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Spin flip spectra as a probe to the structure of cosmic strings

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The application of the Casimir effect to the field surrounding a spinning particle in a space time with deficit angle leads to a modification of the magnetic moment of the particle. In the following we discuss corrections to spin-flip spectra of spinning particles in the field of both global and gauge strings. Such spectra can be used to distinguish global strings from gauge strings. Spin flip spectra can also be used to probe for modifications of gravity due to a Brans Dicke scalar, higher curvature terms, as well as other modifications of gravitational theory.

Keywords: Spin flip; cosmic strings

El uso del efecto Casimir al campo que rodea a una partícula que gira en el espacio-tiempo con deficiencia angular conduce a la modificación del momento magnético de la partícula. A continuación discutimos las correcciones al espectro de cambio de espín de una partícula que gira en un campo de cuerdas globales o de norma. Este espectro se puede usar para distinguir entre cuerdas globales y cuerdas de norma. El espectro de cambio de espín también puede ser usado como sondeo a las modificaciones de la gravedad debidas al escalar de Brans Dicke, a términos de curvatura superior así como otras modificaciones de la teoría de gravitación.

Descriptores: Cambio de espín; cuerdas cósmicas

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1. Introduction

One of the great triumphs of spontaneously broken gauge theories is that they predict topologically stable configurations such as strings [1], monopoles [2] and domain walls [3]. Domain walls have the unpleasant feature that they have an energy density variation with the scale factor that is in conflict with astrophysical data [4]. Strings however can be accommodated within the present limits of cosmological data and provide us with an alternate candidate for how large scale structure forms [5]. The signatures of cosmic strings include the production of wakes in the matter distribution surrounding a string [6], double images of galaxies due to the existence of a deficit angle [7] and the existence of discontinuities in the CMB [8] due to the motion of the string through the cosmic background frame. Anther distinct feature of cosmic strings is that they will produce sharp images of objects located behind the string [9]. Because of the recent experimental observations of how galaxies are distributed in a web-like structure with alternating filaments and voids, cosmic strings have become a very attractive candidate in explaining these filamentry structures [10, 11].

If we try to classify strings we find that they have a number of definite properties according to whether they are global or gauge strings, superconducting, and whether or not they are Abelian or non-Abelian in group theoretic structure. The gravitational field of a string can also be drastically changed by modifications of Einstein gravity, these modifications include Brans Dicke theory [12], higher-order corrections to Einstein gravity [13] and the addition of spin generated torsion for the case of spinning strings [14]. In addition to the cosmological signatures of strings mentioned in Refs. 6–8 in the following we discuss possible ways of detecting the presence of strings through atomic spectral shifts coming from H-I and H-II regions in the nearby vicinity of a string that may exist in the epochs following recombination. In a previous note [15] we discussed the spectral shifts induced by the Casimir effect of an electron spin flip in the field of a string with deficit angle δ. In that particular case we considered a G.U.T. string with constant deficit angle. In the following we elucidate further on the possibility of a deficit angle varying with distance from the string and also varying with certain parameters such as the Brans Dicke constant, the coefficient of the higher curvature corrections to gravity, as well as the scale of symmetry breaking and the possible spin density of the string. Though at present there doesn't seem to be any conclusive evidence for the presence of a cosmic string in a cosmological setting, probing for their presence through induced spin spectral shifts in nearby H-I and H-II regions seems to be a very direct window through which many of the properties of cosmic strings may be observed.

2. Induced spectral shifts in the field of a cosmic string

As mentioned earlier in Ref. 15, an electron (or any charged particle) will experience a correction to the spin-flip (Zeeman effect) energy in a magnetic field in the background of a cosmic string due to the modification of the magnetic moment of the electron induced by the Casimir effect in the non-trivial topological space-time of the string [16, 17]. The correction to the magnetic moment of the electron (magnitude of moment) is

$$\delta\mu = -\frac{\mu_0\left(\frac{1}{a^2}-1\right)e^2\hbar}{48\pi^2 m^2 r^2 C^3}, \quad \left(\mu_0 = \frac{e\hbar}{2mC}\right), \tag{1}$$

(r = distance from string). Here a is found from the metric

$$(dS)^2 = C^2dt^2 - dr^2 - dz^2 - r^2a^2(d\theta)^2,$$

where $a(2\pi) = 2\pi$ '+' λ (λ = dihedral angle). If λ is negative it is called the deficit angle. For a gauge string [18] the metric can be written far from the string core as

$$(ds)^2 = C^2dt^2 - dr^2 - dz^2 - r^2\left(1-\frac{4G\mu}{C^2}\right)^2(d\theta)^2,$$

here μ = mass per unit length and $a = 1 - 4G\mu/C^2$. The limit on $G\mu/C^2$ is $G\mu/C^2 < 2\times10^{-6}$ from the string induced C.M.B. anisotropy [19]. For the Zeeman energy of an electron in a magnetic field B (z component field) we have using Eq. (1):

$$E_\pm = \pm\frac{e\hbar}{2mC}\left[1-\frac{\left(\frac{1}{a^2}-1\right)e^2\hbar}{48\pi^2m^2r^2C^3}\right]B. \tag{2}$$

For the correction term in Eq. (2) to be 10^{-4} of the normal Zeeman energy we have for $a \simeq 1$–10^{-6}, $r \simeq 10^{-13}$ cm which is no longer in the far field region of the string. However there are two ways of circumventing this obstacle, the first is to consider a global string with metric [20]

$$(dS)^2 = \left(1-\frac{4G\mu}{C^2}\ln_e\frac{r}{r_0}\right)(C^2dt^2-dz^2) - dr^2 - \left(1-\frac{8G\mu}{C^2}\ln_e\frac{r}{r_0}\right)r^2(d\theta)^2; \tag{3}$$

here $a^2 = 1 - (8G\mu/C^2)\ln_e(r/r_0)$ and Eq. (3) applies in a region not too far from the string core.

Also μ = effective mass per unit length of string and r_0 is a function of the string parameters. We note at $1 - (8G\mu/C^2)\ln_e(r/r_0) = 0$ [21], the deficit angle becomes 2π and $a \to 0$, from Eq. (2) the wavelength of photons for a spin flip is $E_+ - E_- = hC/\lambda$ giving

$$\lambda = \frac{2\pi mC^2}{eB\left[1-\frac{\left(\frac{1}{a^2}-1\right)e^2\hbar}{48\pi^2m^2r^2C^3}\right]}, \tag{4}$$

thus as we approach $r = r_0e^{c^2/8G\mu}$ we find that the wavelength of a Zeeman spin flip increases, or the spectrum will fade out of the observable window. Thus a plot of wavelength versus distance will show the transition from the X ray spectrum toward the infrared as spin flips further from the string are observed. This of course also necessitates the presence of a z component cosmic magnetic field B along the string axis. The second way of observing the spin flip shifts due to the gravitational field of the string is to note that if gravity grows strong in a domain of the universe [22] $4G\mu/C^2$ can be made close to 1 even for strings less massive than GUT strings, in this case Eq. (4) reads approximately

$$\lambda = \frac{2\pi mC^2}{eB}\left[1+\frac{\left(\frac{1}{a^2}-1\right)e^2\hbar}{48\pi^2m^2r^2C^3}\right], \tag{5}$$

since $a^2 = \left(1-4G\mu/C^2\right)^2$, as $G\mu/C^2$ approaches 1, the second term in Eq. (5) becomes comparable to the first term. Thus in this case an appreciable fractional shift of the normal Zeeman line in the z component magnetic field will be a signature of the presence of a gauge string in the background of strong gravity.

Another approach to probing for a global string is to insert

$$a = \sqrt{1-\frac{8G\mu}{C^2}\ln_e\frac{r}{r_0}}$$

into Eq. (5) giving

$$\lambda = \frac{2\pi mC^2}{eB}\left[1+\frac{\left(\frac{1}{1-\frac{8G\mu}{C^2}\ln_e\frac{r}{r_0}}-1\right)e^2\hbar}{48\pi^2m^2r^2C^3}\right]. \tag{6}$$

In Eq. (6) we find a characteristic variation of λ with r, for $8G\mu/C^2 < 1$, we find

$$\lambda = \frac{2\pi mC^2}{eB}\left[1+\frac{\left(\frac{8G\mu}{C^2}\ln_e\frac{r}{r_0}\right)e^2\hbar}{48\pi^2m^2r^2C^3}\right], \tag{7}$$

a characteristic increase of λ with $\ln_e(r/r_0)/r^2$ in Zeeman-like spin flips would signal the presence of global cosmic strings.

So far we have confined out attention to a gauge string in the far field region and a global string close to the core. Let us now consider a stationary string with spin, Krisch [23] has shown that the metric reads

$$(dS)^2 = (Cdt + kd\theta)^2 - dr^2 - dz^2 - D^2(d\theta)^2, \tag{8}$$

here

$$k = \frac{CS_0}{\bar\lambda}\left(1-\cos\sqrt{\frac{8\pi G\bar\lambda}{C^4}}r\right),$$

$$D = \frac{\sin\left(\sqrt{\frac{8\pi G\bar\lambda}{C^4}}\right)r}{\sqrt{\frac{8\pi G\bar\lambda}{C^4}}},$$

also S_0 = constant of dimensions of spin density, C = speed of light, $\bar\lambda$ = energy density of string and

$$T_4^4 = \bar\lambda, \qquad T_r^r = T_0^0 = 0, \qquad T_z^z = \bar\lambda.$$

For the quantity a we have

$$D^2 - k^2 = a^2 r^2 \quad \text{or} \quad a^2 = \frac{D^2 - k^2}{r^2}. \tag{9}$$

From Eq. (5) we find for the Zeeman spin flip in the field of such a string

$$\lambda = \frac{2\pi m C^2}{eB}\left[1 + \frac{\left(\frac{r^2}{D^2-k^2} - 1\right)e^2\hbar}{48\pi^2 m^2 r^2 C^3}\right]. \tag{10}$$

Since D and k depend on the energy density and spin density of the string we may probe for spinning strings using Eq. (10). Here the particle that experiences the spin flip actually lies within the boundaries of the spinning string configuration. To probe for scalar tensor effects in the field of a string M.E.X. Guimâraes [24] has discussed string solutions in Brans Dicke theory, as above such solutions could be used to calculate the dependence of the quantity a on the Brans Dicke scalar which in turn will determine corrections to the spin-flip frequencies through modifications of a. Also J. Yuanfang *et al.* [25]. Has discussed the corrections induced in the field of a global string by higher curvature corrections to Einstein gravity, the quantity a^2 in this theory is

$$a^2 = 1 - \frac{8G\mu}{C^2}\ln_e\frac{r}{r_0} - \frac{8G\mu(6\bar{a})}{C^2(3\pi)r^2}; \tag{11}$$

here $\bar{a}$ has dimensions of (length)^2 and is the coefficient of the R^2 in the gravitational lagrangian.

$$\frac{C^4}{16\pi G}(R + \bar{a}R^2)\sqrt{-g}.$$

For this case Eq. (5) would read

$$\lambda = \frac{2\pi m C^2}{eB}\left\{1 + \frac{\left[\frac{8G\mu}{C^2}\ln_e\frac{r}{r_0} + \frac{8G\mu(6\bar{a})}{C^2(3\pi r^2)}\right]e^2\hbar}{48\pi^2 m^2 r^2 C^3}\right\}. \tag{12}$$

From Eq. (12) we may probe for any higher curvature term in the lagrangian of gravity by looking for an additional variation of λ with $1/r^4$ which would modify the $1/r^2\ln_e(1/r_0)$ dependence already present for a global string.

There are two other probes to cosmic strings that we would like to discuss here, firstly for a gauge string far from the core (electromagnetic gauge string) the vector potential looks like [26]

$$A_\theta = \frac{\hbar C}{er}\left(1 - e^{-r/r_2}\right)^2, \tag{13}$$

where r_2 depends on the vacuum expectation value of the Higgs field $(\sigma/\sqrt{2})$. The Higgs field is

$$\Phi = \frac{\sigma}{\sqrt{2}}\left(1 - e^{-r/r_1}\right)e^{i\theta} \tag{14}$$

(N= 1 = winding number of Higgs field).

For the z component magnetic field we have

$$B_z = \frac{1}{r}\frac{\partial}{\partial r}\left[\frac{\hbar C}{e}\left(1 - e^{-r/r_2}\right)^2\right] = \frac{2\hbar C}{er}\left(1 - e^{-r/r_2}\right)\left(\frac{1}{r_2}e^{-r/r_2}\right). \tag{15}$$

For the metric for large r we have

$$(dS)^2 = C^2dt^2 - dr^2 - dz^2 - \left(1 - \frac{4G\mu}{C^2}\right)^2 r^2 d\theta^2, \tag{16}$$

where again μ = effective mass per unit length of gauge strings $(a = 1 - 4G\mu/C^2)$. From Eq. (5) we have for Zeeman spin flip in the field of the gauge string

$$\lambda = \frac{2\pi m C^2}{eB}\left\{1 + \frac{\left[\frac{1}{\left(1-\frac{4G\mu}{C^2}\right)^2} - 1\right]e^2\hbar}{48\pi^2 m^2 r^2 C^3}\right\}. \tag{17}$$

or

$$\lambda \simeq \frac{2\pi m C^2 r r_2 e^{r/r_2}}{\left(1 - e^{-r/r_2}\right)2\hbar C}\left\{1 + \frac{\left[\frac{1}{\left(1-\frac{4G\mu}{C^2}\right)^2} - 1\right]e^2\hbar}{48\pi^2 m^2 r^2 C^3}\right\}. \tag{18}$$

In Eq. (18) an increase in λ with r according to $re^{(r/r_2)}$ and a variation in λ with r due to the topological term would signal the presence of electromagnetic gauge strings in a cosmological setting. If another U(1) group defined the group structure of the string, then the corresponding U(1) magnetic moment of the particle in the surrounding field would have to be used in Zeeman like spin flip transitions.

The last probe to a gauge string that is motivated by a paper by Bezerra *et al.* [27] is that of the spectral shifts in transitions of a scalar particle moving in the field of a string under the influence of an infinite square well potential between a and b $(a < r < b)$, here the particle is forbidden to move outside the region. The eigenvalues calculated by Bezerra *et al.* [27] are

$$E = \left[m^2C^4 + \hbar^2C^2k^2 + \frac{\hbar^2C^2l^2}{ab\alpha^2} + \hbar^2C^2\left(\frac{4ab(n\pi)^2 - (b-a)^2}{4ab(b-a)^2}\right)\right]^{1/2}, \tag{19}$$

here k = wave number in z direction, lh/α = z component angular momentum in z direction, (l = integer), $\alpha^2 = 1 - 4G\mu/C^2$.

The wave function is

$$\Psi(t, r, \theta, z) = e^{i[kz + l\theta - (Et/\hbar)]}R(r), \tag{20}$$

where $R(a) = R(b) = 0$, and the metric is

$$(dS)^2 = C^2dt^2 - dr^2 - dz^2 - \left(1 - \frac{4G\mu}{C^2}\right)^2 r^2 d\theta^2, \tag{21}$$

μ = mass per unit length of string. We see from Eq. (19) that any quantum jumps specified by

$$E_{n',l'} - E_{n,l} = \frac{hC}{\lambda} \tag{22}$$

will be sensitive to α and thus if these spectral shifts can be observed they can be used to probe for the presence of gauge string through the dependence of λ on α through Eq. (19) and Eq. (22).

3. Conclusion

The above discussion has indicated that both global strings and gauge strings can be studied if the r dependence of the Zeeman spin slip has a variation consistent with any of the above models. Also for an electromagnetic gauge string Eq. (18) would give us a specific signature of λ *vs.* r that would signal U(1) strings in a cosmological setting. The problem with all the above models is to try to observe line structure in the radiation coming from cosmological sources after e^+e^- annihilations lines and atomic spectral lines have been separated out. Though cosmic strings most likely formed after the G.U.T. and electroweak phase transitions, some of them may have survived to the period of recombination. Also in specific domains of the universe cosmic springs might still be very recent additions to the local structure of the universe. It is hoped that a closer and more careful analysis of the entire spectrum of the extra-galactic radiation might reveal the variation of λ with r predicted above for Zeeman spin flips so as to lend positive evidence for the presence of global and gauge strings in a cosmological setting.

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