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Implications of the color spin-orbit interaction term on the confinement of excited heavy-light and light-light mesons

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The color spin-orbit (LS) contributions to the energy of the heavy-light $(Q, \bar{q})$ and light-light $(q, \bar{q})$ semi-classical systems confined by both an scalar potential and a time-like vector potential, are calculated. It is found that in the limit of a very high orbital angular momentum, they are very sensitive only to the presence of an scalar potential, and that, if this potential is very strong, they spoil the confinement regime. It is shown also that for a confinement potential dominantly time-like vector, the LS contributions are only neglibile relativistic corrections consistent with the quark confinement. Within this scheme a time-like vector confinement is prefered over one of scalar type.

Keywords: Heavy-light and light-light semi-classical systems; semiclassical hamiltonian

Son calculadas las contribuciones de espín-orbita (LS) de color a la energía de los sistemas quark pesado-quark ligero $(Q, \bar{q})$ y quark ligero-quark ligero $(q, \bar{q})$ confinados por potenciales vectoriales y escalares. Se encuentra que en el límite de un momento angular orbital alto, ellas son muy sensibles solo a la presencia de un potencial escalar, y que si este potencial es muy fuerte, ellas estropean el regimen de confinamiento tambien. Es demostrado que, para un potential de confinamiento dominantemente vector, las contribuciones LS son solo correcciones relativistas consistentes con el confinamiento de los quarks. Dentro de este esquema un potencial semi-clásico vector es preferido sobre uno de tipo escalar.

Descriptores: Sistemas excitados quark pesado-quark ligero y quark ligero-quark ligero; hamiltoniano semi-clásico

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1. Introduction

The study of quark-antiquark systems in high orbital angular momentum states (*i.e.* highly excited mesons) has been used in the past to explore the nature of confinement. Detailed knowledge of the characteristics of the confinement has many implications for a deeper understanding of the mesons properties, hence the importance of such a studies. The main approaches so far used for analyzing excited mesons have been mainly based on two different points of view: a semi-classical one [1] and a second which uses a Dirac-like equation [2, 3]. Although in principle, it seems more realistic to work within a Dirac-like approach for a very excited a quark-antiquark system, a semi-classical approach proves to be valid too.

It is currently believed that the Lorentz nature of the confinement potentials can be either linear scalar or vector linear, embedded in a Dirac equation. In order to investigate which of these two potentials is really the responsible of the confining, in Ref. 2 it was studied a $(Q, \bar{q})$ mesonic system in the limit of very high orbital angular momentum states. By assuming that the heavy quark-light quark system is described by a Dirac Hamiltonian and confined by either a scalar or vector potential, in Ref. 2 it was shown that, only the scalar potential has physically admisible solutions.

Through a different approach, consistent of a semi-classical Hamiltonian, it was pointed out in Ref. 1 that a tower degeneracy is allowed, in the limit of very high orbital angular momentum, only in presence of a scalar confinement potential. This fact helps to support the preference for a scalar confining potential over a vector one in the description of a quark-antiquark system.

However, it is important to emphasize at this point, that the spin-orbit contributions to the mesonic energies have not been calculated so far. These contributions are very important, mainly when the system is in the so-called Regge limit: a very large value, of the orbital angular momentum. Therefore, without taking them into account, any inference about confinement from the Regge limit, could not have validity at all. For that reason, the purpose of the present work is to calculate the LS contributions to the energies of both the $(Q, \bar{q})$ and $(q, \bar{q})$ mesonic systems, in the limit of a very high orbital angular momentum (L). Once these contributions are calculated, it is possible then to have a more complete model Hamiltonian to study the confinement and other related aspects, such as Regge trajectories. Calculations are carried out by using of the semi-classical formalism introduced in Ref. 3 in which the quark-antiquark system is confined by both a time-like vector potential and a scalar potential.

The way we proceed in this work is as follows, in Sect. 2, we calculate the LS contributions while in Sect. 3 we discuss the implications on confinement of such a contributions.

2. Hamiltonian with spin orbit term

The Lagrangian of a classical quark of mass m moving with a velocity $\mathbf{v}$ in presence of an effective scalar potential

$S(r) = \kappa_s r$, a time-like vector potential $V(r) = \kappa_v r$ and a color Coulomb potential $U(r) = -\xi/r$ is

$$L_0 = -[m + S(r)]\sqrt{1 - v^2} - V(r) - U(r), \qquad (1)$$

where $v^2 = \dot{r}^2 + \omega^2 r^2$. Since the nonperturbative linear potentials S and V, in the above equation, are the responsible of the confinement of the two quarks inside the meson, consequently $\kappa_s > 0$ and $\kappa_v > 0$.

Equation (1) describes a $(Q, \bar{q})$ system with the heavy quark at the origin $r = 0$. In the case of a two light quarks system, the center of mass is equidistant between the quarks which have a relative displacement r. The respective Lagrangian describing this $(q, \bar{q})$ system would be, $L_0 = -(2m + S)\sqrt{1 - v^2/4} - V(r) - U(r)$. While these changes do not modify substantially anything of the following discussion, in what follows we will continue working with the Lagrangian (1) and quoting whenever necessary, any difference with the $(q, \bar{q})$ systems.

The canonical momenta obtained from (1) are

$$p_r = \frac{\partial L_0}{\partial \dot{r}} = [m + S(r)]\gamma\dot{r}, \qquad (2)$$

$$J = \frac{\partial L_0}{\partial \omega} = [m + S(r)]\gamma\omega r, \qquad (3)$$

where $\gamma = (1 - v^2)^{-1/2}$.

The respective Hamiltonian derived from (1)–(3) is

$$\begin{aligned} H_0 &= \dot{r}p_r + \omega J - L_0 \\ &= [m + S(r)]\gamma + V(r) + U(r). \end{aligned} \qquad (4)$$

Through the relation

$$\begin{aligned} \mathbf{p}^2 &= p_r^2 + \mathbf{J}^2/r^2 \\ &= [m + S(r)]^2\gamma^2 v^2 \\ &= [m + S(r)]^2(\gamma^2 - 1), \end{aligned} \qquad (5)$$

the velocities can be eliminated in (4) to obtain

$$H_0 = \sqrt{[m + S(r)]^2 + p_r^2 + J^2/r^2} + V(r) + U(r). \qquad (6)$$

In the case of a two light quark system, the corresponding Hamiltonian is: $H_0 = \sqrt{[(2m)^2 + S(r)]^2 + (2\mathbf{p})^2} + V(r) + U(r)$. Let us observe that this hamiltonian can be obtained from (6), simply by making the replacement $m \to 2m$ and $p \to 2p$.

While studying the Hamiltonian (6), in Ref. 3 it was found that, only the scalar confinement leads to meson towers, where mesons of different angular momentum lie on top of each other.

In order to have a further understanding of the Hamiltonian (6) in the limit $L \gg 1$, it is more convenient to make a first order expansion in powers of $p_r^2/[(m + S)^2 + J^2/r^2]$ in such an equation. After doing this, it is obtained the familiar

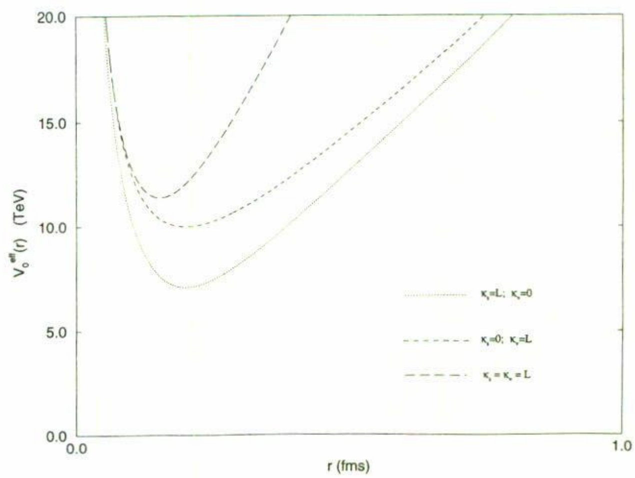


FIGURE 1. Potential $V_0^{\text{eff}}(r)$ plotted when $L = 50$ for three different cases: *i)* $S = 0$, *ii)* $V = 0$, and *iii)* $S = V$. The coupling constants κ_s and κ_v are in units of GeV^2. The value of the strength of the color Coulomb potential is $\xi = 0.445$.

expression

$$H_0 = \frac{p_r^2}{2M_0} + V_0^{\text{eff}}(r), \qquad (7)$$

where

$$V_0^{\text{eff}}(r) \equiv M_0 + V(r) + U(r), \qquad (8)$$

and

$$M_0 \equiv \sqrt{[m + S(r)]^2 + J^2/r^2}. \qquad (9)$$

A primary condition for the confinement potentials, V and S, is that they must be very strong in order to compete with the intense centrifugal forces. This requirement means that their strengths must be of the order of the angular momentum L, *i.e.* κ_s, $\kappa_v \geq L$.

In Fig. 1 the potential (8) is plotted for the three different cases: *i)* $S = 0$, *ii)* $V = 0$, and *iii)* $S = V$. From that figure, it is easily seen that the classical turning points of the light quark are finite. In other words, to first order of approximation, the semiclassical potential $V_0^{\text{eff}}(r)$ allows for perfectly consistent bound states, indepently of which of the two potentials, S or V is the dominant.

Let us investigate now the consequences of the spin-orbit interaction on the confinement of a quark-antiquark system. In order to calculate the spin-orbit term, we assume that the quark has a color magnetic moment

$$\mu = \frac{e_q g}{2m}\mathbf{S}, \qquad (10)$$

where e_q is the light quark color charge, g a color gyromagnetic factor, and $\mathbf{S}$ is the quark spin. This magnetic moment, together with the Coulomb color interaction with the static source at the origin, induces a Thomas precession effect. The corresponding potential associated to this interaction is [4][a]

$$U' = -\mu \cdot H' + \mathbf{S} \cdot \omega_{\text{T}}, \qquad (11)$$

where $\mathbf{H}'$ is the color magnetic induction measured from the quark rest frame and ω_T is the Thomas angular velocity of rotation. These quantities are defined as follows

$$\mathbf{H}' = \gamma\left(\mathbf{H} - \mathbf{v}\times\mathbf{E}\right) - \frac{\gamma^2}{\gamma+1}\mathbf{v}(\mathbf{v}\cdot\mathbf{H}), \tag{12}$$

and

$$\omega_\mathrm{T} = \frac{\gamma^2}{\gamma+1}\mathbf{a}\times\mathbf{v}, \tag{13}$$

where

$$\mathbf{a} = \frac{e_q}{m}\sqrt{1-v^2}\left[\mathbf{E} + \mathbf{v}\times\mathbf{H} - (\mathbf{v}\cdot\mathbf{E})\mathbf{v}\right]. \tag{14}$$

The color electromagnetic fields $\mathbf{E}$ and $\mathbf{H}$ in Eqs. (12)–(14) are measured from the origin.

Since we are not considering any external magnetic field (*i.e.* $\mathbf{H}=\mathbf{0}$) and due that $e_q\mathbf{E} = -\hat{\mathbf{r}}dU(r)/dr$, it is straightforward to see that

$$\mu\cdot\mathbf{H}' = -\frac{g}{2m^2}\mathbf{S}\cdot\mathbf{L}\frac{1}{r}\frac{dU(r)}{dr}, \tag{15}$$

and

$$\mathbf{S}\cdot\omega_\mathrm{T} = -\frac{1}{m^2}\frac{1}{1+\gamma}\mathbf{S}\cdot\mathbf{L}\frac{1}{r}\frac{dU(r)}{dr}. \tag{16}$$

By substituting (15) and (16) in (11) it is obtained the LS interaction potential

$$U' = \frac{1}{m^2}\left(\frac{g}{2} - \frac{1}{\gamma+1}\right)\mathbf{S}\cdot\mathbf{L}\frac{1}{r}\frac{dU(r)}{dr}. \tag{17}$$

We must observe two things at this stage. The first one is that that the spin-orbit interaction potential (17) is valid for both the $(Q,\bar{q})$ and the $(q,\bar{q})$ systems. The other is that Eq. (17), is a complete relativistic expression. In fact, it is straightforward to check that if one turns off the scalar potential (*i.e.* $S=0$) and takes the nonrelativistic limit in (14), the familiar first order expression of ordinary quantum mechanics is obtained [5].

From (6) and (17), we find the complete hamiltonian including the spin-orbit term $H = H_0 + U'$,

$$H = \sqrt{[m+S(r)]^2 + p_r^2 + \frac{J^2}{r^2}} + V(r) + U(r) + \frac{1}{m^2}\left[\frac{g}{2} - \frac{m+S(r)}{\sqrt{[m+S(r)]^2 + p_r^2 + \frac{J^2}{r^2}} + m + S(r)}\right]\left[\frac{1}{r}\frac{dU(r)}{dr}\right]\mathbf{S}\cdot\mathbf{L}. \tag{18}$$

Since the total angular momentum can take only the values $J = L+1/2$ and $J = L-1/2$ then

$$\mathbf{S}\cdot\mathbf{L} = (\mathbf{J}^2 - \mathbf{L}^2 - \mathbf{S}^2)/2 = \begin{cases} L/2 & J = L+1/2 \\ -(L+1)/2 & J = L-1/2 \end{cases} \tag{19}$$

From Eqs. (18)–(19), it can be seen that the spin-orbit contribution is no longer a correction term to the energy when the meson system is in high angular momentum states. Also, it is observed that the LS term is highly sensitive only to the presence of an scalar confining potential.

Now, we calculate the Regge limit (*i.e.* $L \gg 1$) of the Hamiltonian (18). In that limit it can be written as

$$H = \sqrt{[m+S(r)]^2 + p_r^2 + \frac{L^2}{r^2}} + V(r) + U(r) \pm \frac{\xi}{2m^2r^3}\left[\frac{g}{2} - \frac{m+S(r)}{\sqrt{[m+S(r)]^2 + p_r^2 + \frac{L^2}{r^2}} + m + S(r)}\right]L, \tag{20}$$

where we have made $\mathbf{S}\cdot\mathbf{L} \simeq \pm L/2$ and

$$J = \sqrt{\left(L \pm \frac{1}{2}\right)\left(L + 1 \pm \frac{1}{2}\right)} \simeq L.$$

In what follows, we shall assume that in the limit $L \gg 1$, the LS coupling term, can be treated semi-classically. This means that the quantum mechanic operators can be thought as semi-classical canonical variables.

Let us consider the following Hamilton equations[b]:

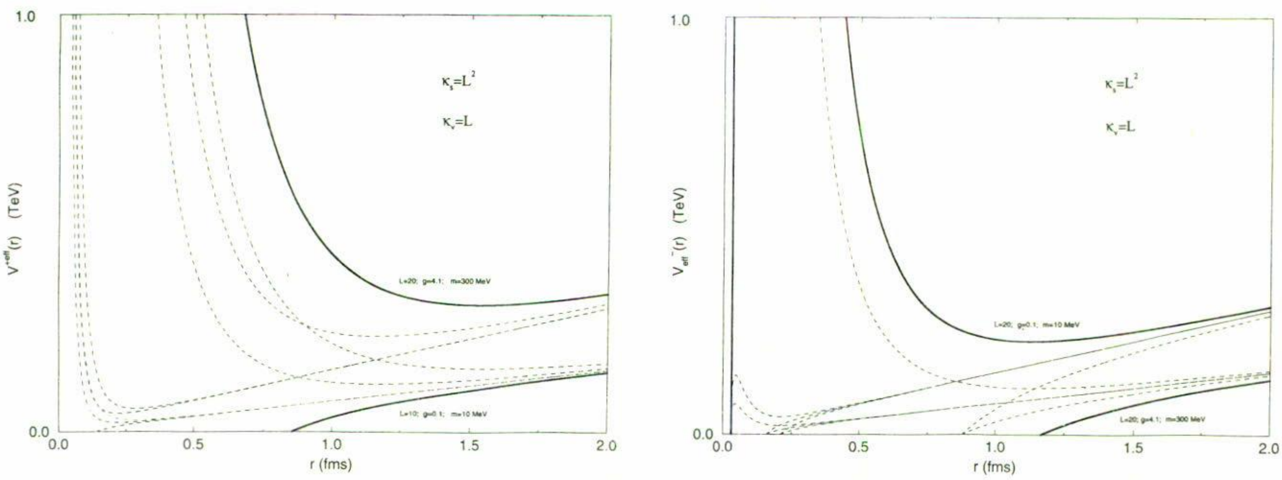
$$\dot{r} = \frac{\partial H}{\partial p_r} = \left[1 \mp \frac{\xi}{2(mr)^2}\frac{m+S}{\left(\sqrt{\phi}+m+S\right)^2}\right]\frac{p_r}{\phi}, \tag{21}$$

$$\dot{\theta} = \frac{\partial H}{\partial L} = \left[1 \mp \frac{\xi}{2(mr)^2}\frac{m+S}{\left(\sqrt{\phi}+m+S\right)^2}\right]\frac{1}{\phi}\frac{L}{r^2}, \tag{22}$$

where $\phi = (m+S)^2 + p_r^2 + (L^2/r^2)$. We shall also assume that the angular velocity,

$$\dot{\theta} = \omega, \tag{23}$$

has a nonzero value, which is a very reasonable assumption in the limit where the orbital angular momentum is very large.



(a) (b)

FIGURE 2. Potentials $V^{\pm}_{\text{eff}}(r)$ plotted in the case $\kappa_s \simeq L^2$ GeV^2 and $\kappa_v \simeq L$ GeV^2, for several values of the orbital angular momentum, color gyromagnetic ratio, and quark mass running in the ranges $10 < L < 20$, $0.1 < g < 4.1$, and $10 < m < 300$ MeV, respectively. The value of the strength of the color Coulomb potential is $\xi = 0.445$. In (a), the $V^{+}_{\text{eff}}(r)$ potentials associated to the states ($L = 10, g = 4.1, m = 10$ MeV and ($L = 10, g = 0.1, m = 300$ MeV) bound from top and bottom respectively, to the rest of the potentials associated to other states. In (b) all of the $V^{-}_{\text{eff}}(r)$ potentials lie between the curves associated to the states ($L = 20, g = 0.1, m = 10$ MeV) and ($L = 20, g = 4.1, m = 10$ MeV).

The quotient of (21) and (22) gives the radial moment as a function of the radial velocity $\dot{r}$, and the tangential velocity $r\omega$

$$p_r = \frac{L}{r}\frac{\dot{r}}{r\omega}. \tag{24}$$

It is not difficult to verify that, through a first order expansion in powers of $p_r^2/[(m+S)^2 + (L/r)^2]$, Eq. (20) can be written as

$$H = \frac{p_r^2}{2M} + V^{\pm}_{\text{eff}}(r), \tag{25}$$

where

$$V^{\pm}_{\text{eff}}(r) \equiv \sqrt{(m+S)^2 + (L/r)^2} + V + U$$
$$\pm \frac{\xi L}{2m^2 r^3}\left[\frac{g}{2} - \frac{m+S}{\sqrt{(m+S)^2 + (L/r)^2} + m + S}\right], \tag{26}$$

$$M = \frac{\sqrt{(m+S)^2 + (L/r)^2}}{1 \pm \frac{\xi L}{2m^2 r^3}\frac{m+S}{\left[\sqrt{(m+S)^2+(L/r)^2}+m+S\right]^2}}. \tag{27}$$

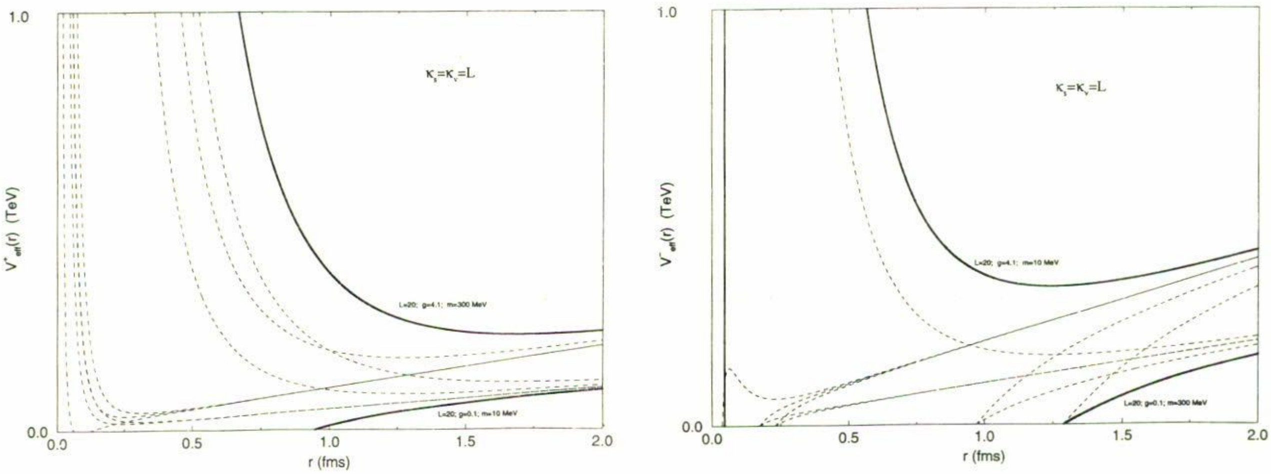
The form of Eq. (25), reminds us the familiar radial equation of a particle of effective mass M, moving in a field of forces $V^{\pm}_{\text{eff}}(r)$.

As it can be appreciated from Eqs. (25)–(27), the LS term is very sensitive only to the presence of a scalar confinement potential. As we shall see below, this single dependence of the LS term, restricts in a considerable way the strength of $S(r)$.

In Figs. 2–4 are plotted the potentials $V^{\pm}_{\text{eff}}(r)$ for several relevant values of κ_s and κ_v. In these figures the quark mass, the orbital angular momentum and the color gyromagnetic ratio are allowed to run in the ranges $10 \leq m \leq 300$ MeV, $10 \leq L \leq 20$, and $0.1 \leq g \leq 4.1$, respectively. In these figures, the strength of color Coulomb-like potential remains fixed in a value $\xi = 0.445$. Thus, in Figs. 2a and 2b the potentials $V^{+}_{\text{eff}}(r)$ and $V^{-}_{\text{eff}}(r)$ are plotted respectively for the case of an scalar potential stronger than the vector potential. With the purpose of taking representative values only, we have considered the case where κ_s and κ_s take the values $\kappa_s \simeq L^2$ GeV^2 and $\kappa_v \simeq L$ GeV^2. In Figs. 3a and 3b we have plotted the same potentials but now for the case in which the potentials $S(r)$ and $V(r)$ compite in strength: $\kappa_s \simeq \kappa_v = L$ GeV^2. Finally in Figs. 4a and 4b, we have plotted the potentials $V^{\pm}_{\text{eff}}(r)$ for the case $\kappa_s \simeq L$ GeV^2 and $\kappa_v \simeq L^2$ GeV^2, *e.g.* when $S(r)$ is weaker than $V(r)$.

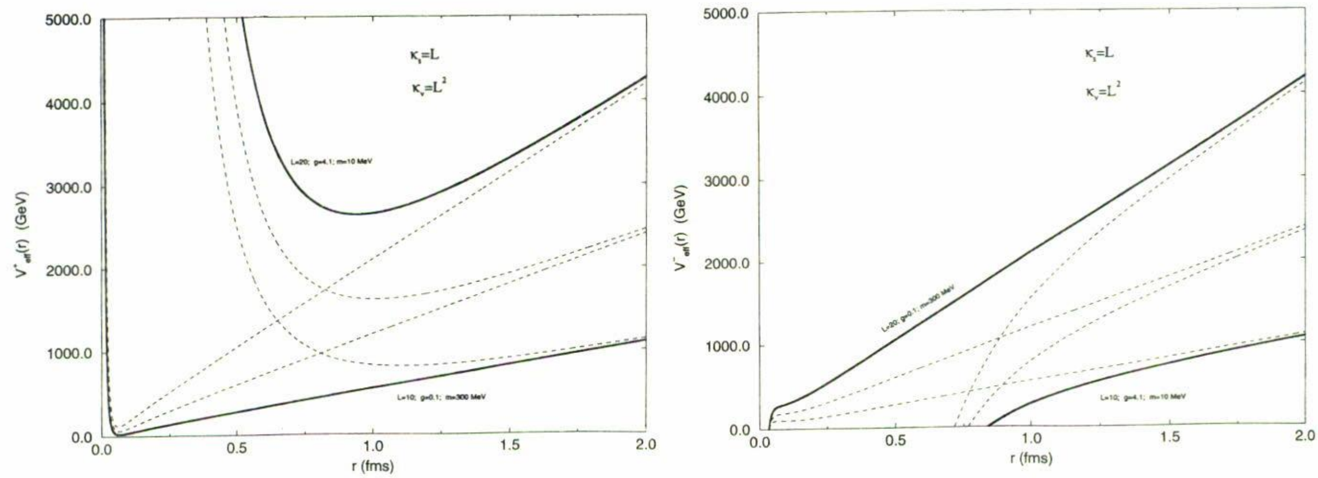
Figures 2 and 3 show that, while more dominant is the scalar potential, the more difficult is to generate a two quarks bound state. In other words, if we consider for instance, mesonic energies of order $E \sim 1$ TeV from Figs. 1a, 1b, 2a, and 2b it is observed that the classical turning points of the quark would be at infinity, for any value of g and m in the ranges stated above[c]. This behavior means that if the meson is confined by a dominantly scalar potential, the LS interaction would break the meson. This unphysical behavior discards the scalar potential as a good confinement potential.

In Figs. 4a and 4b, we have plotted the potentials $V^{\pm}_{\text{eff}}(r)$ in the situation where now, the vector potential is the dominant one, *i.e.* $\kappa_v > \kappa_s$. With the purpose of illustrating the corresponding behavior, we have taking the representative values: $\kappa_v = L^2$ GeV^2 and $\kappa_s = L$ GeV^2. From these figures, it is evident that, when the confinement potential is



(a) (b)

FIGURE 3. Potentials $V^{\pm}_{\rm eff}(r)$ plotted in the case $\kappa_s \simeq \kappa_v \simeq L\,{\rm GeV}^2$, for several values of the orbital angular momentum, color gyromagnetic ratio, and quark mass running in the ranges $10 < L < 20$, $0.1 < g < 4.1$, and $10 < m < 300$ MeV, respectively. The value of the strength of the color Coulomb potential is $\xi = 0.445$. In (a), the $V^{+}_{\rm eff}(r)$ potentials associated to the states ($L = 20, g = 4.1, m = 300$ MeV) and ($L = 20$, $g = 0.1$, $m = 300$ MeV) bound from top and bottom respectively, to the rest of the potentials associated to other states. In (b) all of the $V^{-}_{\rm eff}(r)$ potentials lie between the curves associated to the states ($L = 20$, $g = 4.1$, $m = 10$ MeV) and ($L = 20$, $g = 0.1$, $m = 10$ MeV).



(a) (b)

FIGURE 4. Potentials $V^{\pm}_{\rm eff}(r)$ plotted in the case $\kappa_s \simeq L$ GeV^2 and $\kappa_v \simeq L^2$ GeV^2, for several values of the orbital angular momentum, color gyromagnetic ratio, and quark mass running in the ranges $10 < L < 20$, $0.1 < g < 4.1$, and $10 < m < 300$ MeV, respectively. The value of the strength of the color Coulomb potential is $\xi = 0.445$. In (a), the $V^{+}_{\rm eff}(r)$ potentials associated to the states ($L = 20$, $g = 4.1$, $m = 10$ MeV) and ($L = 10$, $g = 0.1$, $m = 300$ MeV) bound from top and bottom respectively, to the rest of the potentials associated to other states. In (b) all of the $V^{-}_{\rm eff}(r)$ potentials lie between the curves associated to the states ($L = 20$, $g = 0.1$, $m = 300$ MeV) and ($L = 10$, $g = 4.1$, $m = 10$ MeV).

dominantly vector, the classical turning points for a given positive quark energy, are finite. This result implies that the LS interaction does not fragment the meson, and still it allows for physically admisible bound states. This behavior of the potentials $V^{\pm}_{\rm eff}(r)$ assures that the vector potential resist the most important test, the one of confinement.

3. Conclusions

From Eq. (18) it is possible to conclude that, within a scheme of a very excited quark-antiquark system described by the Lagrangian (1), the color LS contributions are only very sensitive to the presence of a scalar confining potential.

As can be seen from Figs. 2 and 3, the dependence of the LS term on $S(r)$ makes that, with a dominant confinement potential of scalar nature, the LS contributions would lead to the unphysical fragmentation the meson into free quarks. The confinement regime might be restored if the color gyromagnetic ratio in (26) acquires an unusual value, $g \sim S/(S + \sqrt{S^2 + (L/r)^2})$, to cancel out the term proportional to ξ in such an equation. Since this possibility is very remote, we can conclude that within the semiclassical approach of Eq. (1), the scalar potential is discarded as a good confinement potential.

On the other hand, the vector potential $V(r)$ is less restricted than the scalar potential $S(r)$. The only restriction it has, is to be not weak. This restriction is very reasonable and it is just the necessary for avoiding the quark deconfinement, due to the intense interquarks centrifugal forces. As it can be appreciated from Fig. 4, with a time-like vector confinement potential and large values of L, the LS contribution do not put at risk the confinement regime.

To conclude, let us observe from Eq. (27), that the confinement of a light quark in a region smaller that its Compton wavelenght, $\Lambda_C = 1/m$ is only an apparent paradox. This fact is due that, as a consequence of the confinement interaction, the light quark acquires an effective mass greater than its current mass. Therefore, we can conclude that within this semi-classical approach a time-like vector confinement is prefered over one of scalar type.

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[a] In the case of a $(q, \bar{q})$ system there exist an interaction term of this kind for each light quark degree of freedom.

[b] The other two Hamilton equations $\dot{p}_r = -(\partial H/\partial r)$ and $\dot{J} = -(\partial H/\partial \omega)$ are not necessary for the present discussion. However, it should be observed from (17) that consistently it is obtained, $\dot{J} = 0$.

[c] These energies could be available in a near future with the advent of the LHC.

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