Analog modulation and demodulation of a chaotic oscillator

Jesús Urías
Instituto de Investigación en Comunicación Optica, Universidad Autónoma de San Luis Potosí
78000 San Luis Potosí, S.L.P., Mexico

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A method of parametric modulation and demodulation of chaotic signals is presented. The method uses chaotic synchronization for detecting parameter mismatches bewteen the transmitter and receiver. Practical considerations to implement a modem using the double scroll oscillator are discussed.

Keywords: Chaotic synchronization; comunications

Se presenta un método para la modulación y demodulación paramétricas de señales caóticas. El método usa la sincronización caótica para detectar las diferencias en uno de los parámetros del transmisor y el receptor. Se discuten algunas de las consideraciones necesarias para llevar a la práctica un *modem* usando el oscilador de doble enroscado.

Descriptores: Sincronización caótica; comunicaciones

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1. Introduction

The use of chaotic waveforms as carriers for communication systems based on the phenomenon of synchronization in chaotic systems [1, 2] is offering new fundamentals for the design of practical secure communication systems. Analog and digital modulation methods for chaotic oscillators have been proposed and the evaluation of their performance in communication systems is in progress [3, 12].

For transmission of binary data two modulation methods are found in the literature [3, 8]. A direct approach [3, 7] is to generate a chaotic carrier by switching between two different chaotic sources, following the sequence of 0's and 1's in the binary data stream. The demodulator identifies the stream of symbols in the chaotic carrier by means of a set of synchronous subsystems that are "tuned" to follow each of the chaotic sources. A second approach, seemingly more robust than the direct one, would use the binary data stream of information to drive the orbit of a chaotic system in a way that is compatible with its own (symbolic) dynamics [8]. A synchronous subsystem at the receiving end recreates the orbit of the transmitting system and then the information is retrieved.

For analog information sources, the direct approach to secure communications is to combine the information with chaotic signals in conventional mixers or scramblers [9]. The receiver consists of a synchronous subsystem that reproduces the masking chaotic signal that is necessary for recovering the information signal. A second approach [10, 12] is to detect parameter mismatches between the transmitter and the receiver. We adopt this approach in the form of the parametric modulation method proposed by Corron and Hahs in Ref. 11.

The principles of the method of parametric modulation [11] are outlined in Sect. 2. Section 3 describes the implementation of the double scroll oscillator (DSO, also known as Chua's circuit) [13] and the conditions that the parameters in the DSO circuit must met to operate in the chaotic regime are exposed. Then, in Sect. 4, the principles of the method of parametric modulation are applied to the DSO. In Sect. 5 the relevant time constants of the DSO are discussed and the results obtained for simple modulating signals are reported in Sect. 6.

2. The method of parametric modulation

Proposed by Corron and Hahs [11], in this approach the transmitter is a chaotic oscillator with a parameter that is modulated by the information signal. The receiver is a synchronous chaotic subsystem that is followed by a nonlinear filter for recovering the information signal. The fundamentals are detailed in the following.

The transmitter

Considering an oscillator of order three, the transmitter has the form

$$\begin{aligned} \dot{x} &= f_0(x,y,z) + \lambda(t) f_1(x,y,z), \\ \dot{y} &= g(x,y,z), \\ \dot{z} &= h(x,y,z). \end{aligned} \tag{1}$$

The parameter $\lambda(t)$ is a prescribed function of time that represents the information to be communicated. The transmitter (1) has to satisfy the following requirements.

(A) The carrier or synchronizing signal [1, 2] is $x(t)$, such that the two-dimensional subsystem on (y, z) is independent of the modulated parameter $\lambda(t)$. Then, the response subsystem on (y, z) at the receiver can synchronize for all "acceptable" values of $\lambda(t)$.

(B) The acceptable values of $\lambda(t)$ satisfy the condition $|\lambda(t)| < \epsilon$. The constant ϵ is selected such that the oscillator (1) operates in a chaotic regime.

The receiver

The original signals y and z are reconstructed from signal x as y_r and z_r by means of chaotic synchronization [1, 2] at the receiving end. The naive approach to recover $\lambda(t)$ would be to use the estimate

$$\lambda(t) = \frac{\dot{x} - f_0(x, y_r, z_r)}{f_1(x, y_r, z_r)}. \tag{2}$$

There are two sources of trouble when using Eq. (2) to implement the demodulator. (*i*) Singularities are encountered whenever $f_1(x, y_r, z_r) = 0$. (*ii*) The carrier signal x has a wide spectrum and further it may get the receiver contaminated with some noise introducing large errors when the time derivative $\dot{x}$ is estimated.

Alternatively to the estimate (2), Corron and Hahs [11] propose the uncoupled pair of first order filters

$$\dot{w}_0 = f_0(x, y_r, z_r) + (x - w_0)/k, \tag{3}$$

$$\dot{w}_1 = f_1(x, y_r, z_r) - w_1/k. \tag{4}$$

Constant $k > 0$ in the filters (3) and (4) is a tuning parameter which is adjusted to reduce residues of the carrier in the output of the receiver. Then, the information signal is estimated to be $\hat{\lambda} = (x - w_0)/w_1$, with w_0 and w_1 the outputs of the filters (3) and (4). This signal $\hat{\lambda}$ is further passed through the low-pass filter

$$\tau_f \dot{\lambda}_f = \hat{\lambda} - \lambda_f. \tag{5}$$

The full receiver consists of a synchronous subsystem on (y_r, z_r) that is driven by the incoming carrier x, generated by the transmitter (1), and followed by the filters (3)-(5). The demodulated signal is λ_f and the quantities k and τ_f are tuning parameters.

The double scroll oscillator (DSO) is a very handy and cheap electronic system to make a practical evaluation of the parametric modulation approach to secure communications.

3. The DSO

The electric circuit known as the double scroll oscillator is described by the set of equations

$$\begin{aligned} C_1\dot{V}_1 &= (V_2 - V_1)G - i(V_1), \\ C_2\dot{V}_2 &= (V_1 - V_2)G + i_L, \\ L\dot{i}_L &= -V_2. \end{aligned} \tag{6}$$

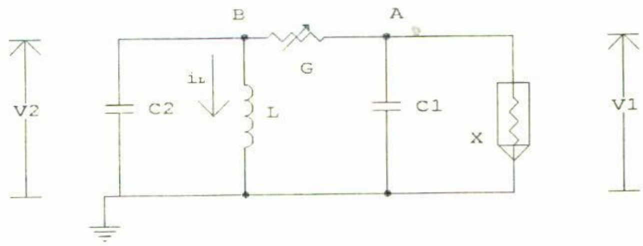

FIGURE 1. Schematics of the double scroll oscillator. The non linear element is marked with an X.

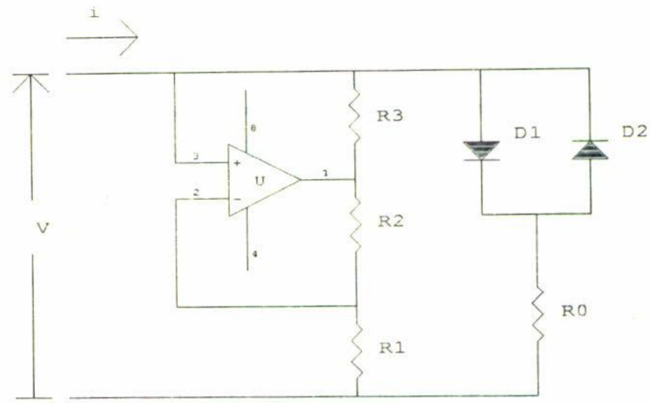

FIGURE 2. The non linear element in the DSO. Diodes D1 and D2 are the non linear components.

A point in phase space has coordinates (V_1, V_2, i_L). The schematics of the circuit is shown in Fig. 1. The first two Eq. (6) express current conservation at nodes A and B. The third one is the induction law for the coil that we are assuming is lossless. The rightmost element in the schematics in Fig. 1 is a non linear (negative) resistor with the following I-V characteristic,

$$i(v) = \begin{cases} -G_1 v - V_B(G_0 - G_1) & v > V_B \\ -G_0 v, & |v| \leq V_B \\ -G_1 v + V_B(G_0 - G_1) & v < -V_B. \end{cases} \tag{7}$$

At the breaking points $v = \pm V_B$, $i(\pm V_B) = \mp G_0 V_B$. The I-V characteristic of the non linear element (7) is implemented with a pair of diodes and an operational amplifier. The schematics is shown in Fig. 2. Diodes D1 and D2 are off whenever $|v| < V_B \sim 0.7$ V. The exact value of the break point of diodes, V_B, depends on the nature of diodes (Ge or Si). The two diodes in Fig. 2, D1 and D2, are off for $|v| < V_B$ and the i-v relationship is $i = -vR_2/(R_1R_3)$. For $v > V_B$, D1 is on and D2 is off and the relation is $i = -v(R_0R_2 - R_1R_3)/(R_0R_1R_3) - V_B/R_0$. The parameters in the i-v characteristic of the nonlinear element (7)

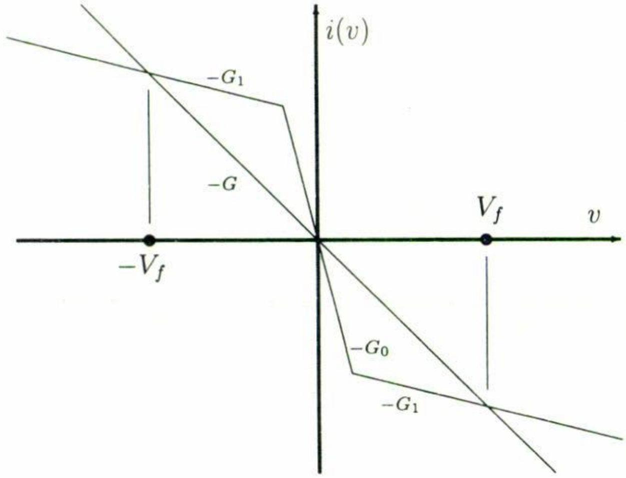

FIGURE 3. Graphical solution to $i(v) = -Gv$. The slopes of the line segments are as marked. The inequalities $G_1 < G < G_0$ are necessary to have three crossings.

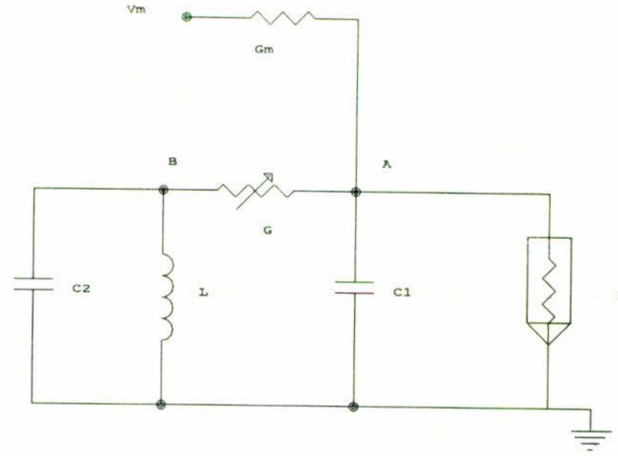

FIGURE 4. Schematics of the DSO with a tap for modulation through node A.

are identified from these relations as follows,

$$V_B = \text{break point of diodes},$$
$$G_0 = \frac{R_2}{R_1 R_3}, \tag{8}$$
$$G_1 = \frac{R_0 R_2 - R_1 R_3}{R_0 R_1 R_3}.$$

The circuit in Fig. 1 has either one or three equilibria, depending on the admittance parameters. It behaves as a real DSO when $G_1 < G < G_0$. In this case the oscillator has three equilibria, one of them, N, is located at the origin of phase space and the other two are located at the points $P = (-V_f, 0, V_f G)$ and $Q = (V_f, 0, -V_f G)$, where

$$V_f = V_B \frac{G_0 - G_1}{G - G_1} \tag{9}$$

satisfies $i(V_f) = -GV_f$. This relation is shown graphically in Fig. 3.

4. Modulating the DSO

The information signal is coupled to the DSO (6) by injecting it in the form of a time varying current into node A. In the schematics of Fig. 4 modulation is implemented by injecting a current through resistor R_m, driven by V_m. The set of equations for the modulated DSO is

$$C_1 \dot{V}_1 = (V_2 - \mu V_1)G - i(V_1) + \Lambda(t),$$
$$C_2 \dot{V}_2 = (V_1 - V_2)G + i_L, \tag{10}$$
$$L \dot{i}_L = -V_2.$$

where $\mu = 1 + G_m/G > 1$ and $\Lambda(t) = G_m V_m(t)$. In the limit of zero admittance, $G_m \to 0$, we recover the unmodulated DSO.

The modulated DSO has three equilibria if the conditions $G_1 < \mu G < G_0$ and $|V_m(t)| < V_B(G_0 - \mu G)/G_m$ are met. The first condition is on the parameters of the circuit and the second one is on the amplitude of the modulation $V_m(t)$.

The modulation signal $\Lambda(t)$ makes the equilibrium points P, Q and N move along the line $i_L = -GV_1$ on the plane $V_2 = 0$,

$$N(t) = \left(\frac{\Lambda(t)}{\mu G - G_0}, 0, \frac{-G\Lambda(t)}{\mu G - G_0}\right),$$
$$P(t) = \left(-V_f' + \frac{\Lambda(t)}{\mu G - G_1}, 0, -G[-V_f' + \frac{\Lambda(t)}{\mu G - G_1}]\right),$$
$$Q(t) = \left(V_f' + \frac{\Lambda(t)}{\mu G - G_1}, 0, -G[V_f' + \frac{\Lambda(t)}{\mu G - G_1}]\right), \tag{11}$$

where $V_f' = V_f(G - G_1)/(\mu G - G_1)$ is the V_1 coordinate of the unmodulated, $\Lambda = 0$, equilibrium point Q.

5. The DSO parameters

By combining the DSO parameters we can construct the following three time constants,

$$\tau_L = \sqrt{LC_2},$$
$$\tau_0 = C_1/G_0,$$
$$\tau_1 = C_1/G_1. \tag{12}$$

These time constants have an interpretation in the case that $G = 0$, when the DSO decouples into two subsystems. In this limit, the last two equations of set (6) become the equations of a harmonic oscillator with period τ_L and the first equation has solution

$$V_1(t) = V_B \begin{cases} K \exp\left(\dfrac{t}{\tau_0}\right), & 0 \le t \le t_B \\ \left(\dfrac{G_0}{G_1}\right)\left[\exp\left(\dfrac{t - t_B}{\tau_1}\right) - 1\right] + 1, & t \le t_B \end{cases} \tag{13}$$

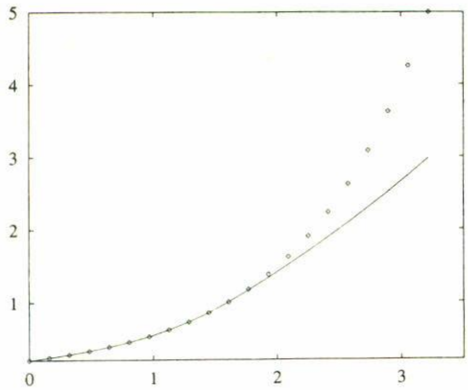

FIGURE 5. Time behavior of the voltage across a capacitor, $V_1(t)/V_B$, that is connected in parallel with the non linear element. The initial voltage is $V_1(0)/V_B = 0.2$ and the time to get the break point is $t_B/\tau_0 = 1.609$. For comparison, the exponential $0.2\exp(t/\tau_0)$ is plotted in points.

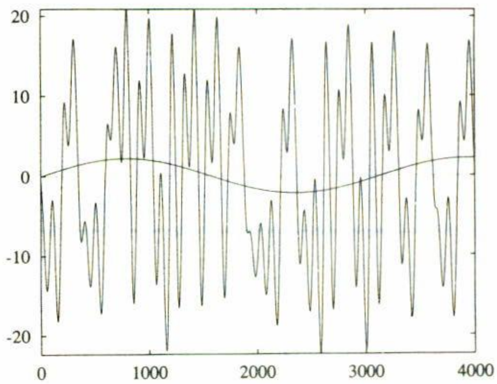

FIGURE 6. The carrier $x(t)$ for a modulation signal $V_m(t)/V_B = \sin(\omega t)$ of angular frequency $\omega = 1/(250\tau_0)$. The signals are evenly sampled at time intervals of $\tau_0/2$, 4,000 samples are shown. The parameter values are as follows, $c = 0.1$, $\gamma = 0.3125$, $g_1 = 0.376$, $g = 0.441$, and $\mu = 1.05$. The time constants are $\tau_1 = 2.66\tau_0$ and $\tau_L = 3.16\tau_0$.

that grows exponentially with two time constants, $\tau_0 < \tau_1$. The constant K in Eq. (13) is determined by the initial conditions, restricted to $|V_1(0)| < V_B$, and $t_B = -\tau_0 \log K$. Figure 5 shows in continuous line the solution for $V_1(0) = 0.2\, V_B$ ($G = 0$). For comparison, the exponential $V_1(0)\exp(t/\tau_0)$ is plotted in points. For $t > t_B$ the solution grows at a slower rate than the points.

For $|V_1(0)| > V_B$ the solution is

$$V_1(t) = V_B \left[K \exp\left(\frac{t}{\tau_1}\right) - \frac{G_0 - G_1}{G_1} \right]. \qquad (14)$$

When $G > 0$, one expects that the transit time for the orbit switching scrolling point is of the order of the time constant τ_0. Usually τ_0 is the shortest time and we take it as the time unit for the new variable $t' = t/\tau_0$. Further define the following dimensionless parameters,

$$g = \frac{G}{G_0}, \qquad g_1 = \frac{G_1}{G_0} = \frac{\tau_0}{\tau_1}, \qquad g_m = \frac{G_m}{G_0}, \qquad (15)$$

$$c = \frac{C_1}{C_2}, \qquad \gamma = \frac{C_1}{LG_0^2} = \frac{1}{c}\left(\frac{\tau_0}{\tau_L}\right)^2. \qquad (16)$$

Using these definitions the set of equations for the DSO, Eq. (6), take the following normal form.

$$\begin{aligned} x' &= g(y - \mu x) - \varphi(x) + \lambda(t) \\ y' &= gc(x - y) + cz \\ z' &= -\gamma y \end{aligned} \qquad (17)$$

The new dimensionless variables are $x = V_1/V_B$, $y = V_2/V_B$, $z = i_L/(G_0 V_B)$, and $\lambda(t) = (\mu - 1) g V_m(t)/V_B$. The I-V characteristics of the non linear element has the form

$$\varphi(x) = \begin{cases} -g_1 x - (1 - g_1), & x \geq 1 \\ -x, & |x| \leq 1 \\ -g_1 x + (1 - g_1), & x \leq -1 \end{cases} . \qquad (18)$$

The conditions to have double scrolling are $g_1 < \mu g < 1$ and $|\lambda(t)| < g(\mu - 1)(1 - \mu g)/g_m$. A typical modulated carrier is shown in Fig. 6.

6. Results and conclusions

Figure 7 shows the carrier $x(t)$ modulated with a simple sine function for $\lambda(t)$. Two estimations for λ extracted from the carrier using the filters Eqs. (3)–(5) after the synchronizing subsystem are shown in Fig. 7. The inputs to the filters Eqs. (3) and (4) are prepared with $f_0(x, y, z) = g(y - \mu x) - \varphi(x)$ and $f_1(x, y, z,) = 1$ for the DSO, Eq. (17).

The results obtained with a modulating square signal are shown in Fig. 8. The smoothening of the transitions in the demodulated signal are introduced by the filters. If the tuning parameters k and τ_f are made smaller the transitions in the demodulated signal are made sharper but the flat portions appear more contaminated with residues of the carrier. Figure 8 is a clear illustration of how the useful bandwidth of the modem is limited by the filtering of the chaotic carrier at the receiver.

The bandwidth limitation introduced by filtering would not be present if the parameter that is modulated is part of the synchronizing subsystem and the information signal is then estimated by error detection. This method has been tested for binary valued bit streams [4] but has not been tested for analog signals yet.

In conclusion, chaotic signals generated by parametric modulation of the DSO can be used as information carriers. The filters to recover the information signal are much simpler and stable than the approach that reconstructs the attractor [12]. A major problem in the reconstruction approach is the introduction of large errors in evaluating the time derivative of the carrier.

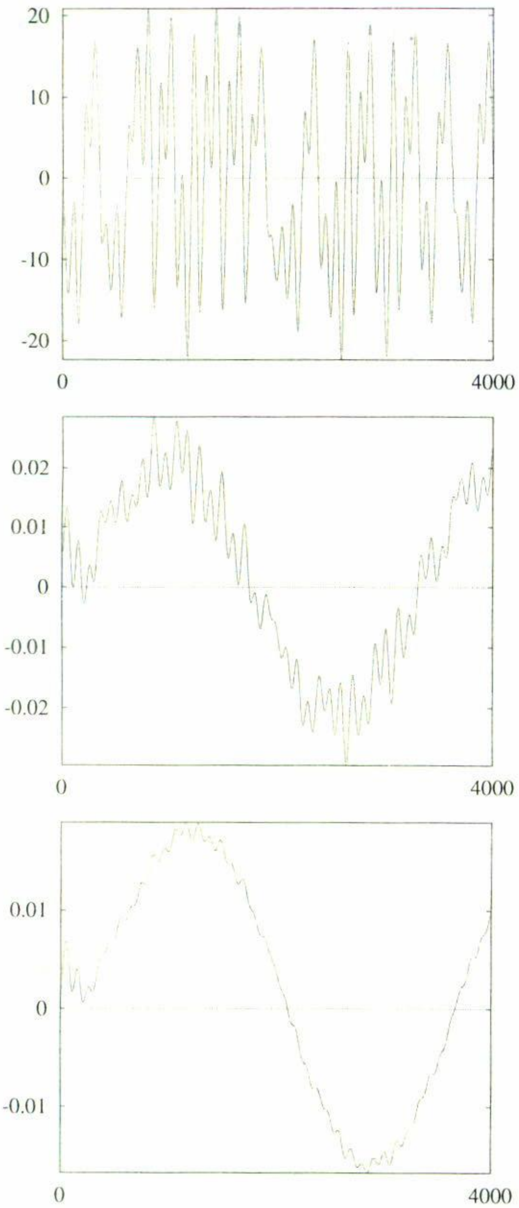

FIGURE 7. (a) The carrier $x(t)$ as in Fig. 6. The recovered signal with tuning parameters (b) $k = 50\ \tau_0$ and $\tau_f = 50\ \tau_0$ and (c) $k = 200\ \tau_0$ and $\tau_f = 100\ \tau_0$.

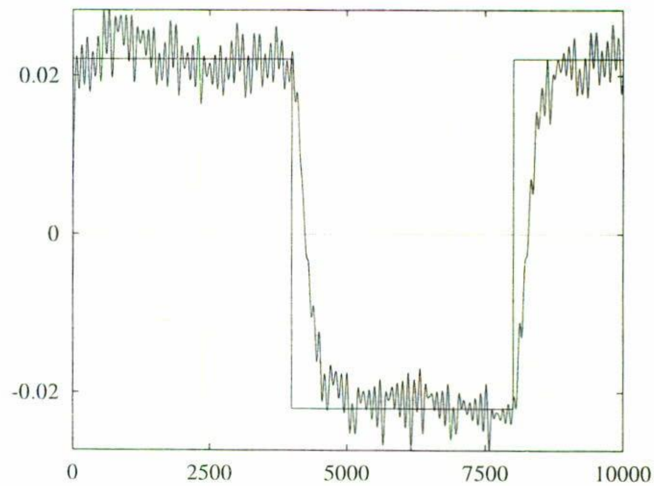

FIGURE 8. A modulating square signal and the demodulated signal recovered with tuning parameters $k = 50\ \tau_0$ and $\tau_f = 200\ \tau_0$. The sampling rate is $2/\tau_0$ samples/s and 10000 samples are shown.

Given the random aspect of the carrier, chaotic modems are being proposed to secure communications. However, the actual capabilities of chaotic signals as information carriers have not been evaluated. Parameters such as channel capacity and the minimum signal to noise ratio for a reliable connection through a noisy channel are not known and all the methods proposed so far to recover the information signal impose strong restrictions on the useful bandwidth.

A practical implementation of the symbolic modulation of the DSO proposed in Ref. 8 has been overlooked. The symbolic method promises to make a more efficient use of the information capacity of the DSO than the parametric modulation methods.

Acknowledgments

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