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Patterns of synthetic seismicity and recurrence times in a spring-block earthquake model

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In this paper, we investigated the spring-block model proposed by Olami, Feder and Christensen (OFC) [15], to find properties of real seismicity, following the same procedure that Brown, Scholz and Rundle (BSR) [19]. We find that the results of the OFC model can reproduce the real seismicity staircase graphics and produce seismicity patterns similar to real patterns. We discuss the advantages of synthetic seismicity catalogues and about the possibility of recognizing seismic precursory quiescence in these catalogues. We find that the distribution of large earthquake recurrence times in the OFC model is log-normal for the elastic parameters values used in that model.

Keywords: Spring-block model; self-organized criticality; seismicity patterns; precursory seismic quiescence and synthetic seismicity

En este artículo se investiga el modelo propuesto por Olami, Feder y Christensen (OFC) [15], para encontrar propiedades de la sismicidad real procediendo en la misma forma que Brown, Scholz y Rundle [19]. Se encuentra que los resultados del modelo reproducen las gráficas en escalera de la sismicidad real y producen patrones de sismicidad análogos a los reales. Se discuten las ventajas de los catálogos de sismicidad sintética y de la posibilidad de reconocer en ellos quietudes precursoras de sismos. Se encuentra que la distribución de tiempos de recurrencia para sismos grandes en el modelo OFC es log-normal para los valores de los parámetros elásticos usados en ese modelo.

Descriptores: Modelo resorte bloque; criticalidad auto organizada; patrones de sismicidad; quietud sísmica precursora y sismicidad sintética

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1. Introduction

Recently much attention has been devoted to the phenomenon known as self-organized criticality (SOC). In 1987, Bak, Tang and Weisenfeld, (BTW) [1], introduced a cellular automaton model [2] called the sand pile, as a paradigm for the explanation of two phenomena that occur very frequently in nature, fractal structures [3] and the temporary effect known as $1/f$ noise [4].

Bak, Tang and Weisenfeld proposed that spatially extended open dynamical systems evolve naturally, by themselves, to a critical state, without intrinsic time or length scales. In such a state, a smaller event often begins a chain reaction that can lead to a catastrophe. Although small events are produced much more than catastrophes, the mechanism that leads to small events is the same that leads to those of greater size. More yet, these systems never reach the equilibrium but evolve from one metastable state to the following [5].

In 1989, Bak and Tang [6] asserted that earthquakes can be the most direct example of self-organized criticality in nature. Ito and Matzuzaki [7] proposed a cellular automaton model similar to the sand pile, but adapted to the study of earthquakes occurrence. In the same year, Sornette and Sornette [8], suggested that if earthquakes are a natural consequence of the stationary dynamical state of the crust subjected to growing tensions, these also organize the crust in a self-consistent way. When the continental plate is distorted so strong that the deformation exceeds a certain limit, (that can change depending of the place), then the rupture occurs and an earthquake (avalanche in terms of SOC) follows. They asserted that not only the earthquake features are a consequence of the organization of the crust and its fault arrangement, but also the long scale structure of the crust is a result of the total history of previous earthquakes, which have organized the crust. Earthquakes can be seen as if they would be part of the long scale total organization of the crust, which evolves spontaneously toward a SOC state.

Using basically SOC concepts other models were elaborated for earthquake simulation, as those of Carlson and Langer [9, 10] and Nakanishi [11, 12]. Most of these simulations were limited until 1991 to conservative models, but in real earthquakes there are losses of energy by friction and not all the released energy contributes to the slipping. It was believed that an indispensable condition for systems to be SOC were that they will be totally conservative [13]. However, Feder and Feder showed that a special nonconservative model with a global perturbation exhibits SOC [14].

In 1992, Olami, Feder and Christensen [15, 16] introduced a continuous cellular automaton model where the conservation level can be controlled. The proposed model is a version in two dimensions of the spring-block model of Burridge and Knopoff (BK) for earthquakes [17]. They obtained a very robust SOC behavior and found results that permitted them to find the Gutenberg-Richter law exponents [18]. In 1991 a more simplified model was proposed by Brown, Scholz and Rundle (BSR) [19] by which they obtained some properties apparently related to real seismicity. The OFC model is more complete than the BSR model, in the sense that the BSR model has certain nonrealistic assumptions such as the fact that the model is a conservative one, and it works with only integer variables. We analyzed the results of the OFC model to see if it is possible to obtain those properties related to real seismicity that were obtained in the BSR model; additionally we looked for some further seismic properties of the model. We find that the model qualitatively reproduces the staircase graphics of real seismicity, we analyzed such graphics and also we obtained some other results, for example that recurrence times distribution for large earthquakes is log-normal, such as it was found for real seismicity by Nishenko and Buland [20] and for synthetic seismicity by Brown, Scholz and Rundle. Our best results were found for elastic parameter values around 0.2 such as Olami, Feder and Christensen had found. In summary, our main contribution is the study of further and new seismic properties of the OFC model.

This paper is organized as follows: In Sect. 2 we briefly describe the OFC model with its principal results; in Sect. 3 we discuss about the staircase graphics for the cumulative seismicity with real data; in Sect. 4 we present the results of synthetic seismicity obtained with the OFC model. We make a discussion about the synthetic seismicity catalogues in Sect. 5 and we describe the results obtained for recurrence times in Sect. 6.

2. The model of Olami, Feder and Christensen

In the OFC model the fault is represented by a two-dimensional array of blocks interconnected by springs. Each block is connected to its four nearest neighbors and also to a driving rigid plate by another set of springs, and by the friction force to a fixed rigid plate. The blocks are let to move by the relative movement of the two rigid plates (see Fig. 1). When the force over one of the blocks is larger than some threshold value $F_{\rm th}$ (the maximal static friction), the block slips. Olami *et al.* assumed that the block that is moved will slip to the zero force position. The block slipping will redefine the forces in its nearest neighbors. This can result in more slippings and a chain reaction can evolve.

If we define an $L \times L$ arrangement of blocks by (i, j), where $i$and j are integers whose values are between 1 and L and if the displacement of each block from its relaxed position on the lattice is $x_{i,j}$, then the total force exerted by the springs on a given block (i, j) is expressed by [16]

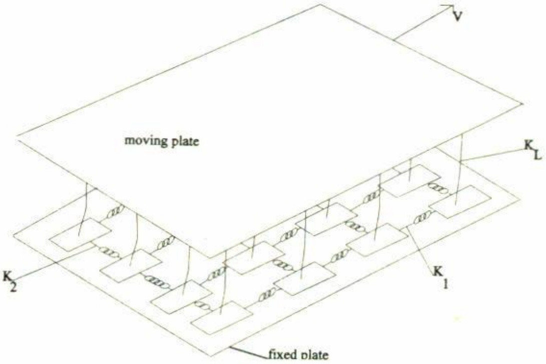

FIGURE 1. The geometry of the spring-block model. The force on the blocks increases uniformly as a response to the relative movement of the two plates.

$$F_{i,j} = K_1\left[2X_{i,j} - X_{i-1,j} - X_{i+1,j}\right] + K_2\left[2X_{i,j} - X_{i,j-1} - X_{i,j+1}\right] + K_L X_{i,j}, \quad (1)$$

where K_1, K_2 and K_L are the elastic constants. The force redistribution in the position (i, j) is given by the following relationship,

$$\begin{aligned} F_{i\pm1,j} &\to F_{i\pm1,j} + \delta F_{i\pm1,j};\\ F_{i,j\pm1} &\to F_{i,j\pm1} + \delta F_{i,j\pm1};\\ F_{i,j} &\to 0, \end{aligned} \quad (2)$$

where the force increase in the nearest neighbors is given by,

$$\begin{aligned} \delta F_{i\pm1,j} &= \frac{K_1}{2K_1 + 2K_2 + K_L} F_{i,j} = \alpha_1 F_{i,j};\\ \delta F_{i,j\pm1} &= \frac{K_2}{2K_1 + 2K_2 + K_L} F_{i,j} = \alpha_2 F_{i,j}. \end{aligned} \quad (3)$$

where α_1 and α_2, are the elastic ratios. Observe that the force redistribution is not conservative.

Olami *et al.* first restricted the simulation to the isotropic case, $K_1 = K_2$ $(\alpha_1 = \alpha_2 = \alpha)$ with a rigid frontier condition, implying that $F = 0$ in it. They made the mapping of the spring-block model into a continuous, nonconservative cellular automaton modeling earthquakes which is described by the following algorithm:

(1) Initialize all the sites to a random value between 0 and $F_{\rm th}$.

(2) Locate the block with the largest force, $Fmax$. Add $F_{\rm th} - F_{\rm max}$ to all sites (global perturbation).

(3) For all $F_{i,j}$ ($F_{\rm th}$ redistribute the force in the neighbors of $F_{i,j}$ according to the rule

$$F_{n,n} \to F_{n,n} + \alpha F_{i,j}; F_{i,j} \to 0, \quad (4)$$

where $F_{n,n}$ are the forces for the four nearest neighbors. An earthquake is in process.

(4) Repeat step 3 until the earthquake has totally evolved.

(5) Once the earthquake has thoroughly ended, return to step 2.

For these conditions, Olami *et al.* obtained a robust SOC behavior for the probability distribution of the earthquakes size. However, we believe that the most important result of Olami *et al.* is that it is possible to calculate the exponents of the Gutenberg-Richter law [18]. The Gutenberg-Richter law establishes that the earthquake occurrence frequency is related to the magnitude m by means of the relationship

$$\log_{10} N(M > m) = a - bm, \tag{5}$$

where a and b are constants and $N(m)$ is the number of earthquakes larger than m in a specific time interval. Although the relationship (5) is universal, the values of a and b depend on each region. The constant a specifies a regional level of seismicity. The values of b are approximately between 0.75 and 1.54. Very large earthquakes ($m > 7$) have different values of b ($1.2 < b < 1.54$) that the smaller ($0.75 < b < 1.2$). This difference can be the result of the large earthquakes exclusion to the ductile region of the lithosphere [21]. The energy (seismic moment) E released during the earthquake is believed to increase exponentially with the earthquake magnitude [22],

$$\log_{10} E = c + dm, \tag{6}$$

where the parameter d is 1 and $3/2$ for small and large earthquakes respectively. If we substitute m of the Eq. (6) into the Eq. (5) the Gutenberg-Ritcher law becomes a power law for the number of earthquakes with energy greater than E,

$$N(E_0 > E) \approx E^{-b/d} = E^{-B}. \tag{7}$$

If we make the calculation of B from minimal and maximum values of b and d we find that approximately B is in the same range for small and large earthquakes, namely, 0.75–1.06.

Olami *et al.* calculated B and concluded that the most approximate values to real seismicity are produced for α values around 0.2. This is reasonable, because if we assume that all the elastic constants are in the same scale ($K_1 \approx K_2 \approx K_L$) then $\alpha \approx 0.20$ [see Eq. (3)]. In fact, Olami *et al.* showed that anisotropy does not seem to be a determinant factor in the modeling of the fault, since similar behavior to the isotropic case is obtained. For this reason, we believe that a most complex model of spring-block simulating a seismic fault must consider values of the elastic parameter α around 0.2.

A seismic fault model must be able to produce power laws of the type of the Gutenberg-Richter law. However, the ability to produce a power law does not mean that the model will be useful, because it also must be able to reproduce other phenomena and lead to predictions that the seismologists could observe in real faults [23]. Analyzing successful models, we can arrive to a better theoretical understanding about the earthquakes. Because of this, we look for new insights in the analysis of the OFC model results.

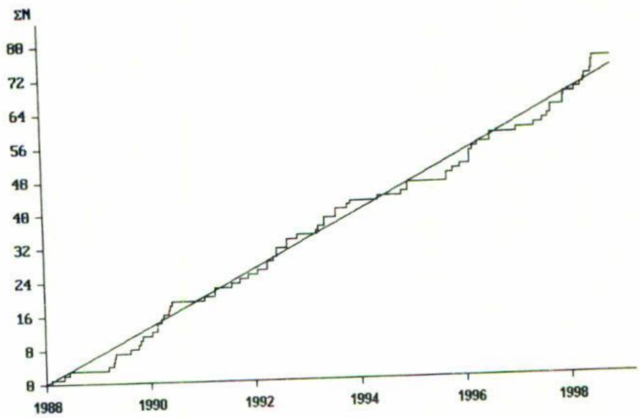

FIGURE 2. Cumulative number of earthquakes for a region of the Mexican Pacific coast, with $5 \leq M_c \leq 8.2$, between 15 and 19° lat. N and 98 and 102° long. W, from. January 1, 1988 to November 19, 1999. M_c is the code magnitude.

3. The staircase graphics of the cumulative seismicity

If we plot real data of the cumulative seismicity *versus* time, we obtain staircase graphics. Habermann [24] and MacNally [25] have reported many of these graphics for the seismiciy in many places of the world. In particular, for Mexico. NacNally accomplished such graphics for earthquakes with magnitude greater than 4.0; on those we can observe precursory seismic quiescence [25]. As an example, we show in Fig. 2 the real cumulative seismicity for a region in the Mexican Pacific coast [26].

In synthetic seismicity, these staircase graphics were first obtained as a result of the BSR model [19]. This model is also a cellular automaton based on the BK spring-block model and exhibits SOC behavior. However, when the automaton was implemented computationally, the positions, forces, displacements and time were taken with integer values and all the calculations were done in integer arithmetic. In the spring-block model one of the dimensions of the array models the fault length and the other the depth, because a fault has a length much greater than the depth. Brown *et al.* supposed that $L_x \gg L_y$.

To change the state of a block they supposed that when a block slides, jumps to its minimal potential energy position plus a random increase. They took $K_L \ll K_C$ so they ignored the elongation in the above spring and determined the position of minimal potential energy only with the position of the four nearest neighbors. The mentioned increase was chosen as a random number uniformly distributed with a minimal value of 1 in the displacement, and a maximum value equal to the quantity that the block is outside of alignment with respect to its minimal potential energy position before the sliding.

In spite of the limitations of this model, Brown *et al.* obtained a decreasing power law akin to the Gutenberg-Ritcher law, with exponent approximately equal to 1.5. They also obtained a large number of events that involve the complete lattice and as an interesting observation they found that large events are commonly preceded by periods of quiescence. Obviously, the OFC model has several advantages on the BSR model, however, Brown *et al.* deepened more and obtained other properties of the model that seem to be related with real seismicity. This work has motivated other authors to propose more realistic models [27]. In particular, it motivated us to ask ourselves: Does the model OFC produce staircase graphics for the cumulative seismicity?

4. The cumulative seismicity in the OFC model

We calculated the cumulative seismicity in the OFC model, then we plotted it as a function of time. The results show that staircase graphics are a characteristic of the model, for all values of elastic parameters α between 0.10 to 0.25 and for system sizes from 15×15 to 200×200 we always obtained such graphics.

The graphics of cumulative seismicity of the OFC model have the aspect shown in Fig. 3, similar staircases to those of real seismicity are obtained taking only events above certain magnitude. In Fig. 3, graph (a) corresponds to the total cumulative seismicity, graph (b) was obtained from graph (a) subtracting the events that were less than $1/16$ of the maximum event and graph (c) corresponds to the subtraction of the total cumulative seismicity of all the events whose magnitude are $1/8$ smaller than the maximum.

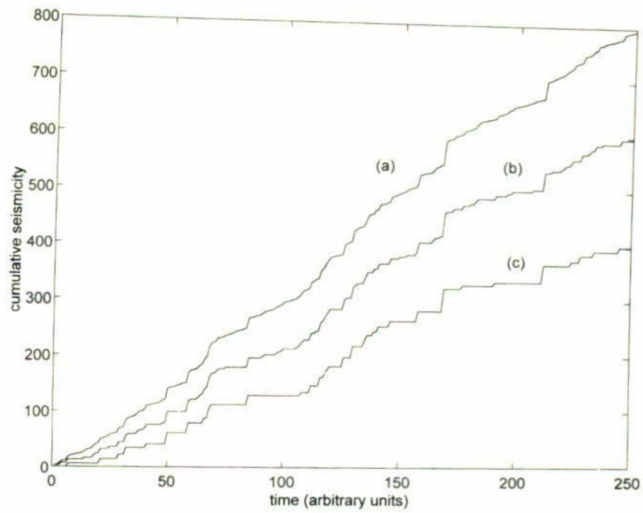

FIGURE 3. The staircase graphs structure for the cumulative seismicity, $\alpha = 0.2$, $L = 80$. (a) Total cumulative seismicity. (b) This graph was obtained when we subtract from total cumulative seismicity the events less than 1/16 of the maximum event. (c) The same but subtracting all events whose magnitude is less than 1/8 than the maximum. The time axis represents iterative updates of the program.

5. The catalogues of synthetic seismicity

In real seismicity a data summary of the earthquakes including at least minimally the occurrence date, location, size and a brief description of the damage caused, is called a catalogue. Practically all the observatories or seismological nets publish catalogues; however, in many parts of the world measurements with seismographs exist only since a few decades ago. It is important to have more complete and exact catalogues, even including the microseismicity, because the analysis of them has many applications. For example, the hypothesis of the seismic gap implies that major earthquakes are waited throughout sections of tectonic plate frontiers in which large earthquakes have been produced and they have not suffered a rupture during the last few years. The application of this concept gives a first order estimate of where the seismic potential is larger. This means that we can bound the spatial limits of a seismic gap. However, only a very crude estimate of the earthquake occurrence time is possible. To bound the temporal limits it is necessary to continually monitor the recognized gaps to see if it is possible to identify any change in the parameters of the crust that be possibly related to the preparation process of the earthquake. Better results have been obtained studying the patterns of seismicity that preceded past events occurred in the gaps [24].

For example, based on the phenomenon of the seismic quiescence, in 1977 Ohtake *et al.* [28] predicted the occurrence of the Oaxaca 1978 earthquake, and, in fact, the prediction was correct. Other interesting examples that show that this technique can give good results, are those reported by Habermann [24] and McNally [25]. All this work has as a premise the existence of good seismicity catalogues.

The technique is promissory and it has been able to give some good results. However, we can not be very optimistic because the technique presents serious problems that have been reported by Habermann [29]. He has demonstrated that in certain periods of time there are decreases and increases in the earthquake detection. This happens in regions where monitoring stations were dismantled, or seismological nets that did not exist were installed, or where special studies were carried out. The effects of detection changes can generally be avoided using a highest cutoff in the magnitude. However, this reduces too much the amount of available data. On the other hand, Prez and Scholz [30] also reported systematical changes in the magnitude.

Summarizing: these studies have showed that systematical changes in magnitude can cause apparent changes in the seismicity, which can easily be confused with precursory seismic quiescence. Habermann says that the only solution is to recalculate the magnitude for all the events in the catalogues that may show these problems. However, this type of problems do not exist with a synthetic catalogue, which has all the events without risk of being incomplete and without measurement errors. Synthetic catalogues can be generated for different values of elastic parameters and for different sizes of the system. This is important because the graphics as

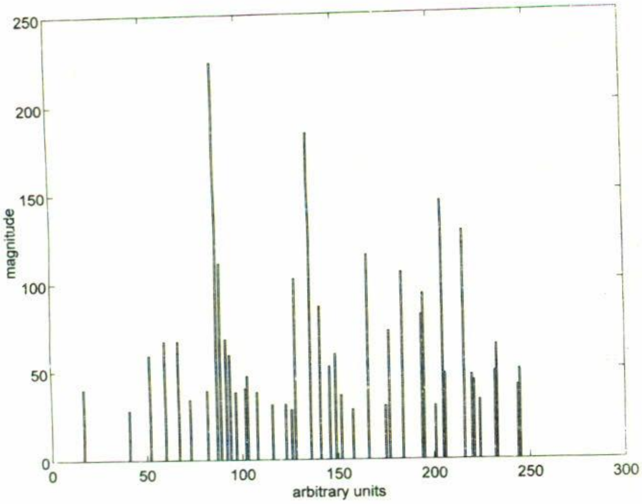

FIGURE 4. Bar graph for a case only with the largest events. Note the precursory quietude previous to the event of greatest magnitude. The time is taken as in the previous figure.

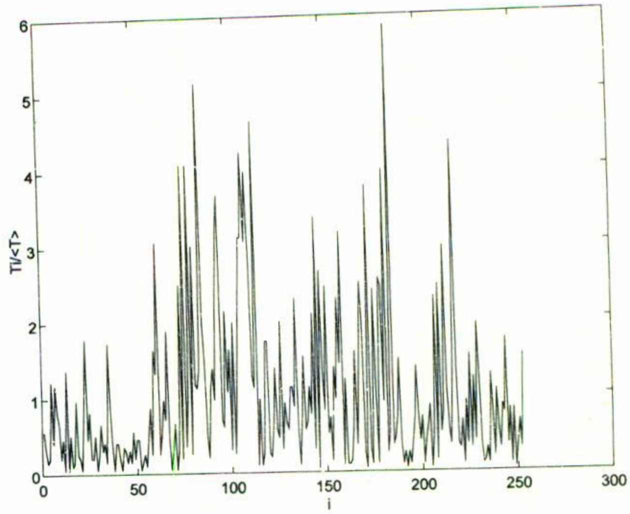

FIGURE 5. Recurrence times normalized by the mean $T_i/\langle T\rangle$ *versus* the number of event i, $\alpha = 0.2$, $L = 35$.

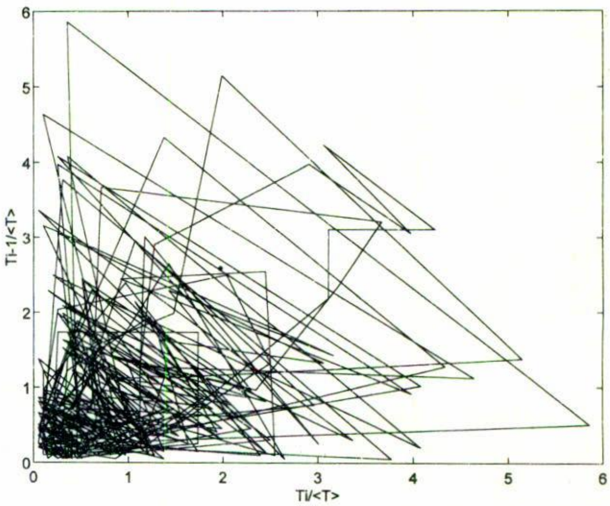

FIGURE 6. $T_i/\langle T\rangle$ *versus* $T_{i-1}/\langle T\rangle$, $\alpha = 0.2$, $L = 35$.

that of Fig. 3a take into consideration all the events in a given period of time, what is equivalent in real seismicity to take all the events with magnitude larger than 0. For very active seismic provinces there are thousands of events with $M \leq 3$, and it is frequent that these data are absent in many catalogues.

However, it is possible to identify possible precursory quiescence in a catalogue of synthetic seismicity, for example in Fig. 4 we show a bar graph for a case in which only we have let the largest events, if we observe the event of greatest magnitude there is a previous quietude to it, also it is easily recognizable the quietude that precedes the second event of greater magnitude. The analysis of synthetic catalogues will give better results for more realistic models, but the complexity of the model should not be very high and the calculations should not be irksome. As an example of the type of study that can be made analyzing the catalogues of synthetic seismicity generated with the OFC model, we report below some interesting characteristics of them.

6. Recurrence times in the OFC model

When a large earthquake is produced we call recurrence time to the time that passes until another earthquake of similar or greater magnitude is produced. In the BSR model, if an earthquake of great magnitude was preceded by a recurrence time larger than the mean recurrence time then the following earthquake will be preceded by a smaller recurrence time [19]. We obtained the same result in the OFC model. Such behavior is shown plotting the recurrence times normalized by the mean *versus* i (the number of event). We show a case in Fig. 5.

To make more evident this situation we also plotted $T_i/\langle T\rangle$ *versus* $T_{i-1}/\langle T\rangle$. In such a return map the diagonal lines prevail, which is also an indication that recurrence times greater than the mean are followed by recurrence times smaller than the mean. See an example in Fig. 6. In the OFC model, a large earthquake is produced because many blocks were near the threshold in a given moment, if this was preceded by a recurrence time greater than the mean, it seems to be that once the large event is produced it still remains certain quantity of blocks near the threshold, in such a form that it will be required a smaller time to produce an equal or greater event. Nishenko and Buland [20] analyzed the times of recurrence T, normalized by the mean recurrence time $\langle T\rangle$ of approximately 50 well characterized real earthquakes and found that they have a log-normal distribution. The normal distribution for earthquake prediction means that 68% of the events are contained within $\pm\sigma$ (standard deviation) of the mean.

The following part of the work is a study of recurrence times in the staircase graphics obtained by the OFC model. First, we made cutoffs in cumulative seismicity, then we cal-

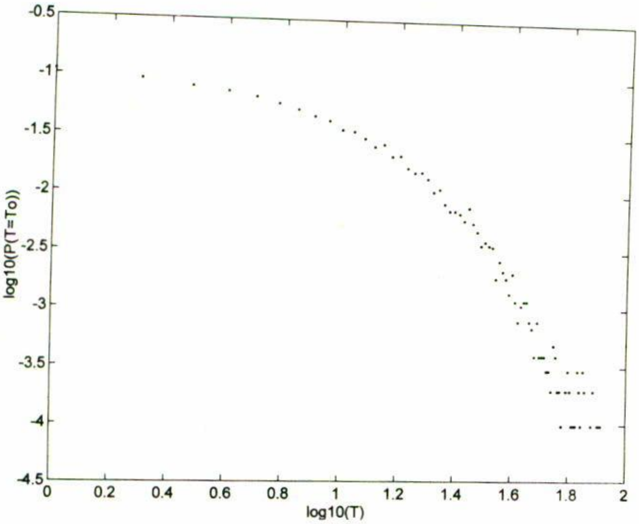

FIGURE 7. Recurrence times distribution, $\alpha = 0.2$, $L = 20$.

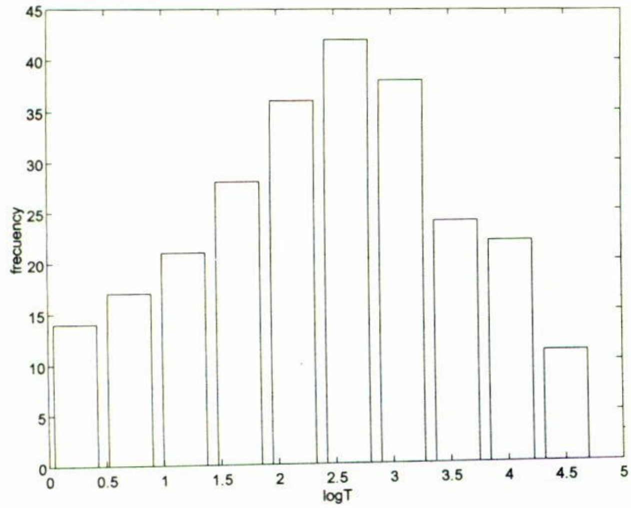

FIGURE 8. Histogram for the logarithm of recurrence times, $\alpha = 0.2$, $L = 35$. Note that it is similar to the normal case.

culated the recurrence times as those times that elapse for having an event of the same magnitude or greater. To know the possible type of distribution of recurrence times is important because we would be able to do some type of forecast, if it is a normal distribution we would make a statistics to know the mean and the standard deviation and then we could bound T. Recurrence times could have a SOC behavior; if this happens we could not give any bound of the type suggested in the previous paragraph. Because of this, the first step was to verify if the recurrence time has or not a SOC behavior. For all simulated cases we concluded that in the OFC model these times do not have a SOC behavior. A case is shown in Fig. 7.

For analyzing the Nishenko and Buland proposal, we made statistics on the logarithm of the recurrence times. The first tests with $\alpha = 0.2$ surprisingly gave histograms of the type shown in Fig. 8, where the similarity with the normal

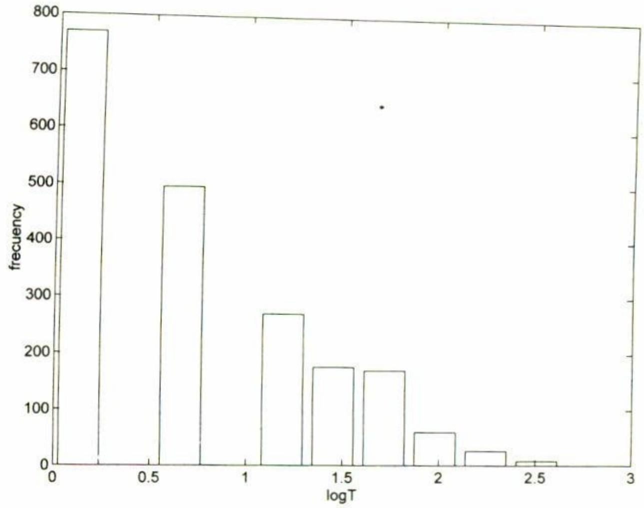

FIGURE 9. Histogram for the logarithm of recurrence times, $\alpha = 0.245$, $L = 20$. Obviously it does not correspond to the normal case.

case is evident. We calculated the mean and the standard deviation σ, because in the case that the logarithm of T had a normal distribution, 68% of the data would have to be contained in the interval , where is the mean of $\log(T_i)$. Then we proceeded to calculate the percentage of data that are found in that interval. For the case shown in Fig. 8 the percentage was 67.19%, which is in agreement with the normal distribution. However, it was noted also that if the same test is made for a different α then the situation changes.

In Fig. 9 we show the same test for the case $\alpha = 0.2450$. Obviously it is not a normal distribution, since if we calculate the previous percentage this gives a value of 38.77%. This would be highly disappointing, but it is not, because this value of α according to Olami *et al.* [15] is almost the conservative case and it does not give good and realistic results. With these α we can not obtain a behavior that at least approaches what happens in real seismicity.

In all tests we made we obtained the same situation, that is, many cases that adjust very well to the log-normal distribution, some more than adjust to it only approximately and others that are far from it. Since in real seismicity the distribution of recurrence times is log-normal, then for a given lattice system size we found the α-values that permit to have this kind of distribution. For different system sizes and for different values of the elastic parameter we found the mentioned percentages. When the percentage was close to 68%, in the histogram we observed a Gaussian behavior. If the behavior of the histogram was not normal then the percentages were far from 68%. In Table I we summarize the results of this analysis. For a better visualization of the results they are presented in Fig. 10.

In Table I, as well as in Fig. 10, we observe that, independently of the size of the system, the better results, that is to say, those where the percentage was close to the 68%, were obtained for values around $\alpha = 0.2$. This confirms what has

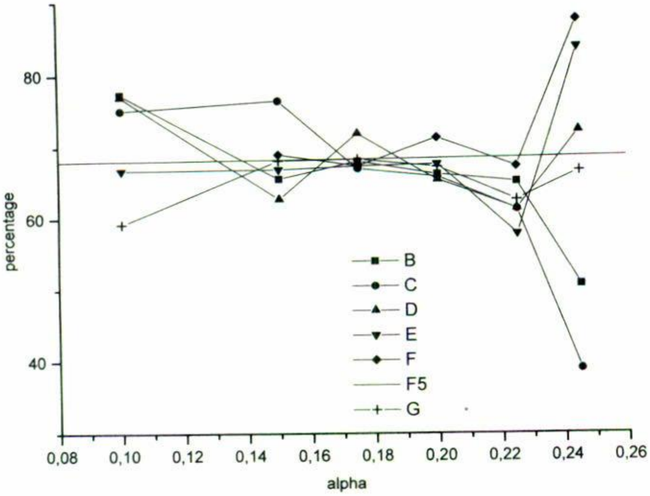

FIGURE 10. Percentage of the data contained in the interval $(\bar{x} - \sigma, \bar{x} + \sigma)$, where $\bar{x}$ is the mean of $\log(T_i)$ for different values of α and system sizes. The straight line F5 belongs to a 68% percentage. The data (B), (C), (D), (E), (F) and (G) belong to $n = 15$, $n = 20$, $n = 25$, $n = 30$, $n = 35$ and $n = 50$ respectively. Note that for α-values around 0.2 the results are close to 68%.

TABLE I. Percentage of the data contained in the interval $(\bar{x}-\sigma, \bar{x}+\sigma)$, where $\bar{x}$ is the mean of $\log(T_i)$ for different values of α and system sizes.

α	$n = 15$	$n = 20$	$n = 25$	$n = 35$	$n = 50$
0.10	77.27	75.00	76.92	66.37	0
0.15	65.31	76.14	62.50	66.67	68.75
0.175	67.70	66.67	71.50	67.01	67.02
0.20	65.74	65.38	64.94	67.19	70.79
0.225	64.62	60.80	60.85	57.38	66.75
0.245	50.37	38.77	38.90	82.87	86.61

already been emphasized: if we use the OFC spring-block model for modeling a fault behavior in real seismicity we should take values of the elastic parameter that are close to 0.2 or, in the anisotropic case, we should take combinations of α_1 y α_2 whose mean is approximately 0.2. Therefore, the OFC model (as also the BSR model) predicts that for some cases, the distribution of recurrence times for large earthquakes is log-normal.

7. Conclusions

We agree with the Olami *et al.* proposal that their model can be used as a basic model to describe at least in a qualitative level the earthquakes occurrence associated with some seismic fault. However, the most difficult task is to establish which are the adequate parameters and then to generate the synthetic catalogue using the model and through the observation of the obtained patterns to describe the behavior of the fault.

For certain values of the OFC model parameters we obtain results that seem to be in qualitative accordance with real seismicity. For example, congruent values with the Gutenberg-Richter law are obtained and also the staircase shape of real seismicity is obtained. From the analysis of these plots it is possible to recognize precursory seismic quiescence using the synthetic catalogues of seismicity obtained from the same model which permits us to assert that for some cases the log-normal distribution of recurrence times is valid. Therefore, we think that models as the OFC or one with some modifications can give some insights about real seismicity. One advantage of having a model that could reproduce with certain approximation the seismicity patterns of the seismic zones is that we can generate a synthetic catalogue where we can have all the events, without the risk of having incomplete catalogues, overestimated or with errors in the measurements. Our results confirm the robustness of synthetic catalogues arising from SOC-models in spite of its simplicity. Finally, we want to emphasize that by means of our study we find that the OFC model has another seismic properties in addition to the previously reported, mainly the staircase graphs and the log-normal distributions of recurrence times.

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