Surface vortex and stream-lines

Alvaro Muñoz Mendoza
Instituto Mexicano de Tecnología del Agua
Paseo Cuauhnáhuac 8532, Jiutepec, Mor., 62550
Mexico

The photograph in Fig. 2 shows a surface vortex formed in the vicinity of the intake of a bulb-type Kaplan turbine, equiped with a 5.35 m diameter propeller and a 10 MW generator, rotating at 93.75 rpm, discharging 275 m^3/s, in the tidal power plant at La Rance, France.

Vortex of the magnitude of the one appearing on the photograph, are difficult to find in nature, nor even in the vicinity of conventional hydroelectric plant intakes, since its formation implies air inclusion in the rotating water column, which can eventually produce devastating effects on the turbine propeller blades.

The photograph also shows the incredible clear natural formation of stream-lines, following very similar paths as those defined by the hydrodynamic theory, which simulates this type of vortex by superimposing a vortex and a sink. Such a spiral vortex is represented with potential and stream-line functions as follows:

$$\phi = \frac{\Gamma}{2\pi} \arctan\frac{y}{x} - \frac{q}{4\pi} \ln\left(x^2 + y^2\right)$$

and

$$\psi = -\frac{\Gamma}{4\pi} \ln\left(x^2 + y^2\right) - \frac{q}{2\pi} \arctan\frac{y}{x},$$

where x and y are catesian coordinates, q is the sink intensity as unitary discharge and Γ is the vortex circulation given by

$$\Gamma = \int_0^{2\pi} v_\theta r\, d\theta = 2\pi k r^2.$$

r and θ are respectivelly, the radius and the angle in polar coordinates, v_θ is the angular velocity and k is the vortex intensity.

FIGURE 2. Surface vortex formed in the vicinity of the intake of a bulb-type Kaplan turbine and the stream-lines defined by the hydrodynamic theory.

1. H. Lamb, *Hydrodynamics*, (Cambridge University Press, Cambridge, United Kingdom, 1932).
2. H.R. Vallentine, *Applied hydrodynamics* (Butterworths, United Kingdom, 1967).