INVESTIGACIÓN REVISTA MEXICANA DE FÍSICA **46** (1) 29–40 FEBRERO 2000

Non divergent formula in the four body region of the Dalitz plot for the precise β energy spectrum in the semileptonic decay of hyperons

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Recibido el 14 de junio de 1999; aceptado el 24 de septiembre de 1999

We have obtained analytical formulas and their corresponding numerical results for the contributions to the electron energy spectrum, due to the bremsstrahlung of semileptonic decays of charged and neutral baryons, in the region of the Dalitz plot that covers the four body-events. We show that the logarithmical singularity at the upper edge of the plot, contained in previous results disappears after performing an analytical integration. The new formulas contain terms of the order α times the momentum transfer. They are applicable to any beta decay process, and are suitable for a model-independent experimental analysis.

Keywords: Hyperons; beta decay; radiative corrections

Hemos obtenido fórmulas analíticas y sus correspondientes valores numéricos para las contribuciones al espectro de energía del electrón, debidas al *bremmstrahlung* en el proceso de decaimiento semileptónico de bariones cargados y neutros, en la región de la gráfica de Dalitz que corresponde a la zona donde tiene lugar el decaimiento a cuatro cuerpos exclusivamente. Mostramos que la singularidad logarítmica que aparece en el borde superior de la gráfica, contenida en resultados anteriores, desaparece después de integrar de manera analítica. Las nuevas fórmulas contienen términos de orden α veces la transferencia de momento. Son aplicables a cualquier proceso de decaimiento beta y son apropiadas para un análisis experimental independiente de modelo.

Descriptores: Hiperones; decaimiento beta; correcciones radiativas

PACS: 13.40.Ks; 13.30.Ce; 14.20.Jn

1. Introduction

The condition of conservation of energy and momentum determines the physical region where the hyperon semileptonic decay (HSD) process takes place. If the products of the decay are three bodies (TB), as in the HSD (without radiative correction), the physical region is delimited by a hyperon minimal and a maximal energy $\left(E_{2,}^{\min}, E_{2,}^{\max}\right)$ for each value of the energy E of the emitted electron. When a real photon is considered as an additional product of the decay, then there are four products of the decay, and the physical four body region (FBR) in which this process is possible, contains the former TB region (TBR) and an additional portion with energies below the $E_2^{\min}(E)$ in such a way that $M_2 < E_2 < E_2^{\min}$ for $m < E < E_c$, where M_2 and m are the masses of the produced hyperon and electron, respectively; and E_c is the energy of the electron for which the hyperon is at rest. The upper boundary of E_2 for the region where the TB decay is forbidden is evaluated with the condition $k = 0$, this means, without the emission of a real photon, it is the case where the neutrino balances the total momentum of the residual hyperon and electron, which are emitted in colinear directions, and is characterized by $\cos\theta = +1$ in the TBR.

The knowledge of the *radiative corrections* (RC) in the four body region (FBR) of the Dalitz plot (DP) is required to obtain formulas for the energy spectrum of the decay products (fermions) in the HSD. The precision of the formulas is increased with the inclusion of all terms of the order α-times the momentum transfer in the RC. The total decay rate of the processes can be computed directly through analytical formulas for the RC, if they do not contain any divergences.

In previous papers we have analyzed the TB decays with radiative corrections (virtual and bremsstrahlung) in the TBR, Refs. 1, 2, and 3. We have also obtained the energy distributions (E_2, E) for events that take place with $E_2 < E_2^{\min}$, *i.e.* in the region where the decay takes place only with the emission of a real photon, Refs. 4 and 5.

In this paper the contribution of the bremsstrahlung to the energy spectrum of the electron is obtained, in the portion of the FBR region which is not considered in the radiative corrections of the TB decay in the TBR. As it is known, the virtual radiative correction (at the TBR) contains an infrared divergence which is cancelled due the existence of another divergence that arises in the process when the real photon emission (bremsstrahlung) is considered (in the same TBR), therefore the bidimensional distribution with the total RC for events with $E_2 = E_2^{\min}$ becomes finite for the precise TB decay [1–3]. The events that arise with the hyperon energy $E_2^{\min}$ are very special events and are of particular interest, because a logarithmical divergent contribution arises in the bidimensional distribution of energies (E_2,E) when the bremsstrahlung alone is considered in the FBR. The divergence is only present on the boundary between the TBR and FBR, where $\cos\theta = +1$. This is just the infrared divergence of the TBR reached from the FBR side (see Appendix of Ref. 2). The bremsstrahlung decay is connected continuously and smoothly in going from the TBR to the FBR. As the events with $E_2 = E_2^{\min}$ should not be taken twice, the total RC remains finite. On Ref. 6, the radiative corrections in the TBR were evaluated by other means, and no divergences are found in the electron energy spectrum.

In this paper we show how the logarithmic divergences, that are contained in the model-independent results given in Refs. 4 and 5, obtained in an analytical way for the RC to baryon β decays in the FBR of the DP are cancelled after performing the integration over the energy of the final hyperon that emerges in the process. The important feature of the new analytical forms of the RC is that they are written in a simple form, as products of two factors, one of them is a model-independent function and the other one does depend on the (strong-interaction) model through the form factors, which are determined through the experimental data. Such formulas are suitable for a direct evaluation of the RC for any event in the allowed physical region. The new results are valid for photon bremsstrahlung calculations in any charged or neutral hyperon decay and its knowledge is important to obtain the precise description of the semileptonic weak decays, which are relevant, in particular the neutron decay, in cosmology, astrophysics, solar physics, the solar neutrino problem, as well as in other areas of particle physics.

The structure of this paper is the following. In Sect. 2 we exhibit the kinematical region, in which only the four body decays take place, and we display the bremsstrahlung amplitudes for the charged and neutral hyperon semileptonic decays with all the $\alpha q/\pi M_1$ terms included. In Sect. 3 the bidimensional observable energy distributions in the FBR of the bremsstrahlung are described for both cases. We devote Sect. 4 to give the energy spectrum of the charged lepton in a non divergent form. In Sect. 5 we present our final results. The purpose of the numerical evaluation is to perform a comparison between our results and other previously published numerical values. Finally, we include six appendices which contain definitions of the coefficients and model-independent functions that appear in the analytical result, the procedure we follow to integrate the divergent integrands, and other relevant relations.

2. Kinematics and amplitudes

The theoretical framework of our calculations can be found in Refs. 4 and 5. In this section we present the main features and the notation to describe the four-body process we are interested in

$$A^s(p_1) \to B(p_2) + e^-(\ell) + \bar{\nu}_e(p_\nu) + \gamma(k), \qquad (1)$$

with the emission of a real photon γ. A^s corresponds to the neutral $(s = n)$ or charged $(s = c)$ decaying baryon, B the produced baryon, e^- and $\bar{\nu}_e$ denote the lepton and its neutrino counterpart, respectively and the γ corresponds to the photon. The four-momenta and masses of the particles involved in the baryon semileptonic decay are denoted by

$$p_1 = (E_1, \vec{p_1}), \quad p_2 = (E_2, \vec{p_2}), \quad \ell = (E, \vec{\ell}),$$
$$p_\nu = (E_\nu, \vec{p_\nu}), \quad \text{and} \quad k = (k_0, \vec{k}), \qquad (2)$$

and by M_1, M_2, m, m_ν, and m_k , respectively. We assume throughout this paper that $m_\nu = 0$, and $m_k = 0$ as corresponds to real photons.

The Four Body Region (FBR) for the process in Eq. (1), in the rest frame of A^s, is defined by

$$M_2 \le E_2 \le E_2^t, \quad E_2^t = E_2^{\min} = \frac{M_2^2 + [M_1 - E - |\bar{\ell}|]^2}{2(M_1 - E - |\bar{\ell}|)},$$
$$m \le E \le E_c, \quad E_c = \frac{(M_1 - M_2)^2 + m^2}{2(M_1 - M_2)}. \qquad (3)$$

The z-axis is chosen along the electron three-momentum and the x-axis oriented so that the final baryon three-momentum is in the first or fourth quadrants of the x-z plane.

The emission of the real photon in Eq. (1) is described as a radiative correction to the semileptonic decay of the hyperon. The uncorrected matrix element M_0 (without the emission of the real photon) for this decay is given by the product of the matrix elements of the baryonic weak current and of the leptonic current:

$$M_0 = \frac{G_v}{\sqrt{2}} \bar{u}_B W_\mu u_A \bar{u}_\ell O_\mu v_\nu, \qquad (4)$$

where $G_v = G_\mu V_{ij}$ and G_μ is the muon decay coupling constant, V_{ij} is the corresponding Cabibbo-Kobayashi-Maskawa (CKM) matrix element. We have

$$W_\mu = f_1(q^2)\gamma_\mu + \frac{f_2(q^2)}{M_1}\sigma_{\mu\nu}q_\nu + \frac{f_3(q^2)}{M_1}q_\mu$$
$$+ \left[g_1(q^2)\gamma_\mu + \frac{g_2(q^2)}{M_1}\sigma_{\mu\nu}q_\nu + \frac{g_3(q^2)}{M_1}q_\mu\right]\gamma_5, \qquad (5)$$

and

$$O_\mu = \gamma_\mu(1 + \gamma_5). \qquad (6)$$

The $q = p_1 - p_2$ denotes the four-momentum transfer. Our metric and γ-matrix conventions are those of Ref. 2.

To obtain the decay rate one has first to obtain the amplitude of these processes. It has been shown in Ref. 7 that the order α bremsstrahlung amplitude can be obtained in a model independent fashion by using the Low-theorem [8, 9].

We reproduce here the model independent amplitudes, in terms of the Dirac form factors, given in Eqs. (18)–(20) of Ref. 2, and Eq. (23) of Ref. 3.

The total transition amplitudes can be written as

$$M_B^s = M_1^s + M_2^s + M_3^s,$$
$$s = c(\text{charged}), \ \text{or} \ n(\text{neutral}) \qquad (7)$$

with

$$M_1^c = eM_0\left(\frac{\epsilon\cdot\ell}{\ell\cdot k} - \frac{\epsilon\cdot p_1}{p_1\cdot k}\right), \qquad (8)$$

$$M_2^c = \frac{eG_v}{\sqrt{2}}\epsilon_\mu \bar{u}_B W_\lambda u_A \bar{u}_\ell \frac{\gamma_\mu \not{k}}{2\ell\cdot k} O_\lambda v_\nu, \qquad (9)$$

$$M_3^c = \frac{G_v}{\sqrt{2}}\bar{u}_\ell O_\lambda v_\nu \epsilon_\mu \bar{u}_B \left\{ \frac{eW_\lambda \not{k}\gamma_\mu}{2p_1\cdot k} - \kappa_1 W_\lambda \frac{\not{p}_1 + M_1}{2p_1\cdot k}\sigma_{\mu\nu}k_\nu + \kappa_2\sigma_{\mu\nu}k_\nu \frac{\not{p}_2 + M_2}{2p_2\cdot k}W_\lambda \right.$$
$$\left. + e\left(\frac{p_{1\mu}k_\lambda}{p_1\cdot k} - g_{\mu\lambda}\right)\left(\frac{f_3 - f_2}{M_1} + \gamma_5\frac{g_3 - g_2}{M_1}\right) + e\left(\frac{p_{1\mu}k_\nu}{p_1\cdot k} - g_{\mu\nu}\right)(\sigma_{\lambda\nu} + g_{\lambda\nu})\left(\frac{f_2 + g_2\gamma_5}{M_1}\right)\right\} u_A, \quad (10)$$

and

$$M_1^n = eM_0\left(\frac{\epsilon\cdot\ell}{\ell\cdot k} - \frac{\epsilon\cdot p_2}{p_2\cdot k}\right), \quad (11)$$

$$M_2^n = \frac{eG_v}{\sqrt{2}}\epsilon_\mu \bar{u}_B W_\lambda u_A \bar{u}_\ell \frac{\gamma_\mu \not{k}}{2\ell\cdot k} O_\lambda v_\nu, \quad (12)$$

$$M_3^n = \frac{G_v}{\sqrt{2}}\bar{u}_\ell O_\lambda v_\nu \epsilon_\mu \bar{u}_B \left\{ -\frac{e\gamma_\mu \not{k} W_\lambda}{2p_2\cdot k} - \kappa_1 W_\lambda \frac{\not{p}_1 + M_1}{2p_1\cdot k}\sigma_{\mu\nu}k_\nu \right.$$
$$+ \kappa_2\sigma_{\mu\nu}k_\nu \frac{\not{p}_2 + M_2}{2p_2\cdot k}W_\lambda + e\left(\frac{p_{2\mu}k_\rho}{p_2\cdot k} - g_{\mu\rho}\right)$$
$$\left. \times\left[\left(\frac{f_2 + g_2\gamma_5}{M_1}\right)\sigma_{\lambda\rho} + g_{\lambda\rho}\left(\frac{f_3 + g_3\gamma_5}{M_1}\right)\right]\right\} u_A. \quad (13)$$

κ_1 and κ_2 are the anomalous magnetic moments of A^s and B given in Eqs. (21) and (22) in Ref. 2. ϵ_μ is the photon polarization four-vector.

3. Bidimensional distribution

After performing the standard trace calculation and the phase space integration according to the kinematical limits given in Eqs. (3), as in Refs. 4 and 5, a high precision model independent and useful result is obtained for processes where the momentum transfer is not small and therefore cannot be neglected. In this result, valid for both types of unpolarized decays, terms of order $\alpha q^2/\pi M_1^2$ and higher are neglected. The result, for the differential bremsstrahlung decay rate of the decay in Eq. (1), which is given in terms of the observable independent variables E_2 and E, the energies of the emitted baryon and the electron, respectively, is relevant for the evaluation of the energy spectrum of the emitted electron.

The differential bremsstrahlung decay rate up to order $\alpha q/\pi M_1$ for HSD is compactly given by

$$d\Gamma_B^s\,(A^s \to Be\nu\,\gamma) = \frac{\alpha}{\pi}d\Omega\sum_{i=0}^{17} H_i^{s\prime}\,\theta_i^{s\mathrm{T}}. \quad (14)$$

$$d\Omega = \frac{G_v^2}{2}\frac{1}{2\pi^3}M_1 dE_2 dE, \quad (15)$$

where the coefficients $H_i^{s\prime}$ (see Appendix I and II) are

$$H_1^{n\prime} = H_1^{c\prime} = H_1', \quad H_i^{c\prime} = H_i',$$
$$H_i^{n\prime} = H_i' + N_i', \quad H_{17}' = 0, \quad (16)$$

and the model independent functions are presented in Appendix III. θ_1^{nT} is given in Eq. (25) and

$$\theta_i^{c\mathrm{T}} = \theta_i^{n\mathrm{T}} = \theta_i^{\mathrm{T}}, \quad \text{for} \quad i = 0, 2, \ldots, 17. \quad (17)$$

The H_i''s depend on the form factors through functions Q_i, $i = 1, \ldots, 4$. The explicit form of the Q_i's is given in Eqs. (16)–(20) in Ref. 1. The H_i''s and the θ_i^{T}'s with $i = 0, \ldots, 16$ are shown in Ref. 4 in Eqs. (37) and in Eqs. (33), respectively. In the Appendices II and III we consider the former coefficients and the model independent functions in a simplified form in order to avoid singular $\theta_i^{s\mathrm{T}}$'s at ($E = E_c, E_2 = M_2$), where $|\vec{p_2}| = 0$. The simplified N_i' 's, which are shown in Appendix IV do also depend on the form factors and are equivalent to the ones given in Ref. 5.

Equation (14) with $s = c$ ($s = n$) corresponds to the analytical result for the bremsstrahlung part of the DP of the HSD of charged (neutral) hyperons, at the FBR. With Eqs. (28) and [Eq. (33)], given in Refs. 10, the full analytic result for the RC at the whole region of the Dalitz plot for semileptonic decay of unpolarized charged (neutral) hyperons is completed.

In order to illustrate and analyze the result in Eq. (14), let us consider the charged (HSD) ($s = c$) and the neutral (HSD) ($s = n$) cases separately.

For the *charged HSD case*: Equation (14) becomes

$$d\Gamma_B^c\,(\,A^- \to B^0 e^- \bar{\nu}_e\,\gamma) = \frac{\alpha}{\pi}d\Omega\sum_{i=0}^{16} H_i'\,\theta_i^{\mathrm{T}}. \quad (18)$$

The H_1' coefficient in the former equation depends on E_2 in the following way:

$$H_1' = \sum_{k=0}^{2}\varepsilon_k E_2^k, \quad (19)$$

and the model-independent function $\theta_1^{c\mathrm{T}}$ is explicitly

$$\theta_1^{c\mathrm{T}} = (I_1 - 2)\,\ln\left|\frac{y_0 + 1}{y_0 - 1}\right|, \quad (20)$$

where

$$I_1 = \frac{2}{\beta}\,\mathrm{arctanh}\,(\beta) \qquad |\vec{\ell}| = \beta E, \quad (21)$$

and

$$y_0 = \frac{\left(E_\nu^0\right)^2 - E^2\beta^2 - |\vec{p_2}|^2}{2\,|\vec{p_2}|\,E\beta}, \quad (22)$$

$$E_\nu^0 = M_1 - E_2 - E. \quad (23)$$

The ε_k's in Eq. (19) are displayed in Appendix I.

Similarly, for the *neutral HSD case*: Eq. (14) becomes

$$d\Gamma_B^n\left(A^0 \to B^+ e^- \bar{\nu}\gamma\right) = \frac{\alpha}{\pi} d\Omega\left[\left(H_0' + N_0'\right)\theta_0^{\mathrm{T}} + H_1'\theta_1^{n\mathrm{T}} + \sum_{i=2}^{16}\left(H_i' + N_i'\right)\theta_i^{\mathrm{T}} + N_{17}'\theta_{17}^{\mathrm{T}}\right]. \quad (24)$$

In this case and for further convenience, we split the $\theta_1^{n\mathrm{T}}$ into two parts as follows:

$$\theta_1^{n\mathrm{T}}\left(E, E_2\right) = \theta_1^{n\mathrm{TD}} + \theta_1^{n\mathrm{TND}}, \quad (25)$$

where

$$\theta_1^{n\mathrm{TD}}\left(E, E_2\right) = -2\ln\left|\frac{y_0+1}{y_0-1}\right|, \quad (26)$$

the equations for the $\theta_1^{n\mathrm{TND}}\left(E, E_2\right)$ and θ_{17}^{T} are included in the Appendix III.

4. Energy spectrum

Pursuing the energy spectrum, the following integration has to be performed

$$\Gamma_B^s(E) = \int_{M_2}^{E_2^t} \frac{d\Gamma(A^s \to Be\nu\gamma)}{dE}. \quad (27)$$

As we have pointed out, the $d\Gamma_B^s(A^s \to Be\nu\gamma)$ in Eq. (14) depends on y_0 through $\theta_1^{s\mathrm{T}}$ in the following way

$$\theta_1^{s\mathrm{T}} \propto \ln\left|\frac{y_0+1}{y_0-1}\right|.$$

Using the definition of y_0, in Eq. (22) one finds that $y_0 \to 1$ for collinear events, *i.e.* the cases when

$$m \leq E \leq E_c, \quad E_2 \to E_2^t. \quad (28)$$

In this special situation, one has to deal with a logarithmical divergence and this divergence is an obstacle to make a direct numerical integration. We solved this difficulty by considering an analytical integration. Our procedure is the following. For simplicity, we consider a new variable z and the parameters a_0 and b_0 such that

$$a_0 = \frac{M_1^2 + M_2^2 + m^2 - 2EM_1}{2M_2E\beta}, \quad b_0 = \frac{E - M_1}{E\beta}, \quad (29)$$

$$z = \frac{E_2}{M_2}, \quad s = \sqrt{z^2 - 1}, \quad z_t^B = \frac{E_2^t}{M_2}, \quad z_b^B = 1, \quad (30)$$

and

$$\frac{(y_0+1)}{(y_0-1)} = \frac{a_0 + b_0 z + s}{a_0 + b_0 z - s}. \quad (31)$$

For the *charged HSD* process we split $\Gamma_B^c(E)$ into the divergent I_C^{TD} and non-divergent I_C^{TND} part

$$\Gamma_B^c(E) = \frac{G_v^2}{2}\frac{1}{2\pi^3}M_1\left[I_C^{\mathrm{TD}}(E) + I_C^{\mathrm{TND}}(E)\right], \quad (32)$$

where

$$I_C^{\mathrm{TD}}(E) = \frac{\alpha}{\pi}M_2\int_{z_b^{\mathrm{B}}}^{z_t^{\mathrm{B}}}\sum_{k=0}^{2}\varepsilon_k^c M_2^k z^k \ln\frac{|a_0 + b_0 z + s|}{|a_0 + b_0 z - s|}dz \quad (33)$$

with

$$\varepsilon_k^c = (I_1 - 2)\,\varepsilon_k, \quad (34)$$

and

$$I_C^{\mathrm{TND}}(E) = \frac{\alpha}{\pi}\int_{M_2}^{E_2^t}\left(H_0'\theta_0^{\mathrm{T}} + \sum_{i=2}^{16}H_i'\theta_i^{\mathrm{T}}\right)dE_2. \quad (35)$$

Following the same strategy for the *neutral HSD* process we obtain:

$$\Gamma_{\mathrm{B}}^n(E) = \frac{G_v^2}{2}\frac{1}{2\pi^3}M_1\left[I_N^{\mathrm{TD}}(E) + I_N^{\mathrm{TND}}(E)\right], \quad (36)$$

where

$$I_N^{\mathrm{TD}}(E) = \frac{\alpha}{\pi}M_2\int_{z_b^{\mathrm{B}}}^{z_t^{\mathrm{B}}}\sum_{k=0}^{2}\varepsilon_k^n M_2^k z^k \ln\frac{|a_0 + b_0 z + s|}{|a_0 + b_0 z - s|}dz \quad (37)$$

with

$$\varepsilon_k^n = -2\varepsilon_k \quad (38)$$

and

$$I_N^{\mathrm{TND}}(E) = \frac{\alpha}{\pi}\int_{M_2}^{E_2^t}\left[\left(H_0' + N_0'\right)\theta_0^{\mathrm{T}} + A_{1N}'\theta_1^{n\mathrm{TND}} + \sum_{i=2}^{16}\left(H_i' + N_i'\right)\theta_i^{\mathrm{T}} + N_{17}'\theta_{17}^{\mathrm{T}}\right]dE_2. \quad (39)$$

We observe that in both cases, one has to integrate the same function

$$R_k^{\mathrm{TB}} = \int_{z_b^{\mathrm{B}}}^{z_t^{\mathrm{B}}} z^k \ln\frac{|a_0 + b_0 z + s|}{|a_0 + b_0 z - s|}dz. \quad (40)$$

After performing a very subtle analysis (see Appendix V) or Ref. 11, we obtain the following non-divergent analytical result:

$$R_k^{\mathrm{TB}} = \frac{1}{k+1}\left[z_t^{k+1} - \left(z_t^{\mathrm{B}}\right)^{k+1}\right]T_0^{\mathrm{B}} - \frac{1}{k+1}\left\{\sum_{r=0}^{k}\left[s_t z_t^r - s_t^{\mathrm{B}}\left(z_t^{\mathrm{B}}\right)^r\right]\int_{z=1}^{z_t^{\mathrm{B}}} z^{k-r}\frac{dz}{s}\right\}, \quad (41)$$

where

$$T_0^{\mathrm{B}} = \ln\left|\frac{M_1\left(E_m - E\right)}{E\beta M_2}\right|,$$

$$E_m = \frac{M_1^2 - M_2^2 + m^2}{2M_1}. \quad (42)$$

and

$$z_t = \frac{1}{2}\left[\frac{M_1 - E(1-\beta)}{M_2} + \frac{M_2}{M_1 - E(1-\beta)}\right],$$

$$z_t^{\mathrm{B}} = \frac{1}{2}\left[\frac{M_1 - E(1+\beta)}{M_2} + \frac{M_2}{M_1 - E(1+\beta)}\right],$$

$$s_t = \frac{1}{2}\left[\frac{M_1 - E(1-\beta)}{M_2} - \frac{M_2}{M_1 - E(1-\beta)}\right],$$

$$s_t^{\mathrm{B}} = \frac{1}{2}\left[\frac{M_1 - E(1+\beta)}{M_2} - \frac{M_2}{M_1 - E(1+\beta)}\right]. \quad (43)$$

5. Final results and conclusions

Gathering and refining previous results, we obtain the whole spectrum of events in the FBR.

For the *charged HSD* process by $\sum^-(p_1) \to n(p_2) + e(\ell) + \bar{\nu}_e(p_\nu) + \gamma(k)$,

$$\Gamma_{\mathrm{B}}^{c}(E) = \frac{G_v^2}{2}\frac{1}{2\pi^3}M_1\left[I_C^{\mathrm{TD}}(E) + I_C^{\mathrm{TND}}(E)\right], \quad (44)$$

where

$$I_C^{\mathrm{TD}}(E) = \frac{\alpha}{\pi}\sum_{k=0}^{2} M_2^{k+1}\varepsilon_k^c R_k^{\mathrm{TB}},$$

and

$$I_C^{\mathrm{TND}}(E) = \frac{\alpha}{\pi}\int_{M_2}^{E_2^t}\left(H_0'\theta_0^{\mathrm{T}} + \sum_{i=2}^{16} H_i'\theta_i^{\mathrm{T}}\right)dE_2.$$

For the *neutral HSD* process as the $\Lambda(p_1) \to p(p_2) + e(\ell) + \bar{\nu}_e(p_\nu) + \gamma(k)$,

$$\Gamma_{\mathrm{B}}^{n}(E) = \frac{G_v^2}{2}\frac{1}{2\pi^3}M_1\left[I_N^{\mathrm{TD}}(E) + I_N^{\mathrm{TND}}(E)\right] \quad (45)$$

with

$$I_N^{\mathrm{TD}}(E) = \frac{\alpha}{\pi}\sum_{k=0}^{2} M_2^{k+1}\varepsilon_k^n R_k^{\mathrm{TB}}$$

and

$$I_N^{\mathrm{TND}}(E) = \frac{\alpha}{\pi}\int_{M_2}^{E_2^t}\Big[(H_0' + N_0')\,\theta_0^{\mathrm{T}} + A_{1N}'\theta_1^{n\mathrm{TND}} + \sum_{i=2}^{16}(H_i' + N_i')\,\theta_i^{\mathrm{T}} + N_{17}'\theta_{17}^{\mathrm{T}}\Big]dE_2.$$

The formula for the lepton energy spectrum becomes in general:

$$d\Gamma_{\mathrm{B}}^{s}(E) = \frac{\alpha}{\pi}\frac{G_v^2}{2}\frac{1}{2\pi^3}M_1M_2\Big[\sum_{k=0}^{2}\varepsilon_k^s M_2^k R_k^{\mathrm{TB}} + \int_1^{z_t^{\mathrm{B}}}\left(H_0^{s\prime}\theta_0^{\mathrm{T}} + \sum_{i=2}^{17} H_i^{s\prime}\theta_i^{\mathrm{T}} + A_{1N}'\theta_1^{n\mathrm{TND}}\delta_s^n\right)dz\Big]. \quad (46)$$

TABLE I

$\Sigma^-(p_1) \to n(p_2) + e^-(\ell) + \bar{\nu}_e(p_\nu) + \gamma(k)$					
$x = E/E_m$	0.1	0.2	0.3	0.4	0.5
% in Ref. 12	7.8	1.5	0.5	0.1	0.02
% from Eq. (46)	7.78	1.53	0.46	0.14	0.02

TABLE II

$\Lambda(p_1) \to p^+(p_2) + e^-(\ell) + \bar{\nu}_e(p_\nu) + \gamma(k)$					
$x = E/E_m$	0.1	0.2	0.3	0.4	0.5
% in Ref. 12	9.5	2.3	0.8	0.25	0.02
% from Eq. (46)	9.28	2.20	0.77	0.24	0.02

where δ_s^n indicates that the last term appears only in the neutral HSD case ($s = n$). $\varepsilon_k^c = (I_1 - 2)\,\varepsilon_k$, and $\varepsilon_k^n = -2\varepsilon_k$, where the ε_k 's are displayed in the Appendices.

In brief, Eq. (46) is a precise formula suitable to be evaluated numerically, without any ambiguity, at any energy in which the charged lepton is emitted, it contains the bremsstrahlung in the four body decay region where the three body decay does not take place and it includes events in which the electron is collinear to the produced hadron (at the edge of the DP). Other authors [12] have published numerical data for the percent contributions due to the *radiative corrections* to the HSD decay for the events at the FBR.

In order to compare with these results, we consider the numerical values obtained with the formula in Eq. (46). In Tables I and II we compare the data given in Ref. 12 for the relative RC in %, caused by bremsstrahlung events, which fall outside the TBR Dalitz plot, with the numerical values obtained by means of Eq. (46). As one can see, the results are in a very good agreement, except for the low energy region in Table II.

In summary, the analytical result is useful to obtain information, from the experimental data, about the underlaying interactions in the decay processes, the basic symmetries, and the internal structure of hadrons, through the derivation of precise values of the form factors involved in the effective interaction.

The knowledge of the energy spectrum of the electron is fundamental for the determination of the decay rate in these processes. For the complete determination of the decay rate it is necessary to add the contributions of the events in the so called three body and in the four body regions. In Ref. 10 we consider the values given in Ref. 12, since the analytical results for the FBR were not available then.

Let us mention that RC were also computed by a Monte Carlo method for photon bremsstrahlung calculations in semileptonic decays Ref. 13. At last, though the evaluation of the *radiative corrections* for the HSD is a complex and an old problem (see list of references in Ref. 13), the result in

Eq. (46) is new, it is worth by itself and it is the culmination of a systematical approach to the problem in the FBR.

Acknowledgments

We are grateful to Professor P. Kielanowski for reading the manuscript and providing valuable suggestions. We also acknowledge partial support by the Comisión de Operación y Fomento de Actividades Académicas (Instituto Politécnico Nacional), and by the Consejo Nacional de Ciencia y Tecnología (CONACYT)-México

Appendix I

The coefficients Q_i's $(i = 1, \ldots, 5)$ which are form factors functions, are given in Ref. 1.

In the H_1' expansion in terms of E_2 we consider the approximation in which $Q_5 = 0$, then

$$H_1' = A_{1N}' = \sum_{k=0}^{2} \varepsilon_k E_2^k,$$

where the ε_k are explicitly given by

$$\begin{aligned}\frac{\varepsilon_0}{M_1^2} &= \left(F_1^2 - G_1^2 + F_1F_2 + G_1G_2 + \frac{F_2^2 - G_2^2}{2}\right)\left[\frac{1}{2}M^- + 2\frac{E}{M_1}\left(1 - \frac{E}{M_1}\right)\right]\frac{M_2}{M_1} - 2\left(F_1^2 - G_1^2\right)\left(1 - \frac{E}{M_1}\right)\frac{M_2E}{M_1^2}\\ &+ \left[(F_1 + G_1)^2 + F_1F_2 - G_1G_2\right]\left\{-\frac{1}{2}M^+ + \frac{E}{M_1}\left[2\left(1 - \frac{E}{M_1}\right) + \frac{m^2}{M_1^2}\right]\right\} + F_1G_1\frac{4E}{M_1}\left[-\frac{1}{2}M^+ + \left(\frac{M_2^2}{M_1^2} + \frac{E}{M_1}\right)\right]\\ &+ (F_1F_3 + F_2F_3 + G_1G_3 - G_2G_3)\left(1 - \frac{E}{M_1}\right)\frac{m^2M_2}{M_1^3} + (F_1F_3 - G_1G_3)\left(1 - \frac{E}{M_1}\right)\frac{m^2}{M_1^2},\\ \frac{\varepsilon_1}{M_1} &= \left(F_1F_2 + G_1G_2 + \frac{F_2^2 - G_2^2}{2}\right)\left(1 - \frac{2E}{M_1}\right)\frac{M_2}{M_1} + \left(F_1^2 - G_1^2\right)\frac{M_2}{M_1} + (F_2F_3 + G_2G_3)\left(1 - \frac{2E}{M_1}\right)\frac{m^2}{2M_1^2}\\ &+ \left(F_2^2 + G_2^2\right)\left(1 - \frac{E}{M_1}\right)\frac{E}{M_1} + \left[(F_1 + G_1)^2 + F_1F_2 - G_1G_2\right]\left[\frac{1}{2}M^+ + \left(1 - 2\frac{E}{M_1}\right)\right]\\ &+ \left(F_1F_2 - G_1G_2 + \frac{F_2^2 + G_2^2}{2}\right)\frac{1}{2}M^- - (F_1F_3 + F_2F_3 + G_1G_3 - G_2G_3)\frac{m^2M_2}{M_1^3}\\ &+ \left[F_2F_3 + G_2G_3 + 2\left(G_1G_3 - F_1F_3\right)\right]\frac{m^2}{2M_1^2},\\ \varepsilon_2 &= \left(\frac{F_2^2 + G_2^2}{2}\right)\left(1 - \frac{2E}{M_1}\right) - (F_1 + G_1)^2 - (F_2F_3 + G_2G_3)\frac{m^2}{M_1^2},\end{aligned}$$

where terms of $O(q^2/M_1^2)$ were neglected and

$$M^- = \frac{m^2 - M_2^2 - M_1^2}{M_1^2} \qquad \text{and} \qquad M^+ = \frac{m^2 + M_2^2 + M_1^2}{M_1^2}.$$

Appendix II

For completeness, we explicitly show the new simplified expressions for the form factor dependent coefficients H_i' s

$$\begin{aligned}H_0' &= E\beta\,|\vec{p}_2|\left\{\frac{1}{2}\left(Q_3 - Q_4E_\nu^0\right) - \frac{E}{2M_1}\left[(f_1 + g_1)^2 + 4f_2g_1 + 2\left(f_1f_3 - g_1g_2\right)\right]\right\},\\ H_1' &= A_{1N}' = E\left[E_\nu^0Q_1 - Q_2\,|\vec{p}_2|^2 - Q_3\beta^2E + \beta\,|\vec{p}_2|\,y_0\left(E_\nu^0Q_4 - Q_3 - EQ_2\right)\right],\\ H_2' &= \frac{m^2\beta}{2E}\left[-\left(Q_1 + Q_3\right)E_\nu^0 + \left(Q_2 + Q_4\right)E\,|\vec{p}_2|\,\beta y_0 + Q_4\left(E + E_\nu^0\right)^2 + Q_2\,|\vec{p}_2|^2\right],\\ H_3' &= \beta\frac{E}{4}\left\{\left[2E_\nu^0 - E\left(1 + \beta^2\right)\right](Q_1 + Q_3) + 2\beta\,|\vec{p}_2|\,y_0\left(E_\nu^0Q_4 - Q_3 - EQ_2\right) - 2\,|\vec{p}_2|^2Q_2\right.\\ &\qquad\left. + \left(E_\nu^0 + E\right)\left[E\left(1 + \beta^2\right)\left(Q_2 + 3Q_4\right) - 2\left(Q_3 + 2EQ_4\right)\right]\right\}\\ &\qquad + m^2\beta\left\{\frac{h^+}{e_i}\left(E_\nu^0 + 2E\right) + \left[{f_1}^2 + {g_1}^2 + 2\left(f_1f_3 - g_1g_2\right)\right]\frac{E_\nu^0}{2M_1} - g_1\left(f_1 + f_2 + g_2\right)\frac{\left(2E + E_\nu^0\right)}{M_1}\right\},\end{aligned}$$

$$H_4' = \frac{E\beta}{2}\left\{\frac{1}{2}E\left[Q_1 - E_\nu^0 Q_2 - E\left(Q_2+Q_4\right) + 3\left(Q_3 - E_\nu^0 Q_4\right)\right] + \frac{1}{2}\left[-E^2\beta^2 Q_2 + \left|\vec{p}_2\right|^2 Q_4 + E_\nu^0\left(2Q_3 - 3E_\nu^0 Q_4\right)\right]\right.$$
$$-\frac{EE_\nu^0}{M_1}\left[\left(f_1-g_1\right)^2 + 2f_1f_3\right] + \frac{1}{M_1}\left[f_1^2 - g_1^2 + 2\left(g_1g_2 + f_1f_3\right)\right]\left|\vec{p}_2\right|\beta E y_0 + \frac{2E}{M_1}g_1g_2\left[2E_\nu^0 + E\left(4-3\beta^2\right)\right]$$
$$\left.+\frac{2E}{M_1}g_1\left[2E\left(f_1+f_2-g_2\right) + f_2\left(E_\nu^0 + \beta^2 E\right)\right] + \frac{2h^-}{e_i}\beta^2E^2 - 2E\left[E_\nu^0 + 2E\left(1-\beta^2\right)\right]\frac{h^+}{e_i}\right\},$$

$$H_5' = \frac{E^2\beta^2}{4}\left\{Q_1 - \left(E+E_\nu^0\right)Q_2 + 3Q_3 - \left(3E+7E_\nu^0\right)Q_4 - 4\left(2E+E_\nu^0\right)\frac{h^+}{e} - 8E_\nu^0\frac{h^-}{e}\right.$$
$$\left.+\frac{4}{M_1}\left[E_\nu^0\left(f_1^2+2f_1f_3\right) + \left(2E-E_\nu^0\right)\left(f_1+f_2\right)g_1 + \left(2E+3E_\nu^0\right)g_1g_2\right]\right\},$$

$$H_6' = \frac{1}{4}\left|\vec{p}_2\right|\left(1-\beta^2\right)E\beta\left[Q_1+Q_3 - \left(Q_2+Q_4\right)\left(E_\nu^0+E\right)\right],$$

$$H_7' = -\frac{E\beta\left|\vec{p}_2\right|}{4}\left\{\frac{\left(2E-E_\nu^0\right)}{E}\left(Q_1+Q_3\right) + \frac{\left|\vec{p}_2\right|^2}{E}\left(Q_2+Q_4\right) + 2\left(\beta^2-1\right)EQ_4 + \left|\vec{p}_2\right|\beta y_0\left(Q_2+3Q_4-2\frac{h^+}{e}\right)\right.$$
$$-2\left(E_\nu^0 + E\beta^2\right)\frac{h^+}{e} + \left[\left(\beta^2-3\right)E - 2E_\nu^0\right]Q_2 + \frac{E}{M_1}\left(1-\beta^2\right)\left[f_1^2 + g_1^2 - g_1\left(2f_1+3f_2+g_2\right)\right]$$
$$\left.+g_1\left(f_2-g_2\right)\left(\frac{M_2}{E}\right)\left(\frac{2E_2-M_1}{M_2} - \frac{M_2}{M_1}\right) + \frac{2m^2}{EM_1}\left(f_1f_3 - g_1g_2\right)\right\},$$

$$H_8' = \frac{E\beta\left|\vec{p}_2\right|}{4}\left\{Q_1+Q_3 - \left(2E+E_\nu^0\right)Q_2 + \left(E_\nu^0-E\right)Q_4 + 2E_\nu^0\left(\frac{2h^- - h^+}{e}\right) + \left[\left(f_1-g_1\right)^2 + 2f_1f_3\right]\left(\frac{E-2E_\nu^0}{M_1}\right)\right.$$
$$\left.+\frac{g_1}{M_1}\left[\left(2f_2+g_2\right)\left(E_\nu^0 - 2E\right) + g_2E_\nu^0\right]\right\},$$

$$H_9' = \frac{\left|\vec{p}_2\right|\beta}{8}\left[-\left(Q_1+Q_3\right) + \left(Q_2+Q_4\right)\left(E+E_\nu^0\right)\right],$$

$$H_{10}' = \frac{\left(E\beta\right)^3}{4}\left\{-\left(Q_2+5Q_4\right) - 4\left(\frac{3h^- + 2h^+}{e}\right) + \frac{2}{M_1}\left[3f_1\left(f_1+2f_3\right) + g_1\left(g_1 - 4f_1 - 6f_2 + 8g_2\right)\right]\right\},$$

$$H_{11}' = 0,$$

$$H_{12}' = \left(E\beta\right)^2\left|\vec{p}_2\right|\left\{\frac{h^+}{e} - \frac{Q_4}{2} + \frac{1}{2M_1}\left[\left(f_1+g_1\right)^2 + 2g_1\left(3f_2 - 2g_2\right) + 2f_1f_3\right]\right\},$$

$$H_{13}' = \frac{\left(E\beta\right)^2\left|\vec{p}_2\right|^2}{2}\left\{Q_4 - 2\frac{h^+}{e} + \frac{1}{M_1}\left[2f_2\left(f_1-g_1\right) + g_1^2 - f_1^2 - 2f_2^2 - 2f_1f_3\right]\right\},$$

$$H_{14}' = \frac{\left(E\beta\right)^2\left|\vec{p}_2\right|}{4M_1}\left[\left(f_1-g_1\right)^2 - 4f_2g_1 + 2\left(f_1f_3 - g_1g_2\right) - M_1Q_2\right],$$

$$H_{15}' = \frac{E\beta\left|\vec{p}_2\right|}{8}\left\{-\left(Q_2+Q_4\right) - 4\frac{h^-}{e} + \frac{2}{M_1}\left[\left(f_1-g_1\right)^2 - 2f_2g_1 + 2f_1f_3\right]\right\},$$

$$H_{16}' = \frac{\left|\vec{p}_2\right|}{4M_1}\beta\left[-M_1\frac{h^+}{e} + g_1\left(g_2-f_2\right)\right],$$

where we have used that

$$h^{\pm} = -g_1^2\left(\kappa_1+\kappa_2\right) \pm f_1g_1\left(\kappa_2-\kappa_1\right).$$

Appendix III

Simplified analytical results obtained for the $\theta_i^{s\mathrm{T}}$'s.

The $\theta_1^{c\mathrm{T}}$ and $\theta_1^{n\mathrm{T}}\left(E, E_2\right)$ are given in Eqs. (20) and (25) respectively,

$$\theta_0^{\mathrm{T}} = 2\left(I_1-2\right), \qquad \theta_1^{c\mathrm{T}} = \left(I_1-2\right)\ln\left|\frac{y_0+1}{y_0-1}\right|, \qquad \text{and} \qquad \theta_i^{c\mathrm{T}} = \theta_i^{n\mathrm{T}} = \theta_i^{\mathrm{T}}, \qquad \text{for} \qquad i = 0, 2, \ldots 17.$$

The $\theta_1^{n\mathrm{TND}}$ in Eqs. (25) is the following

$$\theta_1^{n\mathrm{TND}} = \frac{1}{2}\left(\ln^2 v_{\max}^{+} - \ln^2 v_{\min}^{+}\right) - \frac{1}{2}\left(\ln^2 v_{\min}^{-} - \ln^2 v_{\max}^{-}\right) - \frac{1}{\beta_N}\left[I_1^n - I_2^n + I_3^n - I_4^n\right] + \frac{1}{\beta_N}\left[\ln v^{+}\ln\left|\frac{v^{+} - a\,(1+\beta_N)}{v^{+} - a\,(1-\beta_N)}\right|\,\Bigg|_{v^{+}=v_{\min}^{+}}^{v^{+}=v_{\max}^{+}} + \ln v^{-}\ln\left|\frac{v^{-} - a\,(1+\beta_N)}{v^{-} - a\,(1-\beta_N)}\right|\,\Bigg|_{v^{-}=v_{\min}^{-}}^{v^{-}=v_{\max}^{-}}\right] \tag{47}$$

The θ_i^{T} for $i = 2, \ldots, 17$ are:

$$\theta_2^{\mathrm{T}} = \frac{1}{\beta}\left[\frac{I_2^{-}}{1+\beta a^{-}} - \frac{I_2^{+}}{1+\beta a^{+}} + \frac{E^2}{m^2}\left(I_2^{+} - I_2^{-} + \beta\ln\left|\frac{I_3^{-}}{I_3^{+}}\right|\right)\right] + \frac{2I_1}{E\,(1+\beta a^{-})\,(1+\beta a^{+})},$$

$$\theta_3^{\mathrm{T}} = I_1\ln\left|\frac{1+\beta a^{+}}{1+\beta a^{-}}\right| + \frac{1}{\beta}\left\{L\left[\frac{1-\beta}{1+\beta a^{-}}\right] - L\left[\frac{1-\beta}{1+\beta a^{+}}\right] + L\left[\frac{1+\beta}{1+\beta a^{+}}\right] - L\left[\frac{1+\beta}{1+\beta a^{-}}\right]\right\},$$

$$\theta_4^{\mathrm{T}} = a^{+}I_2^{+} - a^{-}I_2^{-} + \ln\left|\frac{I_3^{-}}{I_3^{+}}\right|, \qquad \theta_5^{\mathrm{T}} = \frac{1}{2}\left\{\left[1-(a^{+})^2\right]I_2^{+} - \left[1-(a^{-})^2\right]I_2^{-} + 4\frac{|\vec{p}_2|}{E\beta}\right\},$$

$$\theta_6^{\mathrm{T}} = 2\frac{(y_0 - a^{-})}{(1+\beta a^{-})^2}\left(I_2^{-} + \beta I_1\right) - 2\frac{(y_0 + a^{+})}{(1+\beta a^{+})^2}\left(I_2^{+} + \beta I_1\right) + 2\left[2 + \beta\left(\frac{y_0 - a^{-}}{1+\beta a^{-}} - \frac{y_0 + a^{+}}{1+\beta a^{+}}\right)\right]I_4,$$

$$\theta_7^{\mathrm{T}} = 2\left[2I_1 + \frac{y_0 - a^{-}}{1+\beta a^{-}}\left(\beta I_1 + I_2^{-}\right) - \frac{y_0 + a^{+}}{1+\beta a^{+}}\left(\beta I_1 + I_2^{+}\right)\right], \qquad \theta_8^{\mathrm{T}} = 2\left[4 + \left(y_0 - a^{-}\right)I_2^{-} - \left(y_0 + a^{+}\right)I_2^{+}\right],$$

$$\theta_9^{\mathrm{T}} = 24\,E + 2\left[6\left(E_\nu^0 - E\right) + \beta\left(G^{T-} + G^{T+}\right)\right]I_1 + 2\left(G^{T-}I_2^{-} + G^{T+}I_2^{+}\right) + 2\,|\vec{p}_2|\left[\frac{\left(y_0 - a^{-}\right)^2 I_3^{-}}{1+\beta a^{-}} - \frac{\left(y_0 + a^{+}\right)^2 I_3^{+}}{1+\beta a^{+}}\right],$$

$$\theta_{10}^{\mathrm{T}} = \frac{1}{3}\left\{2\left[(a^{-})^2 - (a^{+})^2\right] - (a^{-})^3 I_2^{-} + (a^{+})^3 I_2^{+} + \ln\left|\frac{I_3^{-}}{I_3^{+}}\right|\right\}, \qquad \theta_{11}^{\mathrm{T}} = 2\left(I_4 - I_1\right)\frac{1}{\beta},$$

$$\theta_{12}^{\mathrm{T}} = \frac{1}{\beta}\theta_0^{\mathrm{T}}, \qquad \theta_{13}^{\mathrm{T}} = 0, \qquad \theta_{14}^{\mathrm{T}} = 2\left[\left(2 - a^{-}I_2^{-}\right)\left(y_0 - a^{-}\right) - \left(2 - a^{+}I_2^{+}\right)\left(y_0 + a^{+}\right)\right],$$

$$\theta_{15}^{\mathrm{T}} = 24E_\nu^0 + 4\beta E\left[a^{-}\left(y_0 - a^{-}\right)I_2^{-} - a^{+}\left(y_0 + a^{+}\right)I_2^{+}\right] + 2\,|\vec{p}_2|\left[\left(y_0 - a^{-}\right)^2 I_3^{-} - \left(y_0 + a^{+}\right)^2 I_3^{+}\right],$$

$$\theta_{16}^{\mathrm{T}} = 24E^2\left(I_1 - 2\right) + 8\left(E_\nu^{0\,2} - 2E^2\beta^2\right)I_1 + 4E\beta\,|\vec{p}_2|\left[\frac{\left(y_0 - a^{-}\right)^2}{1+\beta a^{-}}\left(\beta I_1 + I_2^{-}\right) - \frac{\left(y_0 + a^{+}\right)^2}{1+\beta a^{+}}\left(\beta I_1 + I_2^{+}\right)\right],$$

$$\theta_{17}^{\mathrm{T}} = 2I_1, \qquad \text{and} \qquad a^{\pm} = \frac{E_\nu^0 \pm |\vec{p}_2|}{E\beta},$$

I_1, $I_2^{\pm}$, $I_3^{\pm}$, I_4, and $G^{\mathrm{T}\pm}$ are given by

$$I_1 = \frac{2}{\beta}\mathrm{arctanh}\beta, \qquad I_2^{\pm} = \ln\left|\frac{1+a^{\pm}}{a^{\pm}-1}\right|,$$

$$I_3^{\pm} = \frac{2}{(a^{\pm})^2 - 1}, \qquad I_4 = \frac{2}{1-\beta^2},$$

$$G^{\mathrm{T}\pm} = \mp\beta\left[\frac{2Ea^{\pm}\left(y_0 \pm a^{\pm}\right)}{\left(1+\beta a^{\pm}\right)} + \frac{|\vec{p}_2|\left(y_0 \pm a^{\pm}\right)^2}{\left(1+\beta a^{\pm}\right)^2}\right],$$

$$v_{\max}^{\pm} = 2\left(E_2 \pm |\vec{p}_2|\right)\left(E \pm |\vec{\ell}|\right),$$

$$v_{\min}^{\pm} = 2\left\{EE_2 - |\vec{p}_2|\,\left|\vec{\ell}\right| \pm \left|E_2\,|\vec{\ell}| - |\vec{p}_2|\,E\right|\right\},$$

$$a = M_1^2 - H^2 - q^2, \quad H^2 = \left(p_1 - \ell\right)^2,$$

$$\beta_N = \left(1 - \frac{4m^2M_2^2}{a^2}\right)^{1/2}, \quad q^2 = \left(p_1 - p_2\right)^2.$$

The I_i^n's, in Eq. (47) are defined in terms of the Heaviside function

$$\theta(x) = \begin{cases} 1 & \text{if} \quad x \geq 0 \\ 0 & \text{if} \quad x < 0 \end{cases}$$

Then for $i = 1, \ldots, 4$

$$I_i^n = I_{iA}\theta\left[r_{Ai}\right] + I_{iB}\theta\left[r_{Bi}\right]\theta\left(r'_{Bi}\right) + I_{iC}\theta\left(r_{Ci}\right),$$

where

$$I_{1A,2A} = L\left[\frac{a\,(1\pm\beta_N)}{v_{\min}^{+}}\right] - L\left[\frac{a\,(1\pm\beta_N)}{v_{\max}^{+}}\right] + \frac{1}{2}\left(\ln^2 v_{\max}^{+} - \ln^2 v_{\min}^{+}\right),$$

$$I_{1B,2B} = \frac{-\pi^2}{3} - L\left[\frac{v_{\min}^{+}}{a\,(1\pm\beta_N)}\right] - L\left[\frac{a\,(1\pm\beta_N)}{v_{\max}^{+}}\right] + \frac{1}{2}\ln^2\frac{a\,(1\pm\beta_N)}{v_{\min}^{+}} + \frac{1}{2}\left(\ln^2 v_{\max}^{+} - \ln^2 v_{\min}^{+}\right),$$

$$I_{1C,2C} = L\left[\frac{v^{+}_{\max}}{a(1\pm\beta_N)}\right] - L\left[\frac{v^{+}_{\min}}{a(1\pm\beta_N)}\right] + \ln|a(1\pm\beta_N)|\ln\left|\frac{v^{+}_{\max}}{v^{+}_{\min}}\right|,$$

$$I_{3A,4A} = L\left[\frac{a(1\pm\beta_N)}{v^{-}_{\min}}\right] - L\left[\frac{a(1\pm\beta_N)}{v^{-}_{\max}}\right] - \frac{1}{2}\left(\ln^2 v^{-}_{\min} - \ln^2 v^{-}_{\max}\right),$$

$$I_{3B,4B} = \frac{\pi^2}{3} + L\left[\frac{v^{-}_{\max}}{a(1\pm\beta_N)}\right] + L\left[\frac{a(1\pm\beta_N)}{v^{-}_{\min}}\right] - \frac{1}{2}\ln^2\frac{a(1\pm\beta_N)}{v^{-}_{\max}} + \frac{1}{2}\left(\ln^2 v^{-}_{\max} - \ln^2 v^{-}_{\min}\right),$$

$$I_{3C,4C} = L\left[\frac{v^{-}_{\max}}{a(1\pm\beta_N)}\right] - L\left[\frac{v^{-}_{\min}}{a(1\pm\beta_N)}\right] + \ln|a(1\pm\beta_N)|\ln\left|\frac{v^{-}_{\max}}{v^{-}_{\min}}\right|,$$

and the arguments of the Heaviside function are given by the expressions

$$r_{A1,A2} = -r_{B1,B2} = v^{+}_{\min} - a(1\pm\beta_N),$$
$$r'_{B1,B2} = -r_{C1,C2} = v^{+}_{\max} - a(1\pm\beta_N),$$
$$r_{A3,A4} = -r_{B3,B4} = v^{-}_{\max} - a(1\pm\beta_N),$$
$$r'_{B3,B4} = -r_{C3,C4} = v^{-}_{\min} - a(1\pm\beta_N),$$

The left (right) subindex in the left hand side (LHS) in the former equations corresponds to the upper (lower) sign in the right hand side (RHS) in the same equations. $L(x)$ is the Spence function.

Appendix IV

The simplified coefficients N_i's are

$$N'_0 = -|\vec{p_2}|\frac{E\beta}{2M_1}\left[2\left(E - E^0_\nu\right)R^+ + \left(E + 2E^0_\nu\right)R^- + (1-y_0)\frac{|\vec{p_2}|}{2\beta}R^-\right],$$

$$N'_2 = N'_6 = N'_9 = N'_{11} = N'_{15} = 0,$$

$$N'_3 = \frac{E\beta m^2}{M_1}R^+,$$

$$N'_4 = -\frac{E\beta}{2M_1}\left[2E^2R^+ + E\beta\left(E\beta + 4|\vec{p_2}|y_0\right)R^-\right],$$

$$N'_5 = -\frac{(E\beta)^2}{M_1}\left[ER^+ + 2E^0_\nu R^-\right],$$

$$N'_7 = |\vec{p_2}|\frac{\beta}{2M_1}\left[2m^2 + EE^0_\nu(1-\beta x_0)\right]R^+,$$

$$N'_8 = -|\vec{p_2}|\frac{E\beta}{2M_1}\left(2E + E^0_\nu\right)R^+,$$

$$N'_{10} = -\frac{3(E\beta)^3}{2M_1}R^-,$$

$$N'_{12} = |\vec{p_2}|\frac{E\beta^2}{M_1}\left(2E - E^0_\nu\right)R^+,$$

$$N'_{13} = -|\vec{p_2}|^2\frac{(E\beta)^2}{2M_1}\left(2R^+ - R^-\right),$$

$$N'_{14} = -|\vec{p_2}|\frac{(E\beta)^2}{M_1}R^+,$$

$$N'_{16} = -|\vec{p_2}|\frac{\beta}{4M_1}R^+,$$

$$N'_{17} = |\vec{p_2}|\frac{E\beta}{4M_2}\left[2E^0_\nu + (1-y_0)|\vec{p_2}|\beta\right]R^-,$$

and

$$R^\pm = |f_1|^2 \pm |g_1|^2.$$

Appendix V

The procedure to integrate Eq. (40) is the following:

$$R^{\rm TB}_k = R^{\rm B}_k\left(z^{\rm B}_t\right) - R^{\rm B}_k(1) = \int_{z^{\rm B}_b=1}^{z^{\rm B}_t} z^k \ln\left|\frac{a_0 + b_0 z + s}{a_0 + b_0 z - s}\right| dz.$$

Integrating by parts

$$\int z^k \ln|a_0 + b_0 z \pm s|\,dz = \frac{z^{k+1}}{k+1}\ln|a_0 + b_0 z \pm s| - \frac{1}{k+1}\int\frac{z^{k+1}}{\sqrt{z^2-1}}\left[\frac{b_0\sqrt{z^2-1}\pm z}{(a_0+b_0z)+\sqrt{z^2-1}}\right]dz,$$

then

$$R^{\rm B}_k(z) = \frac{z^{k+1}}{k+1}\ln\left|\frac{a_0 + b_0 z + s}{a_0 + b_0 z - s}\right| + I^+_k(z) - I^-_k(z),$$

where

$$I^\pm_k(z) = -\frac{1}{k+1}\int\frac{z^{k+1}}{\sqrt{z^2-1}}\left[\frac{b_0\sqrt{z^2-1}\pm z}{(a_0+b_0z)\pm\sqrt{z^2-1}}\right]dz.$$

$$I_k(z) = I^+_k(z) - I^-_k(z) = -\frac{1}{k+1}\int z^{k+1}b_0\left[\frac{-2\sqrt{z^2-1}}{(a_0+b_0z)^2-(z^2-1)}\right]dz - \frac{1}{k+1}\int\frac{z^{k+2}}{\sqrt{z^2-1}}\left[\frac{2(a_0+b_0z)}{(a_0+b_0z)^2-(z^2-1)}\right]dz.$$

To simplify

$$I_k(z) = \frac{2b_0}{k+1}\int \frac{z^{k+1}}{\sqrt{z^2-1}}\left(\frac{z^2-1}{az^2+bz+c}\right)dz - \frac{1}{k+1}\int \frac{z^{k+2}}{\sqrt{z^2-1}}\left[\frac{2(a_0+b_0z)}{az^2+bz+c}\right]dz$$

where

$$(a_0+b_0z)^2-\left(z^2-1\right)^2 = az^2+bz+c,$$

and

$$a=\left(b_0^2-1\right),\quad b=2a_0b_0,\quad c=a_0^2+1.$$

Then

$$I_k(z) = \frac{2b_0}{k+1}\int \frac{-z^{k+1}}{az^2+bz+c}\frac{dz}{s} - \frac{2a_0}{k+1}\int \frac{z^{k+2}}{az^2+bz+c}\frac{dz}{s},$$

where

$$az^2+bz+c = a\left(z-z_t\right)\left(z-z_t^{\mathrm{B}}\right).$$

The z_t and z_t^{B} are given in Eqs. (43). Now we find that

$$I_k(z) = \frac{2}{k+1}\int\left[\frac{-b_0}{a}\frac{1}{\left(z-z_t\right)\left(z-z_t^{\mathrm{B}}\right)}\right]z^{k+1}\frac{dz}{s} + \frac{2}{k+1}\int\left[-\frac{a_0}{a}\frac{z}{\left(z-z_t\right)\left(z-z_t^{\mathrm{B}}\right)}\right]z^{k+1}\frac{dz}{s}$$

and

$$I_k(z) = -\frac{2}{k+1}\frac{1}{\Delta_z}\int\left[\left(\frac{b_0}{a}+\frac{a_0}{a}z_t\right)\frac{1}{z-z_t}\right]z^{k+1}\frac{dz}{s} + \frac{2}{k+1}\frac{1}{\Delta_z}\int\left[\left(\frac{b_0}{a}+\frac{a_0}{a}z_t^{\mathrm{B}}\right)\frac{1}{z-z_t^{\mathrm{B}}}\right]z^{k+1}\frac{dz}{s}.$$

To simplify we use

$$\Delta_z = \left(z_t - z_t^{\mathrm{B}}\right),\quad \left(\frac{b_0}{a}+\frac{a_0}{a}z_t\right) = \frac{1}{2}s_t\Delta_z,$$

and

$$\left(\frac{b_0}{a}+\frac{a_0}{a}z_t^{\mathrm{B}}\right) = \frac{1}{2}s_t^{\mathrm{B}}\Delta_z,$$

then

$$I_k(z) = \frac{1}{k+1}\int\left(\frac{s_t}{z_t-z}-\frac{s_t^{\mathrm{B}}}{z_t^{\mathrm{B}}-z}\right)z^{k+1}\frac{dz}{s},$$

$$I_k(z) = \frac{1}{k+1}\int D_{\mathrm{B}}^{(k)}\frac{dz}{s},$$

where

$$D_{\mathrm{B}}^{(k)}(z) = z^{k+1}\left[\frac{s_t}{(z_t-z)}-\frac{s_t^{\mathrm{B}}}{z_t^{\mathrm{B}}-z}\right].$$

After performing several algebraic operations in order to get simple integrals, we obtain that

$$D_{\mathrm{B}}^{(k)}(z) = \left[\frac{s_t z_t^{k+1}}{z_t-z}-\frac{s_t^{\mathrm{B}}\left(z_t^{\mathrm{B}}\right)^{k+1}}{z_t^{\mathrm{B}}-z}\right] - \sum_{r=0}^{k}\left[s_t z_t^r - s_t^{\mathrm{B}}\left(z_t^{\mathrm{B}}\right)^r\right]z^{k-r}.$$

Therefore

$$R_k^{\mathrm{B}}(z) = \frac{z^{k+1}}{k+1}\ln\left|\frac{a_0+b_0z+\sqrt{z^2-1}}{a_0+b_0z-\sqrt{z^2-1}}\right| + \frac{1}{k+1}\int\left[\frac{s_t z_t^{k+1}}{z_t-z}-\frac{s_t^{\mathrm{B}}\left(z_t^{\mathrm{B}}\right)^{k+1}}{z_t^{\mathrm{B}}-z}\right]\frac{dz}{s} - \frac{1}{k+1}\int\sum_{r=0}^{k}\left[s_t z_t^r - s_t^{\mathrm{B}}\left(z_t^{\mathrm{B}}\right)^r\right]z^{k-r}\frac{dz}{s}.$$

Now

$$R_k^{\mathrm{TB}} = R_k^{\mathrm{B}}\left(z_t^{\mathrm{B}}\right) - R_k^{\mathrm{B}}(1) = \frac{z^{k+1}}{k+1}\ln\left|\frac{a_0+b_0z+\sqrt{z^2-1}}{a_0+b_0z-\sqrt{z^2-1}}\right|\Bigg|_{z=1}^{z_t^{\mathrm{B}}} + \frac{1}{k+1}s_t z_t^{k+1}\int_{z=1}^{z_t^{\mathrm{B}}}\frac{1}{s}\frac{dz}{z_t-z} - \frac{1}{k+1}s_t^{\mathrm{B}}\left(z_t^{\mathrm{B}}\right)^{k+1}\int_{z=1}^{z_t^{\mathrm{B}}}\frac{1}{s}\frac{dz}{z_t^{\mathrm{B}}-z} - \frac{1}{k+1}\left\{\sum_{r=0}^{k}\left[s_t z_t^r - s_t^{\mathrm{B}}\left(z_t^{\mathrm{B}}\right)^r\right]\int_{z=1}^{z_t^{\mathrm{B}}} z^{k-r}\frac{dz}{s}\right\}.$$

Recalling that

$$I_2^t(z) = \int\frac{dz}{(z_t-z)\sqrt{z^2-1}} = \mp\frac{1}{\sqrt{z_t^2-1}}\ln\left|\frac{z_t\,z-1\mp\sqrt{z_t^2-1}\sqrt{z^2-1}}{z_t-z}\right|,$$

and

$$I_2^t(z_t^{\mathrm{B}}) - I_2^t(1) = \mp\frac{1}{\sqrt{z_t^2-1}}\ln\left|\frac{z_t z_t^{\mathrm{B}}-1\mp s_t^{\mathrm{B}}s_t}{z_t-z_t^{\mathrm{B}}}\right|,$$

then

$$R_k^{\mathrm{TB}} = \frac{\left(z_t^{\mathrm{B}}\right)^{k+1}}{k+1}\left(\ln\left|a_0+b_0z_t^{\mathrm{B}}+s_t^{\mathrm{B}}\right| - \ln\left|a_0+b_0z_t^{\mathrm{B}}-s_t^{\mathrm{B}}\right|\right) \mp \frac{z_t^{k+1}}{k+1}\ln\left|\frac{z_t z_t^{\mathrm{B}}-1\mp s_t^{\mathrm{B}}s_t}{z_t-z_t^{\mathrm{B}}}\right| - \frac{1}{k+1}\left[s_t^{\mathrm{B}}\left(z_t^{\mathrm{B}}\right)^{k+1}\int_{z=1}^{z_t^{\mathrm{B}}}\frac{1}{s}\frac{dz}{z_t^{\mathrm{B}}-z}\right] - \frac{1}{k+1}\left\{\sum_{r=0}^{k}\left[s_t z_t^r - s_t^{\mathrm{B}}\left(z_t^{\mathrm{B}}\right)^r\right]\int_{z=1}^{z_t^{\mathrm{B}}} z^{k-r}\frac{dz}{s}\right\},$$

where

$$\int_1^{z_t^{\rm B}} \frac{dz}{\sqrt{z^2-1}} = \ln\left|z_t^{\rm B}+s_t^{\rm B}\right|,$$

$$\int_{z_b^{\rm B}=1}^{z_t^{\rm B}} \frac{z\,dz}{\sqrt{z^2-1}} = s_t^{\rm B},$$

$$\int_1^{z_t^{\rm B}} \frac{z^2dz}{\sqrt{z^2-1}} = \frac{z_t^{\rm B}s_t^{\rm B}}{2} + \frac{\ln\left|z_t^{\rm B}+s_t^{\rm B}\right|}{2}.$$

To consider carefully the divergences, we split terms in the following way

$$R_k^{\rm TB} = T_D^{\rm B} + \frac{1}{k+1}\left(z_t^{\rm B}\right)^{k+1}\ln\left|a_0+b_0z_t^{\rm B}+s_t^{\rm B}\right| \mp \frac{z_t^{k+1}}{k+1}\ln\left|\frac{z_tz_t^{\rm B}-1\mp s_t^{\rm B}s_t}{z_t-z_t^{\rm B}}\right| - \frac{1}{k+1}\left[\sum_{r=0}^{k}\left(s_tz_t^r - s_t^{\rm B}\left(z_t^{\rm B}\right)^r\right)\int_{z=1}^{z_t^{\rm B}} z^{k-r}\frac{dz}{s}\right],$$

where

$$T_D^{\rm B} = -\frac{1}{k+1}\left(z_t^{\rm B}\right)^{k+1}\left[\left(\ln\left|a_0+b_0z_t^{\rm B}-s_t^{\rm B}\right| + s_t^{\rm B}\int_1^{z_t^{\rm B}}\frac{1}{s}\frac{dz}{z_t^{\rm B}-z}\right)\right],$$

contains the divergent terms. After performing a subtle analysis for $T_D^{\rm B}$ (see next appendix) we obtain the following finite result

$$T_D^{\rm B} = -\frac{1}{k+1}\left(z_t^{\rm B}\right)^{k+1}\ln\left|\frac{2M_1\left(E_m-E\right)}{E\beta M_2}s_t^{\rm B}\right|.$$

As

$$a_0+b_0z_t^{\rm B}+s_t^{\rm B} = 2s_t^{\rm B}, \quad m<E<E_c,$$

then

$$R_k^{\rm TB} = -\frac{1}{k+1}\left(z_t^{\rm B}\right)^{k+1}\ln\left|\frac{M_1\left(E_m-E\right)}{E\beta M_2}\right| \mp \frac{z_t^{k+1}}{k+1}\ln\left|\frac{z_tz_t^{\rm B}-1\mp s_t^{\rm B}s_t}{z_t-z_t^{\rm B}}\right| - \frac{1}{k+1}\left\{\sum_{r=0}^{k}\left[s_tz_t^r - s_t^{\rm B}\left(z_t^{\rm B}\right)^r\right]\int_{z=1}^{z_t^{\rm B}} z^{k-r}\frac{dz}{s}\right\}.$$

We use now the following relation to simplify

$$\frac{z_tz_t^{\rm B}-1+s_ts_t^{\rm B}}{z_t-z_t^{\rm B}} = \frac{\left(E_m-E\right)M_1}{M_2E\beta}.$$

Finally

$$R_k^{\rm TB} = \frac{1}{k+1}\left[z_t^{k+1}-\left(z_t^{\rm B}\right)^{k+1}\right]T_0^{\rm B} - \frac{1}{k+1}\left\{\sum_{r=0}^{k}\left[s_tz_t^r - s_t^{\rm B}\left(z_t^{\rm B}\right)^r\right]\int_1^{z_t^{\rm B}} z^{k-r}\frac{dz}{s}\right\},$$

where

$$T_0^{\rm B} = \ln\left|\frac{M_1\left(E_m-E\right)}{E\beta M_2}\right|,$$

$$E_m = \frac{M_1^2-M_2^2+m^2}{2M_1}.$$

The explicit values for $R_k^{\rm TB}$, in Eq. (41) for $k=0,1,2$ are:

$$R_0^{\rm TB} = \Delta_zT_0^{\rm B} - \Delta_s\ln\left|z_t^{\rm B}+s_t^{\rm B}\right|,$$

$$R_1^{\rm TB} = \frac{1}{2}\left[z_t^2-\left(z_t^{\rm B}\right)^2\right]T_0^{\rm B} - \frac{1}{2}\Delta_s\,s_t^{\rm B} - \frac{1}{2}\left(s_tz_t - s_t^{\rm B}z_t^{\rm B}\right)\ln\left|z_t^{\rm B}+s_t^{\rm B}\right|,$$

$$R_2^{\rm TB} = \frac{1}{3}\left[z_t^3-\left(z_t^{\rm B}\right)^3\right]T_0^{\rm B} - \frac{1}{3}\left(s_tz_t - s_t^{\rm B}z_t^{\rm B} + \Delta_s\frac{z_t^{\rm B}}{2}\right)s_t^{\rm B} - \frac{1}{3}\left(s_tz_t^2 - s_t^{\rm B}\left(z_t^{\rm B}\right)^2 + \frac{\Delta_s}{2}\right)\ln\left|z_t^{\rm B}+s_t^{\rm B}\right|,$$

where $T_0^{\rm B}$ and E_m, are given in Eqs. (42), and s_t, z_t, $s_t^{\rm B}$, $z_t^{\rm B}$, are given in Eqs. (43), and

$$\Delta_z = z_t - z_t^{\rm B} = \frac{2M_1\left(E_m-E\right)E\beta}{M_2M_1^2\left(1-\frac{2E}{M_1}+\frac{m^2}{M_1^2}\right)},$$

$$\Delta_s = s_t - s_t^{\rm B} = \frac{E\beta}{M_2}\left[1+\frac{M_2^2}{M_1^2\left(1-\frac{2E}{M_1}+\frac{m^2}{M_1^2}\right)}\right].$$

Appendix VI

$$T_D^{\rm B} = -\frac{1}{k+1}\left(z_t^{\rm B}\right)^{k+1}\lim_{z\to z_t^{\rm B}}\ln\left|a_0+b_0z-s\right| \pm \frac{1}{k+1}\left(z_t^{\rm B}\right)^{k+1}\lim_{z\to z_t^{\rm B}}\left(\ln\left|\frac{z_t^{\rm B}z-1\mp s_t^{\rm B}s}{z_t^{\rm B}-z}\right|\right),$$

due to

$$I_2(z) = \int\frac{dz}{\left(z_t^{\rm B}-z\right)\sqrt{z^2-1}} = \mp\frac{1}{\sqrt{\left(z_t^{\rm B}\right)^2-1}}\ln\left|\frac{z_t^{\rm B}\,z-1\mp\sqrt{\left(z_t^{\rm B}\right)^2-1}\sqrt{z^2-1}}{z_t^{\rm B}-z}\right|$$

Then

$$T_D^{\rm B} = -\frac{1}{k+1}\left(z_t^{\rm B}\right)^{k+1}\lim_{z\to z_t^{\rm B}}\ln\left|\frac{a_0+b_0z-s}{z_t^{\rm B}-z}\right| - \frac{1}{k+1}\left(z_t^{\rm B}\right)^{k+1}\lim_{z\to z_t^{\rm B}}\left(\ln\left|z_t^{\rm B}z-1+s_t^{\rm B}s\right|\right),$$

and

$$T_D^{\rm B} = -\frac{1}{k+1}\left(z_t^{\rm B}\right)^{k+1} \lim_{z\to z_t^{\rm B}} \ln\left|\frac{a_0 + b_0 z - s}{z_t^{\rm B} - z}\right| - \frac{1}{k+1}\left(z_t^{\rm B}\right)^{k+1} \ln\left|2\left(s_t^{\rm B}\right)^2\right|.$$

Here we apply L'Hospital rule

$$\lim_{z\to z_t^{\rm B}} \left|\frac{a_0 + b_0 z - s}{z_t^{\rm B} - z}\right| = \left|\frac{b_0 - z_t^{\rm B}/s_t^{\rm B}}{-1}\right| = \left|\frac{z_t^{\rm B} - b_0 s_t^{\rm B}}{s_t^{\rm B}}\right|,$$

and substitute

$$z_t^{\rm B} - b_0 s_t^{\rm B} = \frac{M_1\left(E_m - E\right)}{E\beta M_2},$$

to obtain the following result

$$T_D^{\rm B} = -\frac{1}{k+1}\left(z_t^{\rm B}\right)^{k+1} \ln\left|\frac{2M_1\left(E_m - E\right)}{E\beta M_2} s_t^{\rm B}\right|.$$

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