INVESTIGACIÓN REVISTA MEXICANA DE FÍSICA **46** (1) 41–44 FEBRERO 2000

Some optical fields for nonlinear optics

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Recibido el 2 de diciembre de 1998; aceptado el 13 de octubre de 1999

We show the generation of optical field distributions with well defined amplitude and phase characteristics using computer-generated holograms. Some examples of these field distributions applied to nonlinear optics are shown.

Keywords: Nonlinear optics; holography; computer generated holograms

Se muestra la generación de campos ópticos con distribuciones de amplitud y fase bien definidas empleando hologramas generados por computadora. Se dan algunos ejemplos con distribuciones de campo utilizados comúnmente en óptica no lineal.

Descriptores: Óptica no lineal; holografía; hologramas generados por computadora

PACS: 42.50.Rh

1. Introduction

Nonlinear optics is the study of phenomena that occur as due to the modification of the optical properties of a material in presence of intense light. Typically a laser is used to modify these optical properties [1].

Some experiments in nonlinear optics require a very well defined optical fields distributions. However, it is almost impossible to have lasers beams with the required amplitude or phase to produce such optical fields. This necessity, led us to considere some methods to modify the typical output of a laser to produce beams with the required amplitude and phase properties. There are different methods to do this using a phase mask [2], a diffraction grating [3] holographic grating [4], mode converters [5] or interference of beams [6]. But each one has restraints such as high power lasers, complex optical system, etc. Here we describe a simple method which employs a computer generated hologram and an adequate optical system to demonstrate how to generate an optical field which has the desired characteristics.

The encoding of complex wavefronts to make computer generated holograms was first demonstrated by Brown and Lohman [7]. Their holograms have only two levels of amplitude transmittance (0 or 1). Their method has been widely used for making binary computer generated holograms. One of the purposes of this paper is to give an intuitive idea of how to generate a hologram without knowing all the physics behind. A more detailed theoretical description and different techniques to generate computer generated holograms is given in Ref. 8.

Our holograms are created by engraving the superposition of two functions on a photographic film. The first function represents the characteristics of the object and the second function represents a plane wave.

Next section describes the holographic technique used to record information of three-dimensional objects. In section three we describe the method to generate computer generated holograms, some examples are shown. In section four we present some examples of such holograms to obtain some optical fields commonly used in nonlinear optics experiments. In the last section we give the conclusion of this work.

2. Holographic technique

Holography is a technique that allows to record in a film all the information (*i.e.* amplitude and phase) of a wavefront coming from a 3-dimensional object. An interference pattern from two optical waves is recorded on a photosensitive plate, which after chemical developing is called a hologram [9].

To obtain the hologram of an object, the pattern resulting from the superposition of two coherent optical waves is recorded on the holographic film. One wave, diffused by the object, known as signal or object wave, interferes with the other one, that comes directly from the source known as the reference wave. Both waves must be emitted by the same laser source due to the required coherence conditions. To reconstruct the recorded object wave it is only necessary to illuminate the hologram with the reference wave. In Fig. 1 we sketch the principles of the recording and reconstruction of a hologram.

When the hologram is reconstructed there are two diffracted waves. One produces a virtual image while the other a real image. The virtual image can be seen by one observer in front of the hologram, at the same position occupied by the object during recording. The real image is obtained in a conjugate position at the other side of the hologram.

In this way it is possible to obtain the hologram of any desired object, regardless of how complicated this object could be.

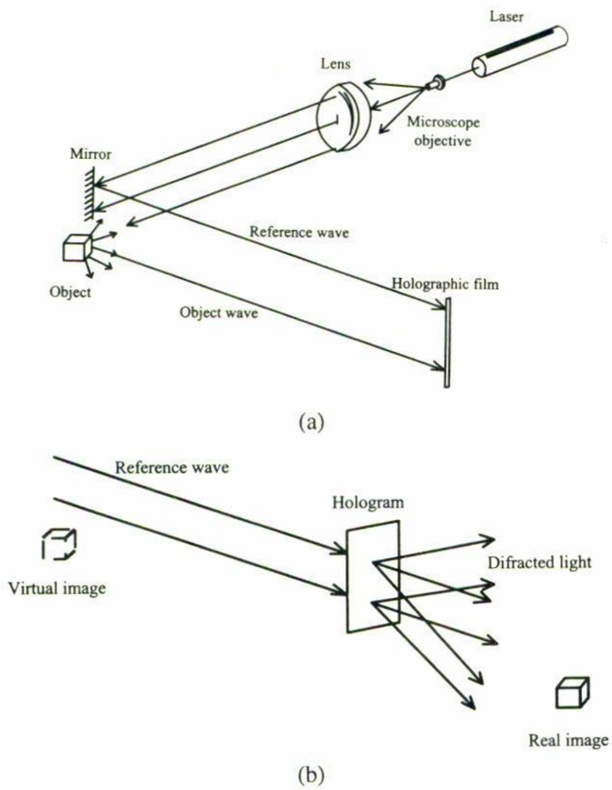


FIGURE 1. Typical configuration to record (a) and reconstruct (b) a hologram of a real object.

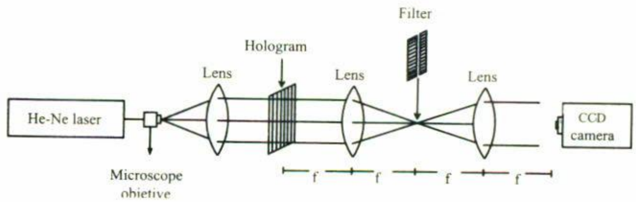


FIGURE 2. 4-f Fourier optical system using a He-Ne laser beam and a CCD to capture the images at the 4-f plane.

3. Computer generated holograms

Following the analogy with holography, we describe a simple method to obtain optical fields with defined amplitude and phase distributions. The main difference with the holographic technique described above, is that the hologram is now obtained in a numerical fashion. This method associates a mathematical function to the desired field f. This function is added numerically to a function that represents a plane wave g. In this way the interference of the object and reference wave are obtained. This process is done using commercial software. Then we multiply the sum of $f + g$ by its complex conjugate and obtained a binary image of the result. Finally, the distribution is printed out and reduced in a photographic film with the adequate size, depending on the wavelengt used. In this way we obtain our computer-generated holograms.

The reconstruction of these holograms is done by placing the hologram at the entry of a 4-f Fourier transform optical system illuminated by a plane wave. The 1 or -1 diffraction order is filtered with an adequate screen, and the desired optical field is reconstructed at the focal length of the second lens of the system, see Fig. 2.

As an example of this method, three functions representing optical fields used in nonlinear optics were chosen.

First we generate initial conditions needed to form a $(1 + 1)$-dimensional dark spatial solitons field, commonly know as a phase jump [10]. Here a beam with a π phase jump is needed. It looks as a bright beam with a dark stripe in the center where the phase difference between the two bright zones separated by the dark stripe is π. The function that represents such field is

$$f_1(x) = \tanh x, \tag{1}$$

where the x is the spatial coordinate.

The graphical representation of this optical field as an function in intensity and phase is shown in Fig. 3a.

As a second example, we consider the initial condition to generate a $(2+1)$-dimensional dark spatial solitons, which is known as an optical vortex. We need a beam with a continuous variation of the phase from 0 to 2π around the center. The cross section of such field is as a bright beam with a dark spot in its center. The function that describes such behavior is given by:

$$f_2(r, \theta) = \tanh r \exp{(im\theta)}, \tag{2}$$

where r and θ are the coordinates in a polar representation, i is the imaginary number and m represents the order of the vortex. The graphical representation of this optical field as an function in intensity and phase is shown in Fig. 3b.

Finally we propose a third function, to obtain an optical field of a zero order Bessel beam. We used the following function:

$$f_3(x, y) = J_0(x, y), \tag{3}$$

where J_0 represents the zero order Bessel function. The graphical representation of this optical field as an function in intensity and phase is shown in Fig. 3c.

The method described here is not restricted to the previous functions, it is possible to use any function to obtain the desired optical field.

4. Results

Each one of previous examples were added to the function that represents a plane wave,

$$g(x) = \exp{(ikx \sin\theta)}, \tag{4}$$

where k is the magnitude of the propagation vector of the wave, and θ represents the angle formed between the plane wave and each one of the objects functions. The magnitude

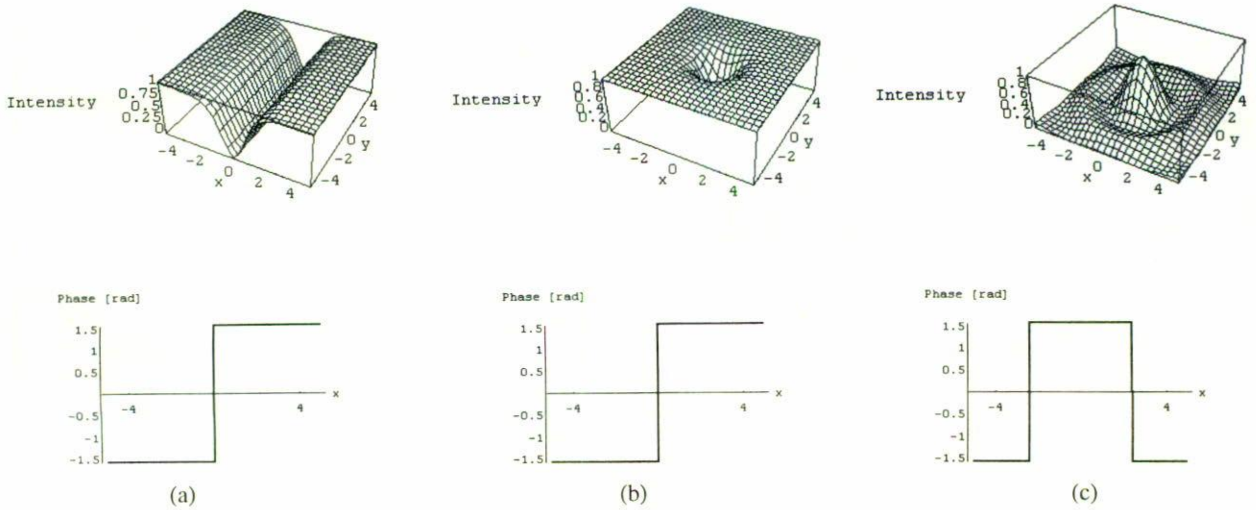


FIGURE 3. Three-dimensional graphs of the intensity $(f \cdot f^*)$ and phase along one axis for the functions: (a) f_1, (b) f_2, and (c) f_3.

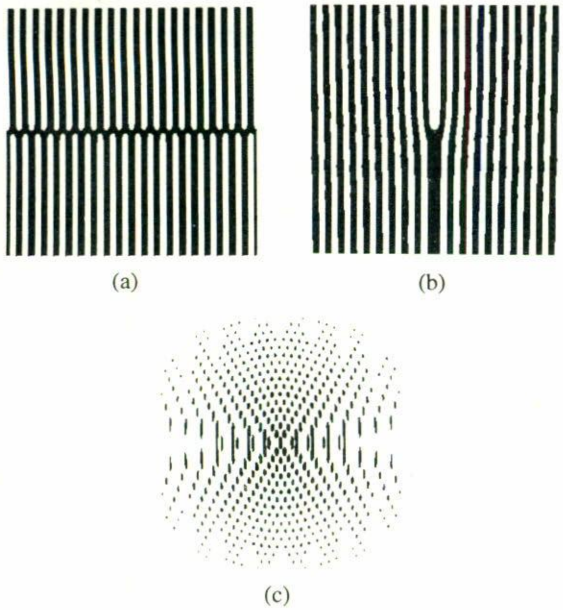


FIGURE 4. Holograms obtained from *Mathematica*® for functions: (a) f_1, (b) f_2 and (c) f_3. With $\theta = \pi/2$, $m = 1$ and $k = 2$.

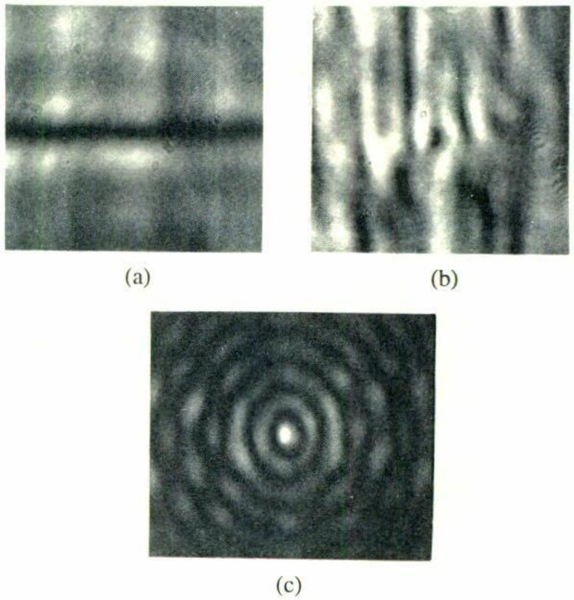
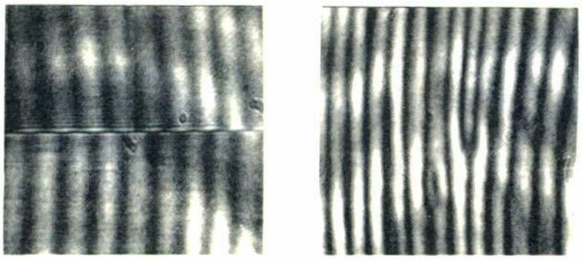


FIGURE 5. Images of the reconstructed fields at the output of the Fourier optical system for functions: (a) f_1, (b) f_2 and (c) f_3.

of θ allows to change the period of the interference pattern. In our case the angle of superposition was chosen to be of $\theta = \pi/2$ and $k = 2$. After adding both functions we multiply the result by its complex conjugate to obtain the patterns shown in Fig. 4. These patterns where obtained using *Matematica*® with the command "Contour" to obtain a binary image. These patterns were printed in paper sheets and later were reduced by 20% into a photographic films. The final size of these holograms was 5×5 mm^2. These holograms were reconstructed in the laboratory, placing them at the entry of a 4-f optical Fourier system and illuminated with a collimated He-Ne laser beam.

The 4-f optical Fourier system consisted of two lenses of focal length equal to 25 cm. With this conditions the diffraction orders were separate 2 mm. Then the first order was selected and filtered using a screen with a slit or a hole.

With a CCD camera we captured the images at the output plane of system for the different functions, obtaining the intensity distributions shown in Fig. 5. In the $(1 + 1)$-dimensional case (Fig. 5a) we obtained a dark stripe with a full width to the half maximum (FWHM) of 150 μm on almost constant bright background. In the $(2 + 1)$-dimensional case (Fig. 5b) we obtained the central dark spot of 100 μm (FWHM), but the background was not uniformly illuminated, other dark spots appeared. This effect can be due to the fact



(a) (b)

FIGURE 6. a) and b) the interference patterns obtained by superposing the optical fields shown in Figs. 5a and 5b with a plane wave, respectively.

the initial image of the interference pattern is not smooth and the diffracted light in the Fourier plane did not pass through the screen used as filter. A similar result was obtained with the Bessel beam, see Fig. 5c.

To verify that the field had the right phase distribution we interfered this field with a plane wave, obtaining patterns with a similar distribution to that of the respectively hologram. As an example, in Fig. 6 we show the interference patter obtained for the optical fields shown in Figs. 5a and 5b, that correspond to functions f_1 and f_2 respectively. The interference patterns confirm that we have the right phase distribution for both fields.

5. Conclusions

We have shown a simple technique that allows to record in a computer generated hologram distributions in amplitude and phase for a given optical field. This method associates to each field a mathematical function that follows the amplitude and phase distributions. Examples of typical distributions used for optical solitons in nonlinear optics experiments, were given.

Acknowledgments

We would like to acknowledge Ruben Ramos Garcia and Carlos Treviño for their supports in the correcting of the language. The anonimous referees their valuable observations and comments. MMA thanks the financial support from CoNaCyT.

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