Cosmological constant and brane new world

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The estimation of the cosmological constant in inflationary brane new world models is done. It is shown that basically it is quite large, of the same order as in anomaly-driven inflation. However, for some fine-tuning of bulk gravitational constant and AdS scale parameter l^2 it maybe reduced to sufficiently small value. Bulk higher derivative AdS gravity with quantum brane matter may also serve as the model where small positive cosmological constant occurs.

Keywords: Cosmological constant; brane-world; quantum gravity

La constante cosmológica es estimada en el contexto de modelos inflacionarios del nuevo mundo de branas. Se muestra esencialmente que es extremadamente grande, del mimso orden que en la inflación dominada por la anomalía. Sin embargo, para un ajuste fino de la constante gravitatoria en el bulto y del parámetro de escala AdS l^2, parece poder ser reducida a un valor suficientemente pequeño. Gravitación con derivadas de orden superior en espacio AdS en el bulto con materia cuántica en la brana, podría también servir como un modelo en donde la constante cosmológica es pequeña y positiva.

Descriptores: Constante cosmológica; mundo de brana; gravedad cuántica

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1. Introduction

It is a quite well-known fact that energy density of the vacuum appears in Einstein equations in the form of an effective cosmological constant. In other words, vacuum (or vacuum polarization) induces the effective cosmological constant which curves the observable 4D-world. Roughly speaking, 4D-curvature is of the order of the square root from the cosmological constant which should include not only vacuum contribution but also other (dark?) matter contributions. According to recent observations (for a review and list of references, see Refs. 1 and 2) the cosmological constant is positive and small. It is interesting that positive small cosmological constant is not what is expected from string theory. Nevertheless, there are some suggestions how to get small cosmological constant within string theory $[3, 4]$

The fundamental question in cosmology is why the observable cosmological constant is so small? QFT considerations predict quite large vacuum energy and hence, quite large cosmological constant. Of course, it could be that the cosmological constant at very early Universe was large. However, due to some dynamical mechanism (supersymmetry? orbifold compactification?) it was reduced to the current small value. It would be interesting to understand the role of quantum effects as concerns to cosmological constant in brane-world physics. In the present work we discuss the cosmological constant value which appears in brane new world suggested in Refs. 7 and 8. Brane new world scenario represents quantum (or AdS/CFT induced) generalization of Randall-Sundrum universe $[10]$ where brane quantum fields are taken into account. It is interesting that quantum-induced brane inflation $[8, 9]$ (for related works, Ref. 10) occurs in the analogy with trace-anomaly driven inflation $[12]$.

2. Quantum-corrected cosmological constant

We start from the FRW-universe equation of motion with quantum corrections (taking into account conformal anomalyinduced effective action). Such quantum-corrected FRW-equation has the form [6]

$$H^2 = -\frac{1}{a^2} + \frac{8\pi G}{3}\frac{E}{V} + \frac{8\pi G}{3}\Bigg\{ - b'\left(4HH_{,tt} + 12H_{,t}H^2 - 2H_{,t}^2 + 6H^4 + \frac{8}{a^2}H^2\right) + \frac{1}{12}\left[b'' + \frac{2}{3}(b+b')\right]\left(- 36H_{,t}^2 + 216H_{,t}H^2 + 72HH_{,tt} - \frac{72}{a^2}H^2 + \frac{36}{a^4}\right) + \frac{\tilde{a}}{a^4}\Bigg\}, \quad (1)$$

where V is the spatial volume of the universe, $\tilde{a} = -8b'$ (a normalization choice [6]), $b'' = 0$, b is not necessary in the subsequent analysis and

$$b' = -\frac{N + 11N_{1/2} + 62N_1 - 28N_{\rm HD} + 1411N_2 + 1566N_{\rm W}}{360(4\pi)^2}. \tag{2}$$

Here $N, N_{1/2}, N_1$, and $N_{\rm HD}$ are the number of scalars, (Dirac) spinors, vectors and higher derivative conformal scalars which are present in conformal QFT filling the Universe. The quantity N_2 denotes the contribution to conformal anomaly from a spin-2 field (Einstein gravity) and $N_{\rm W}$ the contribution from higher-derivative Weyl gravity. As usually, the quantum corrections produce an effective cosmological constant.

In the absence of classical matter energy ($E = 0$), the general FRW equation allows the quantum-induced de Sitter space solution (anomaly-driven inflation [12])

$$\begin{aligned} a(t) =& A\cosh Bt, \\ ds^2 =& dt^2 + A^2\cosh^2\frac{t}{A}\, d\Omega_3^2, \end{aligned} \tag{3}$$

where A is a constant and $B^2 = 1/A^2 = -1/16\pi G b'$. It is evident then that the effective cosmological constant is defined as follows:

$$\Lambda_{\rm eff} = \frac{3}{A^2} = -\frac{3}{16\pi G b'}. \tag{4}$$

If b' is of order unity (what is typical in standard model), we find

$$\Lambda_{\rm eff} \sim \left(10^{19}{\rm GeV}\right)^2; \tag{5}$$

it is quite large.

The natural question now is: what happens for similar inflationary brane-world scenario? Following the approach of Ref. 12 in Ref. 6, the quantum-corrected FRW-type equation was considered as it is predicted by inflationary brane universe in the bulk Schwarzschild-AdS$_5$ spacetime:

$$\begin{aligned} ds^2_{\rm AdS-S} &= \frac{1}{h(a)}\, da^2 - h(a)\, dt^2 + a^2\, d\Omega_3^2, \\ h(a) &= \frac{a^2}{l^2} + 1 - \frac{16\pi G_5 M}{3V_3 a^2}. \end{aligned} \tag{6}$$

Here G_5 is 5D-Newton constant and V_3 is the volume of the unit 3 sphere. The quantum-corrected FRW type equation looks as [7]

$$H^2 = -\frac{1}{a^2} + \frac{8\pi G\rho}{3}, \tag{7}$$

$$\begin{aligned} \rho = \frac{l}{a}\Bigg\{ \frac{M}{V_3 a^3} + \frac{3a}{16\pi G_5}\Bigg[\Bigg(\frac{1}{l} + \frac{\pi G_6}{3}\Bigg\{ -4b'\Big[\left(H_{,\tilde{t}\tilde{t}\tilde{t}} + 4H_{,\tilde{t}}^2 + 7HH_{,\tilde{t}\tilde{t}} + 18H^2H_{,\tilde{t}} + 6H^4\right) + \frac{4}{a^2}\left(H_{,\tilde{t}} + H^2\right)\Big] \\ +4(b+b')\Big[\left(H_{,\tilde{t}\tilde{t}\tilde{t}} + 4H_{,\tilde{t}}^2 + 7HH_{,\tilde{t}\tilde{t}} + 12H^2H_{,\tilde{t}}\right) - \frac{2}{a^2}\left(H_{,\tilde{t}} + H^2\right)\Big]\Bigg\}\Bigg)^2 - \frac{1}{l^2}\Bigg]\Bigg\}. \end{aligned} \tag{8}$$

Here 4D-Newton constant G is given by

$$G = \frac{2G_5}{l}. \tag{9}$$

When $M = 0$, the above Eq. (7) has a solution of the form (3) if $A^2 = 1/B^2$ and

$$0 = -B^2 - \frac{1}{l^2} + \left(\frac{1}{l} - 8\pi G_5 b' B^4\right)^2. \tag{10}$$

For negative b' Eq. (10) has a unique solution. The solution is nothing but the de Sitter brane solution in Refs. 7 and 8. Equation (10) can be rewritten as

$$0 = -1 + 2\beta C + \beta^2 C^3. \tag{11}$$

Here

$$\begin{aligned} C \equiv& l^2 B^2, \\ \beta =& -\frac{8\pi G_5 b'}{l^3} = -\frac{4\pi G b'}{l^2}. \end{aligned} \tag{12}$$

Then the effective cosmological constant is given by

$$\Lambda_{\rm eff} = \frac{3}{A^2} = 3B^2 = \frac{3C}{l^2}. \tag{13}$$

If $|\beta| \gg 1$, a solution of Eq. (11) is given by

$$C = \frac{1}{2\beta}\left[1 + \mathcal{O}\left(\frac{1}{\beta}\right)\right] = -\frac{l^2}{8\pi G b'}\left[1 + \mathcal{O}\left(\frac{1}{\beta}\right)\right], \tag{14}$$

and one gets

$$\Lambda_{\rm eff} = -\frac{3}{8\pi G b'}\left[1 + \mathcal{O}\left(\frac{1}{\beta}\right)\right], \tag{15}$$

which is different from Eq. (4) by factor two but there is no qualitative difference. On the other hand, if $|\beta| \ll 1$, a solution of Eq. (11) is given by

$$C = \frac{1}{\beta^{2/3}}\left[1+\mathcal{O}\left(\beta^{1/3}\right)\right] = \left(-\frac{l^2}{4\pi G b'}\right)^{\frac{2}{3}}\left[1+\mathcal{O}\left(\beta^{1/3}\right)\right], \quad (16)$$

and we find

$$\Lambda_{\rm eff} = -\frac{3}{l^2}\left(-\frac{l^2}{4\pi G b'}\right)^{\frac{2}{3}}\left[1+\mathcal{O}\left(\beta^{1/3}\right)\right]. \quad (17)$$

Since $|\beta| \ll 1$ means $l^3 \gg G_5$ or $l^2 \gg G$, the effective cosmological constant $\Lambda_{\rm eff}$ can be small in this case. Note that one can write other solutions for effective cosmological constant from above cubic equation. However, in most cases it is getting very large.

Motivated by AdS/CFT correspondence (for a review, Ref. 13), we may consider $\mathcal{N} = 4SU(N)$ SYM theory on the brane. Then

$$b = -b' = \frac{N^2-1}{4(4\pi)^2}, \quad (18)$$

and

$$\frac{l^3}{G_5} = \frac{2N^2}{\pi}. \quad (19)$$

In the large N limit, we have

$$\beta = -\frac{8\pi G_5 b'}{l^3} = \frac{1}{16}. \quad (20)$$

Then by numerical solving Eq. (11), one finds

$$C = 4.71804. \quad (21)$$

In this case $\Lambda_{\rm eff} \sim \mathcal{O}\left(l^{-2}\right) \sim \left(10^{19}\ {\rm GeV}\right)^2$ again. Hence, for decreasing the cosmological constant one has to consider QFT which is not exactly conformally invariant (for a recent AdS/CFT discussion of such theories, see Ref. 17). From another point, one may take the arbitrary bulk values for AdS parameter and five-dimensional gravitational constant in order to achieve the smallness of the cosmological constant. The drawback of this is evident: it is kind of fine-tuning.

If we include quantum bulk scalar or spinor, they induce the Casimir effect in orbifold compactification. The corresponding vacuum energy which may stabilize the radius was found in Refs. 14 and 15. In the euclidean signature, de Sitter space is expressed as the sphere. In Ref. 14, the Casimir effect makes the radius smaller or larger, especially the conformal scalar in the bulk makes the radius small and time-dependent. In Minkowski signature, the inverse of the radius corresponds to the expansion rate (*i.e.* B) of the universe. Then from Eq. (13), the conformal scalar in the bulk increases the effective cosmological constant. Note, however, that taking into account the bulk quantum gravity with 5D-cosmological constant may presumably help in resolution of the problem. Unfortunately, the corresponding calculation is quite complicated and it is not done so far.

One may consider 5D-R^2 gravity, whose action is given by:

$$S = \int d^5x\,\sqrt{-\hat{G}} \times\left(a\hat{R}^2 + b\hat{R}_{\mu\nu}\hat{R}^{\mu\nu} + c\hat{R}_{\mu\nu\xi\sigma}\hat{R}^{\mu\nu\xi\sigma} + \frac{1}{\kappa^2}\hat{R} - \Lambda\right), \quad (22)$$

and the brane with quantum matter corrections as in the previous case. When $c = 0$, the net effects can be absorbed into the redefinition of the Newton constant and the length scale l of the AdS_5 [17], given by

$$\frac{1}{16\pi G_5} = \frac{1}{\kappa^2} \to \frac{1}{16\pi\tilde{G}_5} = \frac{1}{\tilde{\kappa}^2} = \frac{1}{\kappa^2} - \frac{40a}{l^2} - \frac{8b}{l^2}, \quad (23)$$

$$0 = \frac{80a}{l^4} + \frac{16b}{l^4} - \frac{12}{\kappa^2 l^2} - \Lambda. \quad (24)$$

Then assuming the brane solution as in Eq. (3) (brane new world in higher derivative gravity), one obtains the analogue of Eq. (10) from corresponding FRW-equation

$$0 = -B^2 - \frac{1}{l^2} + \left(\frac{1}{l} - 8\pi\tilde{G}_5 b' B^4\right)^2. \quad (25)$$

Furthermore if we replace G in Eq. (9) by

$$G = \frac{2\tilde{G}_5}{l}, \quad (26)$$

the arguments from Eqs. (14)–(17) are valid. In other words, using hidden parameters of bulk higher derivative terms one can obtain the 4D-cosmological constant to be reasonably small. This picture maybe generalized for the case of non-zero C, however, the corresponding equation for cosmological constant is more complicated. Nevertheless, the qualitative conclusions will be the same.

3. Discussion

In summary, we considered the effective cosmological constant in brane new world induced by quantum brane matter effects. Rough estimation for inflationary brane indicates that in most cases the cosmological constant is quite large. Fine-tuning of bulk 5D-gravitational and 5D-cosmological constant may sometimes lead to significant decrease of 4D-cosmological constant. Of course, one can imagine the situation that large cosmological constant at the beginning of inflationary era is reduced to current small value by some mechanism in course of evolution. Nevertheless, it looks that new brane world scenario by itself cannot suggest a natural way to solve the cosmological constant problem.

1. S.M. Carroll, astro-ph/0004075.
2. E. Witten, hep-ph/0002297.
3. B. McInnes, hep-th/0105151.
4. A. Chamblin and N.D. Lambert, hep-th/0102159.
5. R. Garattini, gr-qc/0003090.
6. S. Nojiri and S.D. Odintsov, hep-th/0011115, to appear in *Int. J. Mod. Phys. A*; S. Nojiri *et al.*, hep-th/0105052.
7. S. Nojiri and S.D. Odintsov, hep-th/0103078.
8. S.W. Hawking, T. Hertog and H.S. Reall, *Phys. Rev. D* **62** (2000) 043501; hep-th/0003052.
9. S. Nojiri and S.D. Odintsov, *Phys. Lett. B* **484** (2000) 119; hep-th/0004097; S. Nojiri, S.D. Odintsov, and S. Zerbini, *Phys. Rev. D* **62** (2000) 064006; hep-th/0001192; for a review, see S. Nojiri and S.D. Odintsov, hep-th/0105160.
10. L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83** (1999) 4690; hep-th/9906064.
11. L. Anchordoqui, C. Nunez and K. Olsen, hep-th/0007064; K. Koyama and J. Soda, hep-th/0101164; T. Shiromizu, T. Torii, and D. Ida, hep-th/0105256.
12. A. Starobinsky, *Phys. Lett. B* **91** (1980) 99; S.G. Mamaev and V.M. Mostepanenko, *JETP* **51** (1980) 9.
13. I. Savonije and E. Verlinde, hep-th/0102042.
14. O. Aharony *et al.*, *Phys. Rep.* **323** (2000) 183.
15. S. Nojiri, S.D. Odintsov, and S. Zerbini, hep-th/0006115, *Class. Quant. Grav.* **17** (2000) 4855.
16. J. Garriga, O. Pujolas, and T. Tanaka, hep-th/0004109; I. Brevik, K. Milton, S. Nojiri, and S.D. Odintsov, hep-th/0010205, *Nucl. Phys. B* **599** (2001) 305; R. Hoffman, P. Kanti, and M. Pospelov, hep-ph/0012213; A. Flachi, I. Moss, and D. Toms, hep-th/0103138; E. Ponton and E. Poppitz, hep-ph/0105021.
17. S. Nojiri, S.D. Odintsov, and S. Ogushi, hep-th/0105117.
18. M. Pérez-Victoria, hep-th/0105048.