# Hydrodynamical interaction between a shock wave and a cloud. One dimensional approach

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The collision of a plane parallel shock wave with a plane parallel cloud of uniform density is analysed for the case in which magnetic fields and radiative losses are not considered. General analytic solutions are discussed for the case in which the density of the cloud is greater than that of the surrounding environment. This problem generalises one of the classical problems in gas dynamics: the collision between a shock wave and a solid wall.

*Keywords:* Hydrodynamics; shock waves

La colisión de una onda de choque plano-paralela con una nube plano-paralela de densidad uniforme es analizada para el caso en el que campos magnéticos y perdidas por radiación no son consideradas. Se discuten soluciones analíticas generales para el caso en el que la densidad de la nube es mucho mayor que la del gas que le rodea. Este problema generaliza uno de los problemas clásicos en el estudio de la dinámica de gases: la colisión entre una onda de choque y una pared sólida.

*Descriptores:* Hidrodinámica; ondas de choque

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## 1. Introduction

The problem of the collision of a shock wave with a cloud has been intensely investigated in the past by several authors (see, for example, Ref. 1 and references therein). The simplest assumption to make is to consider a cloud for which gravitational effects are not considered, magnetic fields are non-important and radiative losses are negligible. The fact that gravity is not taken into account, makes it possible to consider the density of the cloud as uniform. The complete 3D hydrodynamical problem is extremely complicated, even under the simplifications mentioned above. However, numerical simulations have been done for this case which ultimately give rise to instabilities causing a complete disruption of the cloud [1].

This article describes how the solution of the one dimensional problem can be obtained. It has been argued in the past that at least for the case in which the density contrast is high, *i.e.*, the ratio of the cloud's density to that of the external environment is high, the problem has to be very similar to the one found in the problem of a collision of a plane parallel shock with with a solid wall [2, 3].

Many Astrophysical phenomena give rise to collisions between a shock wave and a cloud. For example, when a supernova explosion occurs, the intense ejection of energy from the supernova into the interstellar medium produces a spherical shock wave which expands into the interstellar medium. Several examples exist for which collisions of this expanding shock have been observed to interact with clouds embedded in the interstellar medium. This interaction is very important, since it seems to induce, under not very well known circumstances, gravitational collapse and star formation [4]. Another scenario is presented by the expansion of jets around active galactic nuclei. A pair of jets expand in opposite directions from the nuclei of the galaxy creating a bow shock which interacts with the intergalactic medium. It is the interaction of this expanding bow shock with clouds or galaxies embedded in clusters of galaxies that provides a mechanism in which shock–cloud interactions take place. It seems that this interaction is able to induce star formation very efficiently.[a]

Having all this considerations in mind, the present paper aims to give a simple way of solving a particular case of the whole problem. This article provides an analytic description of the one dimensional problem of a collision between a plane parallel shock with a plane parallel "cloud" bounded by two tangential discontinuities. It is assumed that the specific volume in the cloud is a quantity of the first order, in other words solutions are given for the case in which the density of the cloud is much greater than that of the surrounding environment.

## 2. General description of the problem

Consider two plane parallel infinite tangential discontinuities. The cloud, or internal region to the tangential discontinuities has uniform pressure $p_c$ and density $\rho_c$. The environment, or external region to the cloud has also uniform values of pressure $p_1$ and density $\rho_1$ respectively. A plane parallel shock wave is travelling in the positive $x$ direction and eventually will collide with the left boundary of the cloud at time $t = t_0 < 0$. For simplicity we assume from now on that the density of the cloud is greater than that of the environment. By knowing the pressure $p_2$ and density $\rho_2$ behind the shock wave, it is possible to solve the hydrodynamical problem thus defined.

The problem of the collision of a shock wave and a tangential discontinuity is well known [5]. Since at the instantaneous time of collision the values of, say, the density in front and behind the shock are $\rho_c$ and $\rho_2$ respectively, the standard jump conditions for a shock no longer hold. A discontinuity in the initial conditions (first initial discontinuity) occurs.

When a discontinuity in the initial conditions occurs, the values of the hydrodynamical quantities need not to have any relation at all between them at the surface of discontinuity. However, certain relations need to be valid in the gas if stable surfaces of discontinuity are to be created. For instance, the Rankine-Hugoniot relations have to be valid in a shock wave. What happens is that this initial discontinuity splits into several discontinuities, which can be of one of the three possible types: shock wave, tangential discontinuity or weak discontinuity. This newly formed discontinuities move apart from each other with respect to the plane of formation of the initial discontinuity.

Very general arguments show that only one shock wave or a pair of weak discontinuities bounding a rarefaction wave can move in opposite directions with respect to the point in which the initial discontinuity took place. For, if two shock waves move in the same direction, the shock at the front would have to move, relative to the gas behind it, with a velocity less than that of sound. However, the shock behind must move with a velocity greater than that of sound with respect to the same gas. In other words, the leading shock will be overtaken by the one behind. For exactly the same reason a shock and a rarefaction wave can not move in the same direction, and this is due to the fact that weak discontinuities move at the velocity of sound relative to the gas they move through. Finally, two rarefaction waves moving in the same direction can not become separated, since the velocities of their boundaries with respect to the gas they move through is the same.

Boundary conditions demand that a tangential discontinuity must remain in the point where the initial discontinuity took place. This follows from the fact that the discontinuities formed as a result of the initial discontinuity must be such that they are able to take the gas from a given state at one side of the initial discontinuity to another state in the opposite side. The state of the gas in any one dimensional problem in hydrodynamics is given by three parameters (say the pressure, the density and the velocity of the gas). A shock wave however, is represented by only one parameter as it seen from the shock adiabatic relation (Hugoniot adiabatic) for a polytropic gas:

$$\frac{V_b}{V_f} = \frac{(\gamma+1)p_f + (\gamma-1)p_b}{(\gamma-1)p_f + (\gamma+1)p_b}, \tag{1}$$

where $p$ and $V$ stand for pressure and specific volumes respectively, $\gamma$ is the polytropic index of the gas and the subscripts $f$ and $b$ label the flow in front of and behind the shock. For a given thermodynamic state of the gas (*i.e.*, for given $p_f$ and $V_f$) the shock wave is determined completely since, for instance, $p_b$ would depend only on $V_b$ according to the shock adiabatic relation. On the other hand, a rarefaction wave is also described by a single parameter. This is seen from the equations which describe the gas inside a rarefaction wave which moves to the left with respect to gas at rest beyond its right boundary [5]:

$$c_R = c_4 + \frac{1}{2}(\gamma_c - 1)w_R, \tag{2}$$

$$\rho_R = \rho_4\Big[1 + \frac{1}{2}\frac{(\gamma_c-1)w_R}{c_4}\Big]^{2/(\gamma_c-1)}, \tag{3}$$

$$p_R = p_4\Big[1 + \frac{1}{2}\frac{(\gamma_c-1)w_R}{c_4}\Big]^{2\gamma_c/(\gamma_c-1)}, \tag{4}$$

$$w_R = -\frac{2}{\gamma_c+1}\Big(c_4 + \frac{x}{t}\Big), \tag{5}$$

where $c_4$ and $c_R$ represent the sound speed behind and inside the rarefaction wave respectively. The magnitude of the velocity of the flow inside the rarefaction wave is $w_R$ in that system of reference. The quantities $p_4$ and $p_R$ are the pressures behind and inside the rarefaction wave respectively. The corresponding values of the density in the regions just mentioned are $\rho_4$ and $\rho_R$.

With only two parameters at hand, it is not possible to give a description of the thermodynamic state of the gas. It is the tangential discontinuity, which remains in the place where the initial discontinuity was produced, that accounts for the third parameter needed to describe the state of the fluid.

When a shock wave hits a tangential discontinuity, a rarefaction wave can not be transmitted to the other side of the gas bounded by the tangential discontinuity. For, if there would be a transmitted rarefaction wave to the other side of the tangential discontinuity, the only possible way the boundary conditions could be satisfied is if a rarefaction wave is reflected back to the gas. In other words, two rarefaction waves separate from each other in opposite directions with respect to the tangential discontinuity that is left after the interaction. In order to show that this is not possible, consider a shock wave travelling in the positive $x$ direction, which compresses gas 1 into gas 2 and collides with a tangential discontinuity. After the interaction two rarefaction waves separate from each other and a tangential discontinuity remains between them. In the system of reference where the tangential discontinuity is at rest, the velocity of gas 2 is $v_2 = -\int_{p_3}^{p_2}\sqrt{-dp\,dV}$, where $p_3$ is the pressure of gas 3 surrounding the tangential discontinuity. Accordingly, the velocity of gas 1 in the same system of reference is $v_1 = -\int_{p_3}^{p_1}\sqrt{-dp\,dV}$. Since the product $-dp\,dV$ is a monotonically increasing function of the pressure and

$0 \le p_3 \le p_1$, then

$$-\int_0^{p_2}\sqrt{-dPdV} \le v_1 - v_2 \le \int_0^{p_1}\sqrt{-dPdV} - \int_{p_1}^{p_2}\sqrt{-dPdV}.$$

The difference in velocities $v_1 - v_2$ has the same value in any system of reference and so, it follows that $v_1 \le v_2$, in particular on a system of reference with the incident shock at rest. However, for the incident shock to exist, it is necessary that $v_1 > v_2$, so two rarefaction waves can not be formed as a result of the interaction.

So far, it has been shown that after the collision between the shock and the boundary of the cloud, a first initial discontinuity is formed. This situation can not occur in nature in any manner and the shock splits into a shock which penetrates the cloud and either one of a shock, or a rarefaction wave (bounded by two weak discontinuities) is reflected from the point of collision. With respect to the point of formation of the initial discontinuity, boundary conditions demand that a tangential discontinuity must reside in the region separating the discontinuities previously formed.

In a shock wave, the velocities ($v$) in front and behind the shock are related to one another by their difference:

$$v_f - v_b = \sqrt{(p_b - p_f)(V_f - V_b)}, \tag{6}$$

where the subscripts $f$ and $b$ label the flow of the gas in front and behind the shock wave.

If after the first initial discontinuity two shock waves separate with respect to the point of collision, then according to Eq. (6) the velocities of their front flows are given by $v_c = -\sqrt{(p_3 - p_1)(V_c - V_{3'})}$ and $v_2 = \sqrt{(p_3 - p_2)(V_2 - V_3)}$, where the regions 3 and 3′ bound the tangential discontinuity which is at rest in this particular system of reference (see top and middle panels of Fig. 1). Due to the fact that $p_3 \ge p_2$ and because the difference $v_2 - v_c$ is a monotonically increasing function of the pressure $p_3$, then:

$$v_2 - v_c > (p_2 - p_1)\sqrt{\frac{2V_c}{(\gamma_c - 1)p_1 + (\gamma_c + 1)p_2}},$$

according to the shock adiabatic relation. Since $v_2 - v_c$ is given by Eq. 6, then:

$$\frac{V_1}{(\gamma - 1) + (\gamma + 1)\left(\frac{p_2}{p_1}\right)} > \frac{V_c}{(\gamma_c - 1) + (\gamma_c + 1)\left(\frac{p_2}{p_1}\right)}, \tag{7}$$

where $\gamma$ and $\gamma_c$ represent the polytropic indexes of the environment and the cloud respectively. $V_1$ and $V_c$ are the specific volumes on the corresponding regions. In other words, a necessary and sufficient condition for having a reflected shock from the boundary of the two media, under the assumption of initial pressure equilibrium between the cloud and the environment, is given by Eq. (7). Since for the problem in question $V_1 > V_c$ and the polytropic indexes are of the same order of magnitude, a reflected shock is produced.

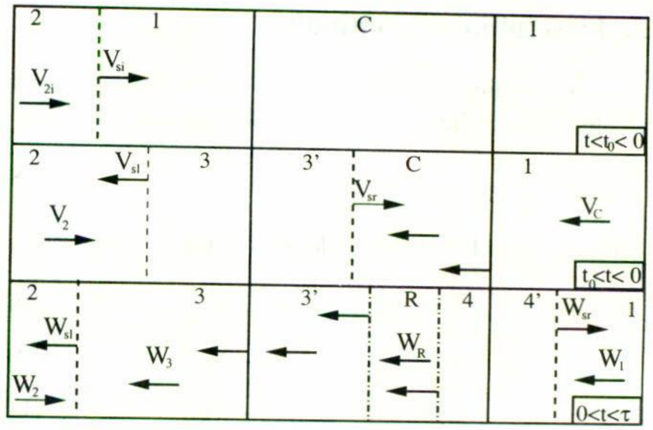


FIGURE 1. An incoming shock travelling to the right (top panel) hits a tangential discontinuity at time $t_0 < 0$. This produces two shocks moving in opposite directions with respect to the place of formation (middle panel). When the transmitted shock into the cloud (region C) collides with its right boundary a reflected rarefaction wave (region R) bounded by two tangential discontinuities and a shock transmitted to the external medium (lower panel) are formed. Arrows represent direction of different boundaries, or the flow itself. The numbers in the figure label different regions of the flow. Dashed lines represent shocks, dash-dot are weak discontinuities and continuous ones are tangential discontinuities. The system of reference is chosen such that the tangential discontinuities which are left as a result of the collisions are always at rest.

In the same form, at time $t = 0$ when the transmitted shock reaches the right tangential discontinuity located at $x = 0$, another (second) initial discontinuity must occur. In this case, we must invert the inequality in Eq. (7), change $\gamma$ by $\gamma_c$ and $p_2$ by $p_3$, where $p_3$ is the pressure behind the shocks produced by the first initial discontinuity. Again, using the same argument for the polytropic indexes, it follows that after this interaction a weak discontinuity bounded by two rarefaction waves must be reflected from the boundary between the two media. As a result of the interaction, once again, boundary conditions demand that a tangential discontinuity remains between the newly formed discontinuities.

This situation continues until the rarefaction wave and the left tangential discontinuity of the cloud collide at time $t = \tau > 0$. At this point, two rarefaction waves separating from each other from the point of formation will be produced once a stationary situation is reached, and a tangential discontinuity will be separating the newly formed discontinuities. One can continue in a somewhat indefinite manner with the solution but, for the sake of simplicity the calculations are stopped at this point. Figure 1 shows a schematic description of the solution described above in a system of reference such that the tangential discontinuities which are left as a result of the different interactions are at rest. The numbers in the figure label different regions in the flow.

## 3. First initial discontinuity

According to Fig. 1, after the first initial discontinuity the absolute values of the velocities ($v$) of the flow are related by

$$v_2 + v_c = v_{2i}. \tag{8}$$

With the aid of Eq. (6), the velocities of Eq. (8) are given by

$$v_{2i}^2 = (p_2 - p_1)(V_1 - V_2), \tag{9}$$

$$v_c^2 = (p_3 - p_1)(V_c - V_{3'}), \tag{10}$$

$$v_2^2 = (p_3 - p_2)(V_2 - V_3). \tag{11}$$

Inserting Eqs. (9)–(11) into Eq. (8) and substituting for the specific volumes from Eq. (1), one ends with a relation which relates the pressure $p_3$ as a function of $p_2$, $p_1$ and the polytropic indexes in an algebraic linear form. Straightforward manipulations show that the resulting equation does not have an easy analytic solution, even for the particular cases in which a strong or weak incident shock collides with the cloud.

In order to find a set of analytic solutions, let us first describe a particular solution to the problem. If we consider a cloud with an initial infinite density -a solid wall, then Eq. (8) takes the form $v_2 = v_{2i}$, and a "zeroth order" solution is found [5]:

$$\frac{p_{3_0}}{p_2} = \frac{(3\gamma - 1)p_2 - (\gamma - 1)p_1}{(\gamma - 1)p_2 + (\gamma + 1)p_1}, \tag{12}$$

where $p_{3_0}$ is the value of the pressure behind the reflected and transmitted shocks for the case in which the cloud has specific volume $V_c = 0$. For this particular case, Eq. (12) determines $p_{3_0}$ as a function of $p_1$ and $p_2$, which are initial conditions to the problem. Due to the fact that the gas is polytropic, this relation is the required solution to the problem.

In order to get a solution more adequate to the general case, we can approximate the whole solution under the assumption that $V_c$ is a quantity of the first order, so

$$p_3 = p_{3_0} + p_3^\star, \tag{13}$$

$$V_3 = V_{3_0} + V_3^\star, \tag{14}$$

$$V_{3'} = V_{3'}^\star, \tag{15}$$

where the quantities with a star are of the first order and the subscript 0 represents the values at zeroth order approximation. Substitution of Eqs. (13)–(15) into Eqs. (10) and (11) gives:

$$v_2^2 = v_{2o}^2 - V_3^\star(p_{3_0} - p_2) + p_3^\star(V_2 - V_{3_0}), \tag{16}$$

$$v_c^2 = (p_{3_0} - p_1)(V_c - V_{3'}^\star). \tag{17}$$

From the shock adiabatic relation, Eq. (1), and Eqs. (13)–(15) it follows that

$$\frac{V_{3_0}}{V_2} = \frac{(\gamma + 1)p_2 + (\gamma - 1)p_{3_0}}{(\gamma - 1)p_2 + (\gamma + 1)p_{3_0}}, \tag{18}$$

$$\frac{V_{3'}^\star}{V_c} = \frac{(\gamma_c + 1)p_1 + (\gamma_c - 1)p_{3_0}}{(\gamma_c - 1)p_1 + (\gamma_c + 1)p_{3_0}}, \tag{19}$$

$$\frac{V_3^\star}{V_2} = -\frac{4\gamma p_2 p_3^\star}{[(\gamma - 1)p_2 + (\gamma + 1)p_{3_0}]^2}. \tag{20}$$

Substitution of Eqs. (16), (17), and (20) in Eq. (8) gives the required solution:

$$\frac{p_3^\star}{p_2} = -\frac{V_c}{V_2}\left(\frac{|\alpha| + \beta}{\eta}\right), \tag{21}$$

where

$$\beta = \left(\frac{p_{3_0}}{p_2} - \frac{p_1}{p_2}\right)\left(1 - \frac{V_{3'}^\star}{V_c}\right),$$

$$\eta = \left(1 - \frac{V_{3_0}}{V_2}\right) - \left(\frac{p_{3_0}}{p_2} - 1\right)\frac{(\gamma - 1) - (\gamma + 1)\left(\dfrac{V_{3_0}}{V_2}\right)}{(\gamma - 1) + (\gamma + 1)\left(\dfrac{p_{3_0}}{p_2}\right)},$$

$$|\alpha|^2 = 4\frac{V_2}{V_c}\left(\frac{p_{3_0}}{p_2} - 1\right)\left(\frac{p_{3_0}}{p_2} - \frac{p_1}{p_2}\right)\left(1 - \frac{V_{3_0}}{V_2}\right)\left(1 - \frac{V_{3'}^\star}{V_c}\right).$$

The specific volumes $V_{3_0}$ and $V_{3'}^\star$ are given by Eq. (18) and Eq. (19) respectively. For completeness, approximations to Eq. (21) for the case of a very strong incident shock and that of a weak incident shock are given

$$\frac{p_3^\star}{p_2} = -\frac{4\gamma^2(\gamma + 1)}{(3\gamma - 1)(\gamma - 1)^2}\frac{V_c}{V_1}\left(\frac{3\gamma - 1}{\gamma_c + 1} + \kappa\right), \tag{22}$$

$$\frac{p_3^\star}{p_1} = -2\zeta\frac{\gamma}{\gamma_c}\sqrt{\frac{V_c}{V_1}}\left(\sqrt{\frac{\gamma_c}{\gamma}} + \sqrt{\frac{V_c}{V_1}}\right), \tag{23}$$

where

$$\kappa = 2\sqrt{\frac{V_1}{V_c}\frac{(3\gamma - 1)(\gamma - 1)}{(\gamma_c + 1)(\gamma + 1)}},$$

and $\zeta \equiv (p_2 - p_1)/p_1 \ll 1$ in the weak limit. Figure 2 shows a plot of the pressure $p_3$ as a function of the strength of the incident shock. It is interesting to note that even for very strong incident shocks the ratio $p_3/p_2$ differs from zero, which follows directly from Eqs. (12) and (22). This simple means that the reflected shock is not strong, no matter the initial conditions chosen.

There are certain important general relations for which the above results are a consequence of. Firstly, by definition the pressure $p_2$ behind the shock is greater than the pressure $p_1$ of the environment. Now consider a strong incident shock, then since $p_3 > p_2 \gg p_1$, it follows that the transmitted shock into the cloud is very strong. Also, the reflected shock does not have to compress too much the gas behind it to acquire the required equilibrium, so it is not a strong shock. This last statement is in agreement with Eq. (22). In general, for any strength of the incident shock, since the inequality

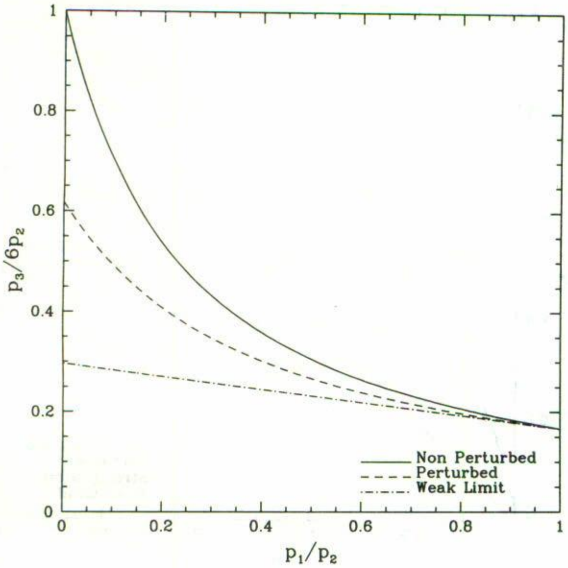


FIGURE 2. Variation of the pressure $p_3$ behind the transmitted shock into the cloud as a function of the strength of the initial incident shock. The continuous line shows the case for which the cloud is a solid wall with infinite density. The dashed curve is the solution at first order approximation for which the cloud's specific volume is a quantity of the first order. The acoustic approximation for which the incident shock is weak, at the same order of accuracy, is represented by a dot-dashed curve. The perturbed solutions were plotted assuming $\rho_c/\rho_1 = 100$ for polytropic indexes $\gamma = \gamma_c = 5/3$, corresponding to a monoatomic gas.

$p_3 > p_2 > p_1$ holds, continuity demands that the reflected shock can not be strong and, more importantly, that the penetrating shock is always stronger than the reflected one.

Secondly, very general inequalities are satisfied by the velocities $v_2$, $v_c$, $v_{sl}$ as defined in Fig. 1. For instance,

$$v_{sl} > v_2. \tag{24}$$

Indeed, since $V_3 v_2/(V_3 - V_2) > v_2$ holds, and the left hand side of this inequality is just $v_{sl}$ according to mass flux conservation across the reflected shock, the result follows.

On the other hand, from Eqs. (10) and (11), since $p_2 > p_1$ it follows that a necessary and sufficient condition for $v_2 > v_c$ to be true is that $V_2 - V_3 > V_c - V_{3'}$. This last condition is satisfied for sufficiently small values of $V_c$. To give an estimate of the smallness of the cloud's specific volume needed, note that a necessary and sufficient condition for $V_2 - V_3 > V_c - V_{3'}$ to be valid is

$$\frac{V_2(p_3 - p_2)}{(\gamma - 1)p_2 + (\gamma + 1)p_3} > \frac{V_c(p_3 - p_1)}{(\gamma_c - 1)p_1 + (\gamma_c + 1)p_3}, \tag{25}$$

according to the shock adiabatic relation for the transmitted and reflected shocks. Since $p_3 > p_2 > p_1$ and $V_2 < V_1$ it follows that

$$\frac{V_1}{(\gamma - 1) + (\gamma + 1)\left(\dfrac{p_3}{p_1}\right)} > \frac{V_c}{(\gamma_c - 1) + (\gamma_c + 1)\left(\dfrac{p_3}{p_1}\right)}, \tag{26}$$

which is very similar to Eq. (7). In the same fashion, under the assumption that the polytropic indexes are of the same order of magnitude, Eq. (26) implies $V_c < V_1$, which was an initial assumption. Although Eq. (26) is not sufficient, due to the fact that $V_c$ is a first order quantity we can use in what follows:

$$v_2 > v_c. \tag{27}$$

The inequalities in Eq. (24) and Eq. (27) will prove to be useful later when we choose a more suitable reference system to describe the problem in question.

## 4. Second initial discontinuity

Let us now analyse the situation for which $0 < t < \tau$. To begin with let us prove that

$$w_1 < v_2 + v_c \equiv u_2, \tag{28}$$

where the velocities $w_1$, $v_2$ and $v_c$ are defined in Fig. 1. Suppose that the inequality in Eq. (28) is not valid, then, by expressing the velocities as function of the specific volumes and pressures by means of Eq. (6) and the fact that $p_2 > p_1$, $p_3 > p_4$ and $V_{4'} > V_3$, it follows that $\rho_3 > \rho_c$; then as the cloud's density grows without limit, so does $\rho_3$. Necessarily, Eq. (28) has to be valid for sufficiently small values of the cloud's specific volume. It is important to point out that since $w_2 = |u_2 - w_1| = u_2 - w_1$, the gas in region 2 as drawn in Fig. 1 travels in the positive $x$ direction. According to Fig. 1 flows in region 1 and 3 are related by

$$w_1 - w_3 = v_c. \tag{29}$$

Let us now prove a very general property of the solution. Regions 2 and 3 are related to one another by the shock adiabatic relation. Since the gas in regions $3'$ and 4 obey a polytropic equation of state $p_3/p_4 = (V_4/V_{3'})^{\gamma_c}$, it follows that

$$\frac{p_4}{p_2} = \left(\frac{V_{3'}}{V_4}\right)^{\gamma_c} \frac{(\gamma + 1)V_2 - (\gamma - 1)V_3}{(\gamma + 1)V_3 - (\gamma - 1)V_2}.$$

Now, due to the fact that $V_{3'} < V_4 < V_1$, $V_3 < V_2 < V_1$ and $\gamma, \gamma_c > 1$ for a reasonable equation of state, this relation can be brought to the form

$$\frac{p_4}{p_2} < \frac{1}{2}\left[-(\gamma - 1) + (\gamma + 1)\frac{V_2}{V_1}\right] \to 0, \quad \text{as} \quad \frac{p_1}{p_2} \to 0, \tag{30}$$

according to the shock adiabatic relation. This result implies that most of the energy from the incoming shock has been injected to the cloud, no matter how strong the initial incident shock is. Only a very small amount of this energy is transmitted to the external gas that lies in the other side of the cloud. Note that this result is of a very general nature since no assumptions about the initial density contrast of the environment were made.

In order to continue with a solution at first order approximation in $V_c$, note that we have to use Eqs. (13)–(15) together with

$$p_4 = p_1 + p_4^\star, \tag{31}$$

$$V_4 = V_4^\star, \tag{32}$$

$$V_{4'} = V_1 + V_{4'}^\star, \tag{33}$$

where the quantities with a star are of the first order. The velocities $w_1$ and $w_3$ can be expressed as functions of the specific volumes and pressures by means of Eq. (6), for which after substitution of Eqs. (31)–(33) it follows:

$$w_1^2 = -p_4^\star V_{4'}^\star, \tag{34}$$

$$w_3 = \frac{2}{\gamma_c - 1}\left(\sqrt{\gamma_c p_{3_0} V_{3'}^\star} - \sqrt{\gamma_c p_1 V_4^\star}\right). \tag{35}$$

The specific volumes behind the transmitted shock and the reflected rarefaction wave are obtained from the shock adiabatic relation and the polytropic equation of state for the gas inside the rarefaction wave:

$$V_{4'}^\star = -V_1 \frac{p_4^\star}{\gamma p_1}, \tag{36}$$

$$V_4^\star = V_{3'}^\star \left(\frac{p_{3_0}}{p_1}\right)^{1/\gamma_c}. \tag{37}$$

By substitution of Eqs. (34)–(36) and Eq. (11) in Eq. (28) the required solution is found:

$$\frac{p_4^\star}{p_2} = \sqrt{\frac{\gamma p_1}{p_2}\frac{V_c}{V_1}}\,(\Gamma + \Psi\Lambda), \tag{38}$$

where

$$\Psi = \frac{2\sqrt{\gamma_c}}{\gamma_c - 1}\sqrt{\frac{(\gamma_c+1)p_1/p_2 + (\gamma_c-1)p_{3_0}/p_2}{(\gamma_c-1)p_1/p_2 + (\gamma_c+1)p_{3_0}/p_2}},$$

$$\Gamma = \frac{\sqrt{2}\,(p_{3_0} - p_1)/p_2}{\sqrt{(\gamma_c-1)p_1/p_2 + (\gamma_c+1)p_{3_0}/p_2}},$$

$$\Lambda = \sqrt{\frac{p_{3_0}}{p_2}} - \sqrt{\frac{p_1}{p_2}\left(\frac{p_{3_0}}{p_1}\right)^{1/\gamma_c}}.$$

For completeness the limits for the case of strong and weak incident shocks are given:

$$\frac{p_4^\star}{p_2} = \sqrt{\frac{\gamma(3\gamma-1)}{(\gamma_c+1)(\gamma-1)}\frac{p_1}{p_2}\frac{V_c}{V_1}}\left(\sqrt{2} + \xi\right), \tag{39}$$

$$\frac{p_4^\star}{p_2} = 6\zeta\sqrt{\frac{\gamma}{\gamma_c}}\sqrt{\frac{V_c}{V_1}}, \tag{40}$$

with:

$$\xi = \frac{2\sqrt{\gamma_c}}{\sqrt{(\gamma_c - 1)}}\left[1 - \left(\frac{p_1}{p_2}\frac{\gamma-1}{3\gamma-1}\right)^{(\gamma_c-1)/2\gamma_c}\right].$$

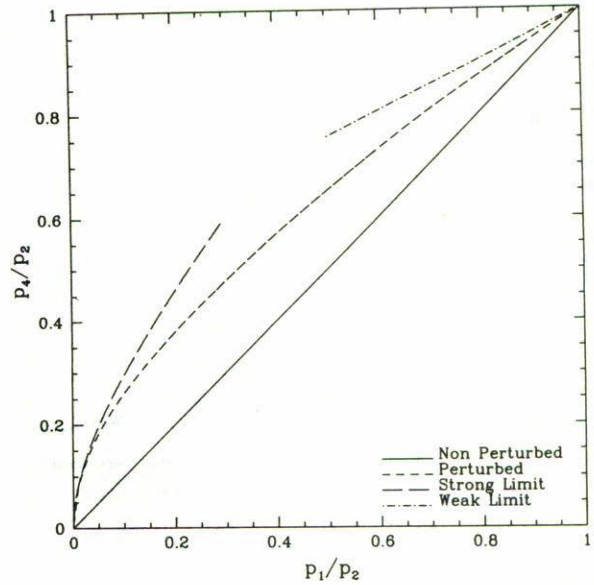


FIGURE 3. Variation of the pressure $p_4$ behind the transmitted shock into the external medium as a function of the strength of the incident shock. The continuous line represents the case for which the cloud has infinite density and so it does not transmit any shock to the external medium. The dashed curve represents the case for which the cloud's specific volume is a quantity of the first order. The long-dashed (dash-dotted) curve is the limit for which a strong (weak) incident shock collides with the cloud at the same order of approximation. The perturbed curves were produced under the assumption that $\rho_c/\rho_1 = 100$ for monoatomic gases.

It follows from Eq. (39) that $p_4 \ll p_2$ as the strength of the incident shock increases without limit. This result was given by a very general argument in Eq. (29). Figure 3 shows the variation of the pressure $p_4$ behind the shock transmitted to the environment as a function of the strength of the initial incident shock, after the second initial discontinuity.

## 5. General solution

Having found values for the pressures $p_3^\star$ and $p_4^\star$ as a function of the initial conditions $p_1$, $p_2$, $V_1$ and $V_c$, the problem is completely solved. Indeed, using the shock adiabatic relation $V_2$ is known. With this, the values of $V_{3'}^\star$, $V_3^\star$, $V_4^\star$ and $V_{4'}^\star$ are determined by means of Eq. (19), (20), (36), and (37) respectively. The complete values for pressure and specific volumes are obtained thus with the aid of Eqs. (13)–(15) and Eqs. (31)–(33). The velocities of the flow, as defined in Fig. 1, are calculated either by mass flux conservation on crossing a shock, or by the formula given for the velocity discontinuity in Eq. (6). The hydrodynamical values of the pressure $p_R$ and density $\rho_R$ inside the rarefaction wave come from Eqs. (2)–(5).

In order to analyse the variations of the hydrodynamical quantities as a function of position and time, let us now describe the problem in a system of reference in which the gas far away to the right of the cloud is always at rest, as presen-

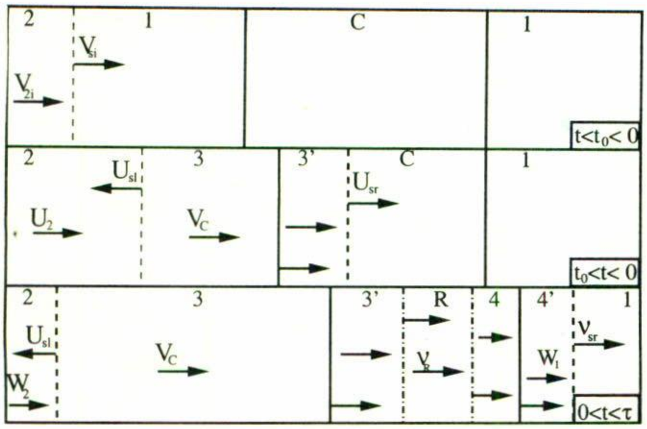


FIGURE 4. Description of the problem of a collision of a shock with a cloud in a system of reference for which the gas far away to the right (at $x = \infty$) is always at rest. Originally a shock is travelling to the right and hits a tangential discontinuity (top panel). This produces a discontinuity in the initial conditions so a reflected and transmitted shock are produced; the gas in the cloud begins to accelerate (middle panel). Eventually the transmitted shock into the cloud collides with its right boundary producing a "reflected" rarefaction wave bounded by two weak discontinuities (region R) and a transmitted shock into the external medium (lower panel). In this system of reference every single discontinuity produced by means of the interaction move to the right, except for the reflected shock produced after the first collision. Arrows represent the direction of motion of various boundaries and direction of flow. Numbers label different regions of the flow. Dashed lines represent shock waves, dash-dotted ones weak discontinuities and continuous ones tangential discontinuities.

ted in Fig. 4. Let $x_{tl}$ and $x_{tr}$ be the coordinates of the left and right tangential discontinuities, $x_{sl}$ and $x_{sr}$ the coordinates of the reflected and transmitted shocks produced after the first initial discontinuity, $\chi_{sr}$ the position of the transmitted shock after the second initial discontinuity and $x_a$ and $x_b$ the left and right weak discontinuities which bound the rarefaction wave. The new velocities are defined by Galilean transformations:

$$u_2 = v_2 + v_c, \tag{41}$$

$$u_{sl} = v_{sl} - v_c, \tag{42}$$

$$u_{sr} = v_{sr} + v_c, \tag{43}$$

$$\nu_R = w_1 - w_R, \tag{44}$$

$$\nu_{sr} = w_1 + w_{sr}. \tag{45}$$

The direction of motion of the flow is shown in Fig. 4 and it follows from Eqs. (24), (27), and (42) that $u_{sl}$ points to the left in this system of reference. Since, in the same frame, $v_c$ and $w_1$ point to the right, continuity across a weak discontinuity demands $\nu_R$ to do it in the same way.

The tangential discontinuities and the shocks produced by the initial discontinuities move with constant velocity throughout the gas. This implies that the time at which the first initial discontinuity takes place is

$$t_0 = -\frac{\Delta}{u_{sr}}, \tag{46}$$

where $\Delta$ represents the initial width of the cloud. Hence, the

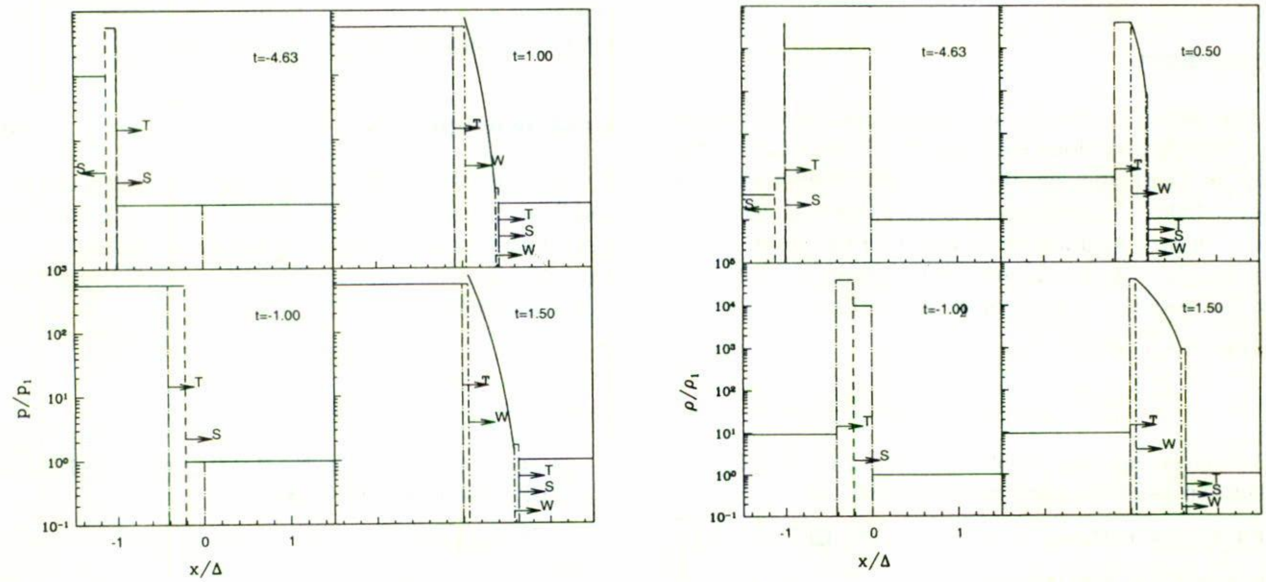


FIGURE 5. Variation of the pressure $p$ and density $\rho$ (with respect to the initial pressure $p_1$ and density $\rho_1$ of the environment) as a function of position $x$ (normalised to the initial width of the cloud $\Delta$) and dimensionless time $t$ (in units of the time $\Delta/c_1$–where $c_1$ is the speed of sound in the external medium). Dashed lines represent shock waves (S), dot-dashed lines are tangential discontinuities (T), which are boundaries of the cloud, and short-long dashed lines represent weak discontinuities (W), which bound a rarefaction wave. The system of reference was chosen so that gas far away to the right of the diagram is at rest. The diagram shows the case for which $\rho_c/\rho_1 = 10^4$, and the polytropic indices correspond to a monoatomic gas.

positions of all different discontinuities for $t_0 < t < 0$ are

$$x_{sr} = u_{sr} t, \tag{47}$$

$$x_{sl} = -\Delta - u_{sl}(t - t_0), \tag{48}$$

$$x_{tl} = -\Delta + v_c(t - t_0). \tag{49}$$

and for $0 < t < \tau$, Eqs. (48) and (49) are valid together with

$$x_a = -t\left(\frac{\gamma_c + 1}{2} w_3 + c_4\right) + w_1 t, \tag{50}$$

$$x_b = (w_1 - c_4)\, t, \tag{51}$$

$$\chi_{sr} = \nu_{sr} t, \tag{52}$$

$$x_{tr} = w_1 t. \tag{53}$$

The time $\tau$ at which the left tangential discontinuity collides with the left boundary of the rarefaction wave is given by $x_{tl} = x_a$, and thus

$$\tau c_{3'} = v_c t_0 + \Delta. \tag{54}$$

Figure 5 shows the variation of the pressure and density as a function of time and position in a system of reference in which the gas far away to the right of the cloud is at rest.

The width of the cloud varies with time, and it follows from Eq. (49) and Eq. (53) that this variation is given by

$$\bar{X}(t) = \Theta(t) w_1 t + \Delta - v_c(t - t_0), \tag{55}$$

where $\Theta(t)$ is the Heaviside step function. This linear relation is plotted in Fig. 6.

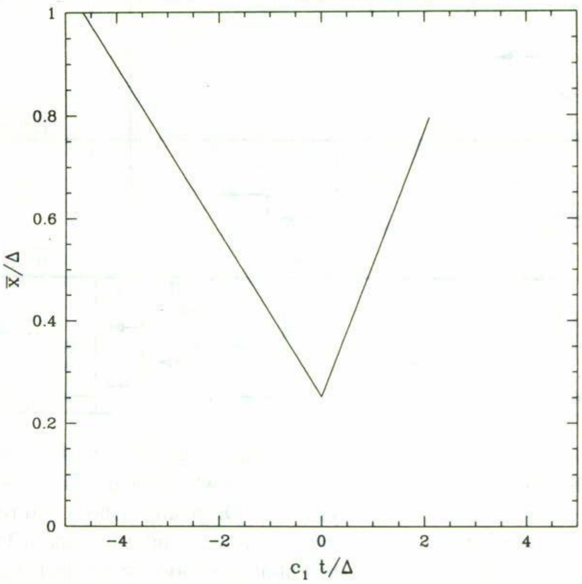


FIGURE 6. Variation of the width of the cloud in units of its original size $\Delta$ as a function of the dimensionless quantity $c_1 t/\Delta$. Where $c_1$ represents the sound speed of the gas for the external environment and $t$ the time. The curve was produced under the assumption that $\rho_c/\rho_1 = 10^4$. The gas was considered to be monoatomic.

When the shock collides with the boundary of the cloud, a discontinuity in the initial conditions is produced. This splits the incoming shock into two shock waves: one which penetrates the cloud and one which is reflected back to the external medium. When the transmitted shock into the cloud reaches the opposite boundary, another discontinuity in the initial conditions is produced, causing the transmission of a shock wave to the external medium and the reflection of a rarefaction wave from the point of collision.

## 6. Summary

The problem of a collision of a plane parallel shock wave with a high density cloud bounded by two plane parallel tangential discontinuities has been discussed. Radiation losses, magnetic fields and self gravity of the cloud were neglected. General analytic solutions were found for the simple case in which the ratio of the environment's density to that of the cloud's density is a quantity of the first order.

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[a] See for example the Hubble Space Telescope WWW site at http://oposite.stsci.edu/pubinfo/pr/1995/30.html.

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