Excitation of gravity modes in the parameter space of the tidal wind system (II): magnetic effects

S. Blanco,[1] M.V. Canullo,[2] A. Costa,[2] and O.A. Rosso[1]
[1] *Instituto de Cálculo, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires Pabellón II, Ciudad Universitaria, 1428 Buenos Aires, Argentina*
[2] *Instituto de Astronomía y Física del Espacio C.C. 67 Suc. 28, 1428 Buenos Aires, Argentina*

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The Navier-Stokes, continuity and energy balance equations including magnetic effects for an inviscid compressible fluid in the ionospheric F-region were perturbed around the tidal phenomenological solution. We imposed adiabaticity and incompressibility to the perturbation. Our results satisfy the internal gravity wave (IGW) dispersion relation. The stable and unstable regions of these modes were derived as a function of two control parameters, the colatitude θ and the slow time evolution for tidal modes τ. These regions were obtained for different values of the magnetic field intensity, showing good agreement with observational data for the South Atlantic anomaly. Our model predictions for other magnetic field intensities corresponding to other latitude dependences need to be contrasted with new observational data. In addition we show for some hours and latitudes that, resonant interaction occurs between low frequency tidal waves and two high frequency gravity waves. In these regions, where tidal modes are linearly unstable, the gravity group velocity is modulated by a function of the tidal phase velocity.

Keywords: Gravity modes; tidal wind; ionospheric F-region; magnetic effects

Hemos perturbado —alrededor de la solución de mareas fenomenológica— las ecuaciones de continuidad y balance de energía que representan la región F de la ionósfera (fluído ideal y compresible). Se han incluido efectos magnéticos e impuesto adiabaticidad e incompresibilidad a las perturbaciones de modo que nuestras soluciones satisfagan la relación de dispersión de las ondas de gravedad internas. Las regiones estables e inestables de estos modos se han representado en términos de dos parámetros de control: la colatitud θ y el tiempo de evolución de los modos de marea τ. Estas regiones fueron obtenidas para diferentes valores de la intensidad de campo magnético mostrando un buen acuerdo para los datos observacionales que representan la armonía del Atlántico Sud. Mostramos también que para ciertas horas y ciertas latitudes se observa interacción resonante entre las ondas de marea de baja frecuencia y dos frecuencias altas de los modos de gravedad. En estas regiones, donde los modos de mareas son linealmente inestables, la velocidad de grupo de las ondas de gravedad se ve modulada por la velocidad de fase de las ondas de marea.

Descriptores: Ondas de gravedad; mareas; región F de la ionósfera; efectos magnéticos

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1. Introduction

In the previous work (from now on Paper I) [1] we obtained the linear stable and unstable regions of a basic quasi-stationary tidal system (perturbative gravitational time of interest $t \ll \tau$ characteristic tidal time). In that model the gravity modes were obtained as a perturbation of a phenomenological solution of the TWS. The stability regions were derived as a function of two control parameters, the colatitude θ and the slow time evolution of the tidal modes τ (day time).

Of various effects which have not been considered in Paper I, the hydromagnetic force would be most complicated for rigorous treatment. This is so as it requires additional electromagnetic equations along with the fluid dynamic ones including the hydromagnetic force. A simple way to deal with this problem is to follow Kato and Matsushita's [2] approach. As an application of the procedure developed in Paper I, we included the hydromagnetic force with the expression given by Kato and Matsushita, to study its influence on the stability regions.

In addition we examined the hypothesis about when gravity modes in the F atmospheric region can be described by stable and independent resonant triad. This is, the linear tidal mode is unstable, giving rise, by a weakly nonlinear interaction, to two stable gravity waves of finite amplitudes.

The paper is organized in the following way: the model, is presented in Sec. 2. In Sec. 3 the procedure and the approximations for solving the equations including magnetic effects are given. Section 4, is devoted to the resonant interaction analysis. Finally, in Sec. 5 we summarize our conclusions.

2. The model

There is an extensive literature devoted to the description of atmospheric tidal winds produced by gravitational and thermal effects [2, 3]. Tidal fields are described by linearizations around the steady state of the Navier-Stokes, continuity and energy balance equations of a compressible non-adiabatic fluid, taking into account Coriolis effects, the gravitational potential due to the earth, the sun and the moon, and thermal effects as well.

Assuming the earth gravity as constant (g) and neglecting viscous effects we can write

$$\frac{D\vec{v}}{Dt} = -\frac{\vec{\nabla}P}{\rho} - 2\vec{\Phi}\times\vec{v} - g\hat{z} + \mathcal{F} + \mathcal{F}_{\rm mag}, \quad (1)$$

$$\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) = 0, \quad (2)$$

$$\frac{DP}{Dt} - s^2\frac{D\rho}{Dt} = (\gamma-1)\mathcal{Q}\rho, \quad (3)$$

where $\mathcal{F}$ is the force associated to the tidal gravitational potential (sun and moon) and $\mathcal{F}_{\rm mag} = (\vec{J}\times\vec{\mathcal{B}_0})/\rho$ includes the magnetohydrodynamic effects. $\vec{J} = \sigma\cdot(\vec{E} + \vec{v}\times\vec{\mathcal{B}_0})$, $\vec{E}$ is the electric field, $\vec{\mathcal{B}_0}$ is the earth magnetic field and σ is the electrical conductivity. $\mathcal{Q}$ are the heat sources, $\vec{\Phi}$ is the earth's rotational velocity, $\gamma = C_P/C_V$, s is the sound velocity and D/Dt is the total derivative.

As we mentioned before, for the treatment of the magnetic contribution we adopted Kato and Matsushita's approach [2]. This simple approximation is to replace $(\vec{J}\times\vec{\mathcal{B}_0})/\rho$ by the term proportional to $\vec{v}$, neglecting the electric field which is only important in the upper dynamo region. Then, in this approach the magnetic force results

$$\mathcal{F}_{\rm mag} = \alpha v_\perp = (\sigma\mathcal{B}_0^2/\rho)v_\perp \quad (4)$$

where $v_\perp$ is the velocity component perpendicular to the magnetic field.

Notice that in this simple approximation for the region of interest, $\mathcal{F}_{\rm mag}$ does not depend on θ and τ. As we said, the neglected term in $(\vec{J}\times\vec{\mathcal{B}_0})/\rho$ is proved to be negligible in the region of interest and $\mathcal{F}_{\rm mag} \ll \mathcal{F}_{\rm Coriolis}$. Then the overall assumptions and hypothesis of Paper I hold in the present calculation. Again, we can obtain the IGW through the perturbation of Eqs. (1) to (3) around the steady tidal solutions, taking into account that for the IGW the atmosphere behaves as an adiabatic and incompressible stratified medium, described as a large-scale quasi-horizontal velocity field. For the tidal fields we take an average phenomenological solution whose functional dependence is the same as in Paper I.

The resulting perturbed equations are

$$\vec{v} = [U(x,y,z,\tau), V(x,y,z,\tau), W(z,\tau)], \quad (5a)$$

$$\rho = \rho(z,\tau), \quad (5b)$$

$$P = P(z,\tau), \quad (5c)$$

where the horizontal dependence of W, ρ and P has been neglected (Paper I). We choose a local coordinate system where $\hat{x}$ is parallel to $\hat{\theta}$, $\hat{y}$ is parallel to $\hat{\varphi}$ and $\hat{z}$ is perpendicular to the earth surface. θ is the colatitude.

Our aim is to obtain the IGW through the perturbation of Eqs. (1) to (3) around the steady tidal solutions, taking into account that for the IGW the atmosphere behaves as an adiabatic and incompressible stratified medium. Thus, we impose adiabaticity and incompressibility to the perturbation. For the tidal fields we take an average phenomenological solution whose functional dependence will be given below.

The expressions of the perturbation amplitudes $\mathbf{A}_i$ of the velocity $\vec{v}$, the density ρ and the pressure P can be written as

$$\begin{pmatrix} U' \\ V' \\ W' \\ \rho' \\ P' \end{pmatrix} = \begin{pmatrix} U \\ V \\ W \\ \rho \\ P \end{pmatrix} + e^{i(\vec{k}\cdot\vec{x}-\omega t)}\begin{pmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \\ \mathbf{A}_3 \\ \mathbf{A}_4 \\ \mathbf{A}_5 \end{pmatrix} \quad (6)$$

respectively, where $\omega = \omega_R + i\omega_I$ and $\vec{k}$ are functions of the parameters θ and τ. We consider that, up to zero order, the amplitudes $\mathbf{A}_i$ are not time dependent.

Following the same hypothesis made in Paper I, the resulting perturbed equations are

$$\rho\left[i\left(\vec{v}\cdot\vec{k}-\omega\right) + \frac{\partial U}{\partial x} - \alpha\right]\mathbf{A}_1 + \rho\left(\frac{\partial U}{\partial y} - 2\phi_z\right)\mathbf{A}_2 + \rho\left(\frac{\partial U}{\partial z} + 2\phi_y\right)\mathbf{A}_3 + \left[\frac{\partial U}{\partial t} + (\vec{v}\cdot\vec{\nabla}U) + 2\left(\phi_y W - \phi_z V\right) - \alpha U\right]\mathbf{A}_4 + ik_x\mathbf{A}_5 = 0, \quad (7a)$$

$$\rho\left(\frac{\partial V}{\partial x} + 2\phi_z\right)\mathbf{A}_1 + \rho\left[i\left(\vec{v}\cdot\vec{k}-\omega\right) + \frac{\partial V}{\partial y} - \alpha\right]\mathbf{A}_2 + \rho\left(\frac{\partial V}{\partial z} - 2\phi_x\right)\mathbf{A}_3 + \left[\frac{\partial V}{\partial t} + (\vec{v}\cdot\vec{\nabla}V) + 2\left(\phi_z U - \phi_x W\right) - \alpha V\right]\mathbf{A}_4 + ik_y\mathbf{A}_5 = 0, \quad (7b)$$

$$-2\rho\phi_y\mathbf{A}_1 + 2\rho\phi_x\mathbf{A}_2 + \rho\left[i\left(\vec{v}\cdot\vec{k}-\omega\right) + \frac{\partial W}{\partial z}\right]\mathbf{A}_3 + \left[\frac{\partial W}{\partial t} + W\frac{\partial W}{\partial z} + 2\left(\phi_x V - \phi_y U\right) + g\right]\mathbf{A}_4 + ik_z\mathbf{A}_5 = 0, \quad (7c)$$

$$\frac{\partial\rho}{\partial z}\mathbf{A}_3 + \left[i(\vec{v}\cdot\vec{k}-\omega) + (\vec{\nabla}\cdot\vec{v})\right]\mathbf{A}_4 = 0, \quad (8)$$

$$s^2(\vec{v}\cdot\vec{k}-\omega)\mathbf{A}_4 - (\vec{v}\cdot\vec{k}-\omega)\mathbf{A}_5 = 0. \quad (9)$$

The functional dependence of the tidal velocity field $\vec{v}$ are given in Paper I.

The amplitudes $\mathbf{B}^{(d),(s)}$, as well as the phases and periods of the diurnal and semidiurnal tides respectively are shown in Table I of Paper I and for σ we took the value $10^{-16}emu$ [2]. Note that due to the proportionality of $\hat{y}$ to τ, the y dependence of these expressions has been absorbed in the τ dependence. The density $\rho(z)$, the amplitudes $\mathbf{B}(z)$, and their derivatives $\partial\rho/\partial z$, $\partial \mathbf{B}_U/\partial z$, $\partial \mathbf{B}_V/\partial z$ and $\partial \mathbf{B}_W/\partial z$ were estimated from tables and graphics as the ones given in Tables I and II of Paper I for a height $z \cong 300$ km.

3. Magnetic effects

The system given by Eqs. (7) to (9), has nontrivial solutions in the amplitudes $\mathbf{A}$ if its complex determinant is equal to zero. After a long but straightforward calculation we obtained the determinant that gives two equations in ω_R and ω_I. The modes that can be excited are those with $\omega_I > 0$. We obtain these excitation regions and the damping ones ($\omega_I < 0$) looking for the boundary curves $\omega_I = 0$. These curves are the separatrixes between damping and excitation zones.

Setting $\omega_I = 0$ we obtain that ω_R must satisfy the following equations:

$$\mathbf{C}_{00} + \mathbf{C}_{10}\omega_R + \mathbf{C}_{20}\omega_R^2 + \mathbf{C}_{30}\omega_R^3 + \mathbf{C}_{40}\omega_R^4 = 0, \quad (10a)$$

$$\mathbf{D}_{00} + \mathbf{D}_{10}\omega_R + \mathbf{D}_{20}\omega_R^2 + \mathbf{D}_{30}\omega_R^3 = 0, \quad (10b)$$

where the coefficients $\mathbf{C}$ and $\mathbf{D}$ depend on the component of the wave vector $\vec{k}$ and the control parameters (θ, τ), their expressions are given in Appendix I. For the resolution of Eq. (10) we followed the procedure described in Paper I. That is, since the dispersion relation is unique either Eq. (10a) and Eq. (10b) are the same (if $\mathbf{C}_{40}\omega_R^4$ is negligible) or the set of roots of Eq. (10b) is included in the set of roots of Eq. (10a).

If we estimate from literature ω_R and $\vec{k}$ characteristic values of the IGW and using the TWS parameters [1], we notice that, for the extreme values of $\omega_R \sim 10^{-2}$ min^{-1} and $4\vec{v}\cdot\vec{k} \sim 10^{-2}\ min^{-1}$ there is an order of magnitude of difference between $\mathbf{C}_{40}\omega_R^4$ and $\mathbf{C}_{30}\omega_R^3$. In the average case this relation gives a difference of two or three orders of magnitude. Then we can say that $\mathbf{C}_{40}\omega_R^4 \ll \mathbf{C}_{30}\omega_R^3$. If we repeat the same procedure for the other terms, we found that again, as in Paper I, $\mathbf{C}_{20}/(\omega_R \mathbf{C}_{30}) < 10^3$. This allows us to neglect the term $\mathbf{C}_{40}\omega_R^4$ in Eq. (10a) and consider that Eq. (10a) and Eq. (10b) are the same term by term.

From this assumption we were able to determine a functional form (denoted by f) for the set of points (θ_b, τ_b) in the parameter space (θ, τ) that satisfy the condition $\omega_I = 0$ and the associated wave vector $\vec{k}(\theta, \tau)$ in selfconsistent way. Note that the consistency of the method will be proved if the resulting values satisfy the hypothesis of the model (about the autonomous condition and the local wave number) and the inequality estimated previously from phenomenological values. From the predicted values of the model given in Table III of Paper I, we have: $\omega_R \sim 510^{-3}\ min^{-1}$; $v \sim \lambda/T \sim 10^{-2}$ m/min; $k_z \sim 10^{-3}$ m^{-1} then $\mathbf{C}_{40}\omega_R^4$ is less than two order of magnitude than $\mathbf{C}_{30}\omega_R^3$.

Then

$$f^{(0)}(\theta_b, \tau_b) + f^{(1)}(\theta_b, \tau_b)\alpha + f^{(2)}(\theta_b, \tau_b)\alpha^2 + \xi\Delta f(\theta_b, \tau_b, \alpha) = 0 \quad (11)$$

and therefore vector $\vec{k}$ in the separatrix is given by

$$k_x = k_x^{(0)}(\theta_b, \tau_b) + k_x^{(1)}(\theta_b, \tau_b)\alpha + k_x^{(2)}(\theta_b, \tau_b)\alpha^2 + \xi\Delta k_x(\theta_b, \tau_b, \alpha), \quad (12a)$$

$$k_y = k_y^{(0)}(\theta_b, \tau_b) + k_y^{(1)}(\theta_b, \tau_b)\alpha + \xi\Delta k_y(\theta_b, \tau_b, \alpha), \quad (12b)$$

$$k_z = k_z^{(0)}(\theta_b, \tau_b) + \xi\Delta k_z(\theta_b, \tau_b, \alpha), \quad (12c)$$

where

$$\xi = \rho\left[s^2 \frac{\partial\rho}{\partial z}\right]^{-1} = \left(-\frac{g}{s^2}\right)\frac{1}{N^2}, \quad (13)$$

$$f^{(0)} = \left(\vec{k}^{(0)}\cdot\vec{v}\right) + \frac{(\vec{\nabla}\cdot\vec{v})}{2}, \quad (14a)$$

$$f^{(1)} = U\cdot k_x^{(1)} + V\cdot k_y^{(1)} - \frac{1}{2}, \quad (14b)$$

$$f^{(2)} = U\cdot k_x^{(2)}, \quad (14c)$$

$$\Delta f = \Delta k_x U + \Delta k_y V + \Delta k_z W, \quad (15)$$

the explicit expressions of which are given in Appendix II.

In Eqs. (11) and (12) the term with parameter ξ is proportional to the stratification of the medium. The parameter ξ is related to the Brunt-Väisälä frequency N [4]. In our model, for the reference height $z \cong 300$ km, $\xi \cong -2.910^{-3}$ min^2/m corresponding to a Brunt-Väisälä frequency of about $N \cong 1.2$ min^{-1}. The terms proportional to ξ can be thought of as corrections, which are proved to be negligible. Note that if $\alpha = 0$ the equations in Paper I are recovered.

The solution of Eq. (11) gives the boundary values (θ_b, τ_b) in the parameter space where $\omega_I(\theta_b, \tau_b) \cong 0$ and they are shown in Fig. 1a for the case $\alpha = 0$. As it was already mentioned these separatrix lines, separate the damping and excitation regions. The values of $\vec{k}$ out of these curves are unknown but close to the boundaries they must coincide with the values previously obtained. Moreover, near the separatrix lines, the $\vec{k}$ values must be continuous and cannot be too different from the set obtained before. Then, for the parameter values $\theta \approx \theta_b$ and $\tau \approx \tau_b$ we can assume $\vec{k}(\theta, \tau) \approx \vec{k}(\theta_b, \tau_b)$ and $\omega_R(\theta, \tau) \approx \omega(\theta_b, \tau_b)$ and solve the original equations (obtained through the complex determinant) so as to obtain ω_I near the borders, outside the separatrix lines. The sign of $\omega_I(+/-)$ indicates respectively the growth or damping of the perturbation in each region. The corresponding results are symbolized with plus and minus signs in Fig. 1.

The earth magnetic field is latitude and longitude dependent and its intensity ranges from 0.25 to 0.40 Gauss around 300 km [5]. This dependence is not taken into account by the ion–drag force in Kato and Matsushita's approach; then in order to study the magnetic influence, we considered different values of $\mathcal{B}_0$ for different colatitudes (see Table I).

In Figs. 1b and 1c we show the damping and excitation zones corresponding to high and low colatitude intensities of $\mathcal{B}_0$, respectively. Figure 1d represents the zones corresponding to the earth magnetic field intensity of the South Atlantic anomaly ($\mathcal{B}_0$=0.25 Gauss, colatitude ranging from 40°–80°).

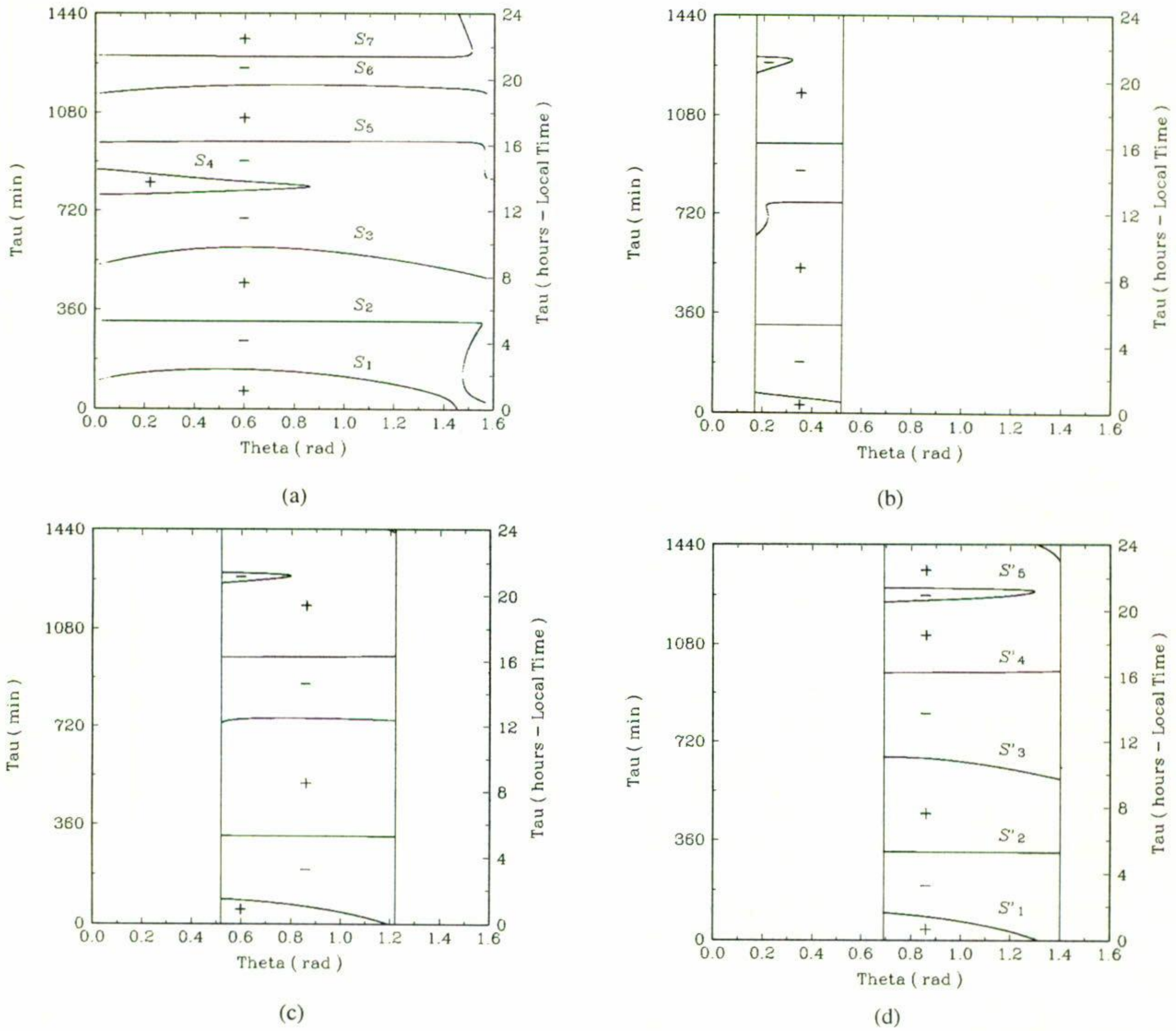

FIGURE 1. Excitation and damping regions for the IGW as a function of θ and τ for (a) $\alpha = 0$, sign + denotes excitation zones and sign − denotes damping zones. The separatrix lines between each zone are denoted by S_i; (b) $\alpha = 8 \times 10^{-4}$ sec^{-1}; (c) $\alpha = 4.5 \times 10^{-4}$ sec^{-1}; and (d) $\alpha = 3.125 \times 10^{-4}$ sec^{-1} (South Atlantic anomaly). The separatrix lines between each zone are denoted by S'_i.

TABLE I. Magnetic field intensity and coefficient α for low and high colatitudes and the South Atlantic anomaly.

θ	$\mathcal{B}_0$ (Gauss)	α (sec^{-1})
10°–30°	0.40	8.000×10^{-4}
30°–70°	0.30	4.500×10^{-4}
40°–80°	0.25	3.125×10^{-4}

As we mentioned the magnetic field is latitude and longitude dependent then, in Fig. 1b–1d we only show the latitude zones where the magnetic intensities used occur.

Comparing Figs. 1b and 1c with Fig. 1a we notice that the damping and excitation regions have varied as a function of the magnetic field intensity. However, comparing Fig. 1a ($\alpha = 0$) with Fig. 1d, we note that the differences are not so pronounced for the South Atlantic anomaly case. In Fig. 2 we contrast the experimental results obtained by Giraldez *et al.* [6] for Argentina (Buenos Aires $\theta = 0.96$ rad, Tucumán $\theta = 1.10$ rad and San Juan $\theta = 1.03$ rad) with the predicted damping and excitation zones calculated both with and without considering magnetic field (Fig. 1d and 1a, respectively). S_i corresponds to separatrix lines between excitation and damping zones with $\alpha = 0$ and S'_i denotes separatrix lines with $\alpha = 3.125 \times 10^{-4}$ sec^{-1}. The agreement with the experimental data is highly satisfactory. A data set more widely spread in latitude is necessary to compare with the predictions of Figs. 1b and 1c.

4. Resonant interaction

As it is well known nonlinearities in the equations and boundary conditions governing any wave motion, lead to wave in-

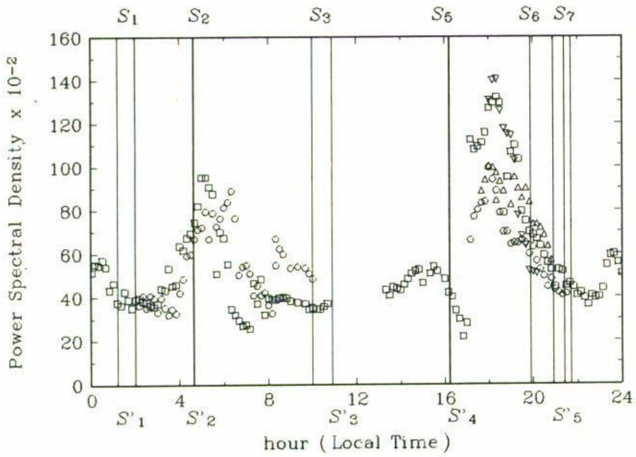

FIGURE 2. Experimental values of spectral power density in the range 10–90 minutes obtained at: □ Buenos Aires, ○ Tucumán and Δ, ∇ San Juan. S_i denotes the separatrix lines between excitation and damping zones with $\alpha = 0$. S'_i denotes the separatrix lines between excitation and damping zones with $\alpha = 3.125 \times 10^{-4}$ sec^{-1} (South Atlantic anomaly).

teraction. Nonlinear terms can be thought of as forcing the linear oscillations, being the response of the same order as the forcing term. An exception takes place when any of the forcing terms has the same period and wavenumber as one of the normal modes. This is called resonant interaction. Two waves may interact at second order to excite a third wave by resonance if not only are their frequencies added to that of the third wave but their wavenumbers are also added [7].

A resonant triad occurs only if the dispersion relation is such that

$$f(\vec{k}_1 + \vec{k}_2) = f(\vec{k}_1) + f(\vec{k}_2) \tag{16}$$

and

$$\vec{k}_3 = \vec{k}_1 + \vec{k}_2. \tag{17}$$

This dispersion relation may be such that no triad of waves can satisfy all the resonance conditions. There are two kinds of resonant conditions

$$\omega_1 = \omega_2 + \omega_3 \quad \text{with} \quad \vec{k}_1 = \vec{k}_2 + \vec{k}_3, \tag{18}$$

$$\omega_1 = \omega_2 - \omega_3 \quad \text{with} \quad \vec{k}_1 = \vec{k}_2 - \vec{k}_3. \tag{19}$$

In order to obtain finite and stable tidal and gravity amplitude disturbances, conditions given by Eq. (19) must hold. If condition of Eq. (18) holds, the amplitude of a wave with frequency ω_1 is nonlinearly unstable [7, 8] . Independent resonant triad are usually associated with problems with one space variable and therefore only a scalar wavenumber [8].

Consider the following triad $(\vec{k}_1, \vec{k}_2, \vec{\varepsilon})$ where $|\vec{k}_1| \sim |\vec{k}_2| \sim |\vec{k}|$ and $|\vec{\varepsilon}| \ll |\vec{k}|$. Then $\vec{k}_1 = \vec{k} + \vec{\varepsilon}$ and $\vec{k}_2 = \vec{k}$ form a resonant stable triad if

$$f(\vec{\varepsilon}) = f(\vec{k}_1) - f(\vec{k}_2) \cong \vec{\nabla}_k f \cdot \vec{\varepsilon}. \tag{20}$$

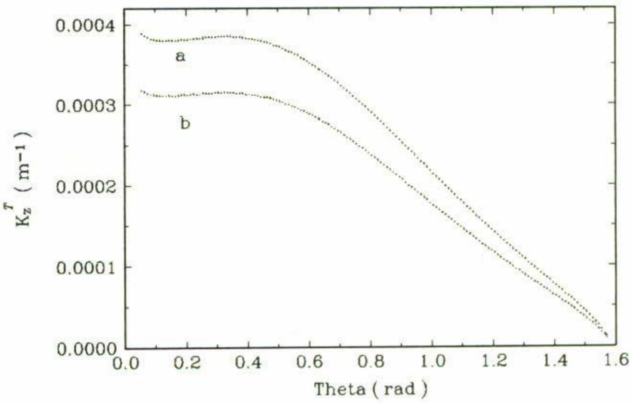

FIGURE 3. Tidal wavenumber k_z as a function of the latitude θ for separatrix S_3.

For the tidal waves the mode identification is normally based on the observation of the vertical structure and the tide temperature. The horizontal components are usually neglected ($k_x^T \simeq k_y^T \simeq 0$) [2]. Typical values in literature are $k_z^T \sim 10^{-4}$ m^{-1} for diurnal tides and $k_z^T \sim 10^{-6}$ m^{-1} for semidiurnal tides.

Equation (20) can be thought of as an equation from which the tidal $\vec{k}^T$ modes can be obtained for each pair of (θ, τ) values. Imposing a relative error $|F(k_z^T)/f(k_z^T)| \leq 10^{-2}$ with

$$F(k_z^T) \equiv f(k_z^T) - \left.\frac{\partial f}{\partial k_z}\right|_{\vec{k}^G} k_z^T = 0 \tag{21}$$

we can obtain the k_z^T values solving numerically Eq. (21).

For simplicity we solve Eq. (21) for the simplified model without magnetic effects. In that model the predicted gravity modes are $k_x^G \sim 10^{-5}$ m^{-1}, $k_y^G \sim 10^{-4}$ m^{-1} and $k_z^G \sim 10^{-3}$ m^{-1} for the separatrix lines between excitation and damping zones, denoted by S_1, S_3 and S_6 in Fig. 1a. We can assume collinearity between gravity and tidal waves which generally implies independence between triad if Eq. (21) is satisfied. For this group the numerical calculation gave us k_z^T values between 10^{-6} m^{-1} and 10^{-4} m^{-1}. A typical situation is displayed in Fig. 3 for the separatrix S_3. Curve a corresponds to frequency ω_1 and curve b to ω_2. These values are in very good agreement with the observational data mentioned above and with the phenomenological tidal solution used in Paper I (a linear combination of diurnal and semidiurnal tides).

For the separatrix S_2, S_5 and S_7 the predicted k_y^G values are of the same order of the $k_z^G \sim 10^{-3}$ m^{-1} values and $k_x^G \sim 10^{-5}$ m^{-1}. For them we could not find the triad which satisfies Eq. (21). Then, for these separatrixes the collinearity condition ($k_x^T = k_y^T = 0$), can be a strong hypothesis or an approximation of higher order must be required. Work in this direction is in progress.

5. Conclusions

The Navier-Stokes, continuity and energy balance equations including magnetic effects for an inviscid compressible fluid in the low atmospheric F-region were perturbed around the tidal phenomenological solution. We imposed adiabaticity and incompressibility to the perturbation.

We derived the linear damping and excitation regions of IGW for different values of the magnetic field intensity, in the parameter space of our model (Figs. 1–5). The separatrix lines (lines between excitation and damping zone of IGW) present a functional dependence with latitude θ and the slow time evolution of the tidal modes τ.

We examined the case of the South Atlantic anomaly (Fig. 4) and found that the predicted time dependence of $\vec{k}$ and ω is in very good agreement with model predictions of Giraldez *et al.* [6] and Canciani [9], and are confirmed by experimental data given in Fig. 5. These values do not differ much from the calculations without taking into account magnetic fields (Fig. 1) and are also indicated in Fig. 5.

However, great differences of the general pattern of regions are observed for low and high latitudes magnetic intensities (Figs. 2 and 3). These model predictions need to be contrasted with data more widely spread in latitude.

We also show how to calculate the tidal and gravity modes as a function of latitude and time, for those hours that it is possible, assuming they form a resonant independent and stable triad. In these regions, where tidal modes are linearly unstable, the gravity group velocity is modulated by a function of the tidal phase velocity.

Acknowledgments

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Appendix I

The coefficients **C** of Eq. (11a) are:

$$\begin{aligned}
\mathbf{C}_{00} = \frac{\partial\rho}{\partial z}\Big\{ &\alpha_1(\vec{v}\cdot\vec{k})^2 + s^2(\vec{v}\cdot\vec{k})\,(k_z\eta_2 - 2k_y\phi_x) + \alpha_2\beta + 2\delta\phi_x - 4\alpha_3\phi_z + 2\alpha_2\eta_1\phi_z - 2\,(2\nu_2 + \gamma\phi_x)\,W \\
&+ \alpha\Big[2k_z s^2(\vec{v}\cdot\vec{k}) + \alpha_2\eta_2 - 2\phi_x\Big(\psi_1 W + \frac{\partial W}{\partial z}\Big)\Big] - \alpha^2\alpha_2\Big\} \\
+ \rho\Big\{ &(\vec{v}\cdot\vec{k})^4 - (\vec{\nabla}\cdot\vec{v})\Big[(\beta + 2\eta_1\phi_z)\frac{\partial W}{\partial z} - 2\,(2\nu_2 + \gamma\phi_x)\Big] \\
&- (\vec{v}\cdot\vec{k})^2\Big[(\vec{\nabla}\cdot\vec{v})^2 + \frac{\partial W}{\partial z}\eta_2 - 2\Big(\frac{\partial V}{\partial z}\phi_x + \eta_1\phi_z\Big) - \beta + 4\Phi^2\Big] \\
&+ \alpha\Big[(\vec{v}\cdot\vec{k})^2\Big(\frac{\partial W}{\partial z} + 3(\vec{\nabla}\cdot\vec{v})\Big) + 2\phi_x\psi_1(\vec{\nabla}\cdot\vec{v}) - \eta_2\frac{\partial W}{\partial z}\Big(\eta_2 + \frac{\partial W}{\partial z}\Big)\Big]\alpha^2\Big[\frac{\partial W}{\partial z}(\vec{\nabla}\cdot\vec{v}) - (\vec{v}\cdot\vec{k})^2\Big]\Big\},
\end{aligned}$$

$$\begin{aligned}
\mathbf{C}_{10} = \frac{\partial\rho}{\partial z}\Big\{ &- 2\alpha_1(\vec{v}\cdot\vec{k}) - s^2\,(k_z\eta_2 - 2k_y\phi_x) + 2\alpha k_z s^2\Big\} \\
+ \rho\Big\{ &2(\vec{v}\cdot\vec{k})\Big[(\vec{\nabla}\cdot\vec{v})^2 - 2(\vec{v}\cdot\vec{k})^2 + \frac{\partial W}{\partial z}\eta_2 - 2\frac{\partial V}{\partial z}\phi_x - 2\eta_1\phi_z - \beta + 4\Phi^2\Big] \\
&- 2\alpha(\vec{v}\cdot\vec{k})\Big[\frac{\partial W}{\partial z} + 3(\vec{\nabla}\cdot\vec{v})\Big] + 2\alpha^2(\vec{v}\cdot\vec{k})\Big\},
\end{aligned}$$

$$\mathbf{C}_{20} = \alpha_1\frac{\partial\rho}{\partial z} + \rho\Big\{6(\vec{v}\cdot\vec{k})^2 - (\vec{\nabla}\cdot\vec{v})^2 - \frac{\partial W}{\partial z}\eta_2 + 2\frac{\partial V}{\partial z}\phi_x + 2\eta_1\phi_z + \beta - 4\Phi^2 + \alpha\Big[\frac{\partial W}{\partial z} + 3(\vec{\nabla}\cdot\vec{v})\Big] - 2\alpha^2\Big\},$$

$$\mathbf{C}_{30} = -4\rho(\vec{v}\cdot\vec{k}),$$

$$\mathbf{C}_{40} = \rho.$$

The coefficients **D** of Eq. (11b) are:

$$\begin{aligned}
\mathbf{D}_{00} = \frac{\partial\rho}{\partial z}\Big\{ &2k_y s^2\phi_x\frac{\partial U}{\partial x} - 2k_x s^2\phi_x\Big(\frac{\partial V}{\partial x} + 2\phi_z\Big) + k_z s^2\big[(\vec{v}\cdot\vec{k})^2 + 2\,(\eta_1 - 2\phi_z)\,\phi_z + \beta\big] \\
&+ (\vec{v}\cdot\vec{k})\,[+2\phi_x\,(\mu + 2\varphi) - \alpha_1\eta_2] + \alpha\big[k_z s^2\eta_2 - 2k_y\phi_x + 2(\vec{v}\cdot\vec{k})\,(\alpha_1 - V\phi_x)\big] - k_z s^2\alpha^2\Big\} \\
+ \rho(\vec{v}\cdot\vec{k})\Big\{ &(\vec{\nabla}\cdot\vec{v})\Big[- 2(\vec{v}\cdot\vec{k})^2 + \frac{\partial W}{\partial z}\eta_2 - 2\frac{\partial V}{\partial z}\phi_x - 2\eta_1\phi_z - \beta + 4\Phi^2\Big] - (2\eta_1\phi_z + \beta)\,\frac{\partial W}{\partial z} + 4\nu_2 \\
&+ 2\gamma\phi_x\alpha(\vec{v}\cdot\vec{k})\Big[2(\vec{v}\cdot\vec{k})^2 + 2\frac{\partial W}{\partial z}\Big(\frac{\partial W}{\partial z} - 2(\vec{\nabla}\cdot\vec{v})\Big) + 2\psi_1\phi_x - {\eta_2}^2\Big] + \alpha^2(\vec{v}\cdot\vec{k})\Big(\frac{\partial W}{\partial z} + (\vec{\nabla}\cdot\vec{v})\Big)\Big\},
\end{aligned}$$

$$\mathbf{D}_{10} = \frac{\partial \rho}{\partial z}\{ - 2k_z s^2(\vec{v}\cdot\vec{k}) + \alpha_1\eta_2 - 2\phi_x(\mu+2\varphi) + 2\alpha(\alpha_1 - V\phi_x)\}$$
$$+ \rho\Big\{(\vec{\nabla}\cdot\vec{v})\Big[6(\vec{v}\cdot\vec{k})^2 - \frac{\partial W}{\partial z}\eta_2 + 2\frac{\partial V}{\partial z}\phi_x + 2\eta_1\phi_z + \beta - 4\Phi^2\Big] + \frac{\partial W}{\partial z}[\beta + 2(\eta_1 - 2\phi_z)\phi_z]$$
$$- 2(\gamma + 2\nu_1)\phi_x + \alpha\Big[6(\vec{v}\cdot\vec{k})^2 - \frac{\partial W}{\partial z}\Big(\frac{\partial W}{\partial z} - 2(\vec{\nabla}\cdot\vec{v})\Big) - 2\psi_1\phi_x + \eta_2^2\Big] - \alpha^2\Big(\frac{\partial W}{\partial z} + (\vec{\nabla}\cdot\vec{v})\Big)\Big\},$$

$$\mathbf{D}_{20} = k_z s^2\frac{\partial \rho}{\partial z} - 6\rho(\vec{v}\cdot\vec{k})\{(\vec{\nabla}\cdot\vec{v}) + \alpha\}, \qquad \mathbf{D}_{30} = 2\rho\{(\vec{\nabla}\cdot\vec{v}) - 2\alpha\},$$

where

$$\alpha_1 = g + 2\phi_x V + \frac{\partial W}{\partial t} + \frac{\partial W}{\partial z}W, \quad \alpha_2 = g + \frac{\partial W}{\partial t} + \frac{\partial W}{\partial z}W, \quad \alpha_3 = \frac{\partial U}{\partial t}\phi_x + \Big(g + \frac{\partial W}{\partial t}\Big)\phi_z,$$
$$\beta = \frac{\partial U}{\partial y}\frac{\partial V}{\partial x} - \frac{\partial U}{\partial x}\frac{\partial V}{\partial y}, \quad \gamma = \frac{\partial U}{\partial z}\frac{\partial V}{\partial x} - \frac{\partial U}{\partial x}\frac{\partial V}{\partial z}, \quad \delta = \frac{\partial U}{\partial x}\frac{\partial V}{\partial t} - \frac{\partial U}{\partial t}\frac{\partial V}{\partial x},$$
$$\eta_1 = \frac{\partial U}{\partial y} - \frac{\partial V}{\partial x}, \quad \eta_2 = \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y},$$
$$\nu_1 = \frac{\partial U}{\partial x}\phi_x + \frac{\partial U}{\partial z}\phi_z, \quad \nu_2 = \frac{\partial U}{\partial x}{\phi_x}^2 + \frac{\partial U}{\partial z}\phi_x\phi_z + \frac{\partial W}{\partial z}\phi_z^2,$$
$$\mu = (\vec{v}\cdot\vec{\nabla}V) + \frac{\partial V}{\partial t}, \quad \varphi = \phi_z U - \phi_x W, \quad \psi_1 = \frac{\partial V}{\partial z} - 2\phi_x.$$

Appendix II

$$k_x^{(0)} = \Big\{\phi_x(\beta V - \delta) + \phi_z(2\alpha_3 - 2\alpha_1\phi_z + 2\eta_1\phi_x V) + \frac{\partial U}{\partial x}[\alpha_1\eta_2 - \phi_x(\mu + 2\varphi)] + (2\nu_2 + \gamma\phi_x)W\Big\} \times \Big\{s^2\phi_x\Big(\frac{\partial V}{\partial x} + 2\phi_z\Big)\Big\}^{-1}$$

$$k_x^{(1)} = \Big\{\frac{\partial U}{\partial x}(V\phi_x - 2\alpha_1) + \phi_x\Big(\frac{\partial V}{\partial t} + \eta_2 V + \psi_1 W + \mu + 2\varphi\Big)\Big\}\Big\{s^2\phi_x\Big(\frac{\partial V}{\partial x} + 2\phi_z\Big)\Big\}^{-1}$$

$$k_x^{(2)} = 2(\alpha_1 - V\phi_x)\Big\{s^2\phi_x\Big(\frac{\partial V}{\partial x} + 2\phi_z\Big)\Big\}^{-1} \qquad k_y^{(0)} = \{\alpha_1\eta_2 - \phi_x(\mu + 2\varphi)\}\{s^2\phi_x\}^{-1}$$

$$k_y^{(1)} = -(2\alpha_1 - V\phi_x)\{s^2\phi_x\}^{-1} \qquad k_z^{(0)} = \alpha_1 s^{-2}$$

1. S. Blanco, A. Costa, G. Domenech, and O.A. Rosso, *Rev. Mex. Fís.* **3** (2000) 46.
2. S. Katto, *Dynamic of the Upper Atmosphere*, (Reidel Publishing Co., Dodrech, 1980) p. 108.
3. S. Chapman and R. Lindzen, *Atmospheric Tides*, (Gordon and Breach Science Pub., Dordecht Holland, 1970) p. 106.
4. M. Lesieur, *Turbulence in Fluids*, (Kluwer Academic Publishers, Dodrech, 1990) p. 40, 80.
5. E.H. Vestine, *Physics of Geomagnetic Phenomena; 1*, edited by Matsushita and Campbell, (Academic Press, New York, 1967), Chap. 2.
6. A. Giraldez, U. Boldes, and J. Colman, *Lat. Am. Applied Res.* **19** (1989) 125.
7. P.G. Dkazin and H. Reid, *Hydrodynamic stability*, (Cambridge University Press, Cambridge, 1981).
8. F.P. Bretherton, *J. Fluid Mech.* **20** (1964) 457.
9. P. Canciani, Ph.D. Thesis, Universidad de Buenos Aires, Argentina, 1991, p. 73.