}^{\left[\frac{N-m}{k}\right]} \left| v(m+ik) - v[m+(i-1)k] \right| \right] \frac{N-1}{\left[\frac{N-m}{k}\right] k}}{k}, \tag{11}$$

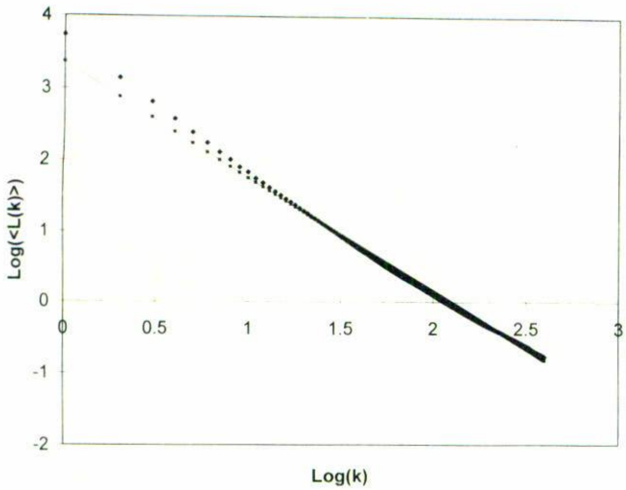

FIGURE 4. Log-log plot of $\langle L(k)\rangle$ against k for the N-S time series. The slope D of the fitted straight line is the corresponding fractal dimension.

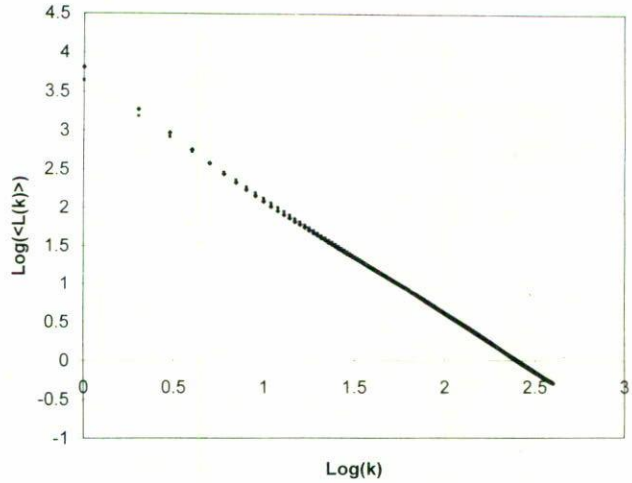

FIGURE 5. Log-log plot of $\langle L(k)\rangle$ against k for the E-W time series. The slope D of the fitted straight line is the corresponding fractal dimension.

where the term $(N-1)/\left[\frac{N-m}{k}\right]k$ represents a normalization factor, then the length of the curve for the time interval k is taken as the average value $\langle L(k)\rangle$ of the lengths associated to the time series given by Eq. (11). If $\langle L(k)\rangle \sim k^{-D}$, then the curve is fractal with dimension D. The Higuchi's algorithm can be applied even over time series that are not stationary and this fact represents an advantage over the spectral techniques. In order to illustrate this, we apply the Higuchi's method over the same time series (Fig. 1) used for calculating the spectral exponent by means of FFT (Figs. 2 and 3). The fractal dimensions obtained by using the Higuchi's method are:

$$D_{\rm NS} = 1.61 \pm 0.0006 \quad \text{and} \quad D_{\rm EW} = 1.52 \pm 0.002 \quad (12)$$

with correlation coefficients

$$R^2_{\rm NS} = 0.995 \quad \text{and} \quad R^2_{\rm EW} = 0.999$$

respectively (see Figs. 4 and 5). That is, the D-calculations have a remarkable better precision than the α-calculations by means of FFT. If we use Eq. (3), we immediately obtain $\alpha_{\rm NS} = 1.78$; $\alpha_{\rm EW} = 1.96$ and $H_{\rm NS} = 0.39$, $H_{\rm EW} = 0.48$. As can be seen the values of D, α, and H for the E-W segment (see Fig. 1) correspond very closely to typical Brownian noise values, that is $\alpha = 2$, $H = 0.5$, and $D = 1.5$. These values can not be obtained by means of FFT-analysis. All the values of D, α, and H obtained by means of Higuchi's method in fact correspond to typical fractional Brownian noise [8]. In Fig. 6, we depict the behavior of $D_{\rm NS}$ and $D_{\rm EW}$ at Ometepec station during the interval May 12 up to December 2, 1993. As can be seen the values correspond to fractional Brownian noise mixed with intervals with $D = 2$, that, is a white noise behavior [13].

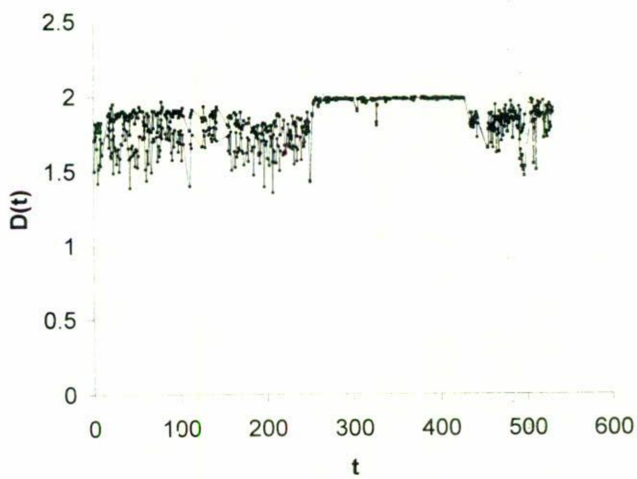

FIGURE 6. Plot of D against time (from May 12 to December 2, 1993) for both N-S and E-W voltage time series taken at Ometepec station. The dominant behavior correspond to fractional Brownian noise and thus to $1/f^{\alpha}$ noise, but as it can be seen, time intervals with white noise ($D = 2$) are also present.

3. Discussion and conclusions

By means of electrotelluric time series taken from a location near of Middle American trench, which is a very seismically active zone, we compare two analysis techniques, the spectral exponent α calculated by means of FFT and the fractal dimension calculated by using the Higuchi's algorithm. We show that the FFT-method leads to very poor correlation coefficients. On the other hand, the usage of the Higuchi's method leads to very precise values of the fractal dimension D of the electrotelluric time series. This fact consequently permits to calculate very precise values of α and H. It should be pointed out that this fact is mainly due to the Higuchi's definition for the length of the time series curve, which corresponds to overlaped averages over several time intervals and this process smooths the estimated lenght.

For the case studied in this article, we find that D, H, and α correspond to typical values of fractional Brownian noise. A possible explanation of these results has to do with the concept of self-organized criticality (SOC) [16]. The SOC-concept was developed for complex systems [3, 16] which are reminiscent of typical geological structures. The notion of SOC was proposed by Bak *et al.* [16] as a general principle governing the behavior of spatially extended dynamical systems with both temporal and spatial degrees of freedom. According to this principle, composite open systems having many interacting elements organize themselves into a stationary critical state with no length or time scales others than those imposed by the finite size of the system. The critical state is characterized by spatial and temporal power laws. According to Bak *et al.*, the temporal "fingerprint" of the SOC-state is the presence of $1/f^{\alpha}$ noise, with α in the range of fractional Brownian noise. Thus, a possible reason of our α-values is that the system generating the electrotelluric fluctuations is in a SOC-state. This assertion must be taken as a conjecture, because our evidences are preliminary. However, the main objective of this paper is the comparison of the nonlinear analysis techniques mentioned. In a previous work [10] we showed that in control stations (Cholula and Mexico City) the predominant noise is of the white-type, while in the stations close to the Middle American trench is common to find fractional Brownian noise. That is, seemingly the levels of seismicity are linked with the type of electrotelluric fluctuations. It is convenient to remark that in the coastal stations also appear intervals with white noise but the possible meaning of these facts are discussed elsewhere [17].

Acknowledgments

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