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Superluminal rates of separation and EPR photons

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We consider rates of separation between two particles greater than c (which is not in contradiction with special relativity) in understanding some conflict between special relativity and quantum mechanics found by moving observers of correlated EPR pairs of photons. The photon frequencies observed in the moving frame have opposite shifts than those found when the detectors are fixed in that frame. By observing from two different frames the arrival of a photon from an EPR pair to a given detector, it is illustrated how the measurement of the simultaneous arrival of the other photon of the EPR pair in one frame disturbs the measurement in the other frame.

Keywords: Superluminal velocities; correlated EPR photon pairs.

Consideramos velocidades de separación entre dos partículas mayores que c (lo que no está en contradicción con la relatividad especial) en la búsqueda de la comprensión del conflicto entre la relatividad especial y la mecánica cuántica, encontrado por observadores móviles de pares correlacionados de fotones EPR en movimiento. Las frecuencias de los fotones observadas en el marco de referencia móvil tiene desplazamiento opuesto al que se encuentra cuando los detectores están fijos en tal marco. Mediante la observación, desde dos marcos diferentes, del arribo de uno de los fotones de un par EPR a un detector dado, se ilustra cómo la medición del arribo simultáneo del fotón contraparte en uno de los marcos, perturba la medición del arribo simultáneo de dicho fotón en cualquier otro marco.

Descriptores: Velocidades superiores a la de la luz; par de fotones correlacionados en EPR.

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1. Introduction

In his well-known book *Speakable and unspeakable in quantum mechanics*, the late John S. Bell [1] states "...*For me then this is the real problem with quantum theory: the apparently conflict between any sharp formulation and fundamental relativity. That is to say, we have an apparent incompatibility, at the deepest level, between the two fundamental pillars of contemporary theory...and of our meeting...It may be that a real synthesis of quantum and relativity theories requires not just technical developments but radical conceptual renewal*." Perhaps the radical conceptual renewal suggested by Bell would come from the consideration of fundamental extended objects, as is string theory. But keeping us in the framework of standard relativity and quantum theory we would like, however, to have a closer look at the *conflict* which arises from the uncompatibity among the ideas of instantaneous reduction of the wavepacket and the relative simultaneity of events.

We start by discussing briefly the problem of superluminal rates of separation between two objects in Sec. 2, and in Sec. 3 we apply such ideas to pairs of photons and detectors in a EPR experiment, as a way for the observer in a moving frame to understand his observations and to make them compatible with those of the observer at rest, from the quantum mechanical and special relativity points of view. In Sec. 4 the shift in frequencies measured by the moving observer is found, and compared to those obtained by him with detectors fixed to his frame. Then by considering the event of arrival of a photon from an EPR pair to a detector, as observed from two different frames of reference, it is discussed the simultaneous measurement of the other member of the EPR pair as made by both observers. Sec. 5 deals with the conclusions.

2. Superluminal velocities and rates of separation

It is usually believed that special relativity forbids any velocity greater than c, which is not right. Some readers would be felt worried even in reading such idea. What is established by special relativity is the constancy of the velocity of light for all observers, or in other words, that the maximum velocity of propagation of a signal in vacuum cannot exceed c in any frame of reference. To be more precise, this point must be related to transport of energy and causality. Then the statement must be made in terms of bounded velocity of transport of energy ($\leq c$) to preserve Einstein causality. Thus, it is possible to conceive systems in which superluminal velocities appear without violating Einstein causality [2, 3], but as pointed out before we would like to consider the problem in the present letter from the kinematical side, *i.e.*, by considering the rate of spatial separation between two objects with regard to a third body with an observer O, which can be produced at a velocity greater than c without violating the postulate of constancy of the velocity of light. Even Einstein in his original paper [4] introduced formally rates of separation $c \pm V$, and Feynman [5] did the same in discussing the Michelson-Morley experiment. On the opposite side is Fock [6], who defined the relative velocity between two moving objects as the velocity of the second with regard to the reference system fixed on the first. In this way he obtained a Lorentz-invariant expression for the relative velocity. However, we will concern with the rate of separation understood in the first sense, which is in general not Lorentz-invariant, but is, however intuitively clear. We name it the Einstein-Feynman (EF) rate of separation

As an extreme case of the EF rate of separation (on which we will deal in the rest of our paper), we give the elementary example which shows that it can be greater than c. For light coming from a point source, we have a spherical wave front propagating with velocity c. If one consider the wavefronts at the extremes of a diameter, we have obviously a relative velocity $v = 2c$, *i.e.*, photons separating at a rate $2c$ with regard to an observer in the system where the source is at rest. Or looking at the problem in another form, the surface joining the wavefronts in the light cone is parallel to the four-dimensional hyperplane perpendicular to the x_4 axis and moves upwards along the t axis at the speed c, and it is obviously spacelike. Opposite photons along a diameter, lying on the cone surface, separate at the speed $2c$ with regard to the observer at rest. Any observer would find that each photon moves at velocity c. With regard to one of the photons, the other photon recedes with velocity c, (this is the Fock relative velocity) and it is impossible for it to inform the second photon, say, about its state of polarization by using a signal propagating at velocity $v \leq c$. For an observer at rest, the spacelike interval grows at two times the velocity of light and is Lorentz-invariant. For particles moving at velocities smaller than c, however, this rate of separation is not Lorentz-invariant (see below). This is of especial interest in considering EPR pairs of correlated photons. The concept might be useful also in inflationary theories [7] which are based on the idea of exponential bubble expansion in which spacelike separated regions may expand at a speed larger than c.

We want to write a general expression for the rate of separation u_{21} of two points 1 and 2, moving in the rest system respectively at velocities u_1, u_2, according to a previous report of one of the present authors [8]. We write, thus, $u_{21} = u_2 - u_1$.If $u_1 = \pm c$, one would have

$$u_{21} = u_2 \mp c \ ,$$

and it is not in contradiction with special relativity at all.

In a frame moving at velocity V, the respective velocities are w_1, w_2, where

$$w_i = \frac{u_i + V}{1 + u_i V/c^2} \ . \tag{1}$$

One can write then

$$w_{21} = \frac{(u_{21})(1-\beta^2)}{(1+\beta_2\beta)(1+\beta_1\beta)} \ , \tag{2}$$

where $\beta_i = u_i/c$, $\beta = V/c$ and $u_{21} = u_2 - u_1$. It is particularly interesting the case in which $u_1 = -u_2$, which *does not mean* that $w_1 = -w_2$ (except if $u_2 = c$). We have

$$w_{21} = \frac{u_{21}(1-\beta^2)}{1-\beta_2^2\beta^2} \ . \tag{3}$$

In consequence, in general $w_{21} \leq u_{21}$. The equality case corresponds obviously to the case $u_2 = c$, leading to

$$w_{21} = u_{21} = 2c \ . \tag{4}$$

Thus, the maximum allowed rate of separation between two bodies with regard a third one is $2c$, which *is* also a relativistic invariant.

3. Measurements of pairs of correlated photons

We want to consider now the problem of the pair of correlated photons in a usual EPR experiment. Let us assume that green photons are sent to the left and red to the right. Calling v and h the vertical and horizontal polarizations, the entangled wavefunction can be written as

$$\psi = |v(x,\tau)> |h(-x,\tau)> + |h(x,\tau)> |v(-x,\tau)> \ . \tag{5}$$

Equation (5) describes an extended wavefunction, which contains the idea of non-locality in a transparent way.

We shall use in what follows the von Neumann - Penrose [9, 10] concepts concerning procedures U (unitary evolution) and R (measurement or reduction of the wavepacket).

The measurements of polarizations at the two symmetric left-hand A and right-hand B polarizers + detectors are two space-like events. If the polarizer B is removed,a measurement at A, say v, means that the wavefunction of the unmeasured photon at B jumps simultaneously to h.

Let us name K, K' the rest and moving systems respectively (K' moving to the left, that is, with velocity $-V$). An essential conflict appears [10] between Quantum Mechanics (if we consider the notion of instantaneous reduction of the wavepacket) and *the spirit* of Special Relativity (covariance, relative simultaneity) when the process of measurement at the left and wavefunction jump at right is observed from the moving system K'. The moving observer would find that the left photon polarization at A is measured at a time t_1' before the right photon arrives at B, and the simultaneous event at the right (jump of the wavefunction) is expected to occur before the arrival of the signal at B; thus, a puzzle appears. An opposite conclusion would be obtained for another observer moving to the right at velocity V, leading to contradictory statements to both observers.

We want to give an operational way of characterizing the two times of arrival of the photons at the detectors. In K, the times of arrival of the photons to the detectors are determined by the quotients of the lengths of the apparatus arms by the velocity of light. Let us name by L the common length of the arms of the polarizer and then $t = L/c$ is the common value of the time the photon takes to travel along both arms in K.

If a measurement is made, the R procedure on (5) leads to either $\psi_1 = |v(L,t) > |h(-L,t) >$ or $\psi_2 = |h(L,t) > |v(-L,t) >$. We want to carefully analize the conditions of both observers concerning the two photon detection trying to find a partial understanding of the source of the puzzle from the point of view of the observer in K'. For observers in K the times of arrival of the photons to A and B can be measured as said before either a) by the quotient of the length of the (equal) arms by the velocity of light *relative to the polarizer*, which is the velocity of light or b) by dividing the coordinates of the detectors by c. One gets the common value $t = L/c$ by following any of the procedures a) and b), which means the simultaneity of both events at K.

However, for observers in K' procedures a) and b) do not lead to the same results. Thus, to have equivalent kinematical conditions for the measurements in K, K', we must have a set of polarizers at rest in K' (which we will consider later) to be allowed to make a complete comparison of the R procedure in both systems.

Let us write the time of arrival of the two photons at the detectors A and B, as seen by an observer in K'.

The time of arrival of the photon to A can be written in K' as

$$t'_1 = \frac{L(c-V)}{c^2\sqrt{1-\beta^2}} = \frac{L\sqrt{1-\beta^2}}{c+V} \ . \tag{6}$$

From this equation it results

$$t'_1 = \frac{L}{c}\sqrt{\frac{1-\beta}{1+\beta}} < \frac{L}{c} = t \ .$$

If we call $L' = \sqrt{1-\beta^2}L$ the length of the arm as measured from the observer K', one can write

$$t'_1 = \frac{L'}{c+V} \ , \tag{7}$$

and for the arrival time at B, we have,

$$t'_2 = \frac{L'}{c-V} \ . \tag{8}$$

Thus, the observer O' in K' concludes that the photon arrives at A earlier than is indicated by the clocks in K, and earlier than the photon at B, since

$$t'_2 = \frac{L'}{c-V} > t > t'_1 = \frac{L'}{c+V} \ .$$

The observer O' in K' argues that this is due to the fact that the photon at the left moved along the length of the arm $L' = L\sqrt{1-\beta^2}$ (defined by the spacetime points $(x'+L',t');(x',t')$) *with regard to the detector* A at the superluminal (tachyonic) velocity $c+V$, faster than the photon in K and faster than the photon moving (with regard to K') to B, which traveled at the relative (sub-luminal or tardyonic) velocity $c-V$. Thus, the observer in K' explains the inequality in the times t'_1, t'_2 of actions of the R operation at A and B as some sort of *tachyonic* effect in A and *tardyonic* effect in B, concerning rates of separation between the light and the detectors. The wavefunction at the right jumps later than that at the left because it approaches B at a relative speed slower than c since the conditions under which the observations are made in the moving frame K' are not the same than the ones in K. (Note that $t > t'_1$ is due to the usual lag of the moving clock C. It is assumed that the rest and moving clocks were synchronized at the space-time coordinates $(0,0), (0',0')$, respectively, the moving clock being compared with two stationary ones at K . The fact that $t < t'_2$ does not contradicts the lag of moving clocks, and it results from the fact that t'_2 is measured in B with *another* clock C' in K', previously synchronized with C.) If the removed polarizer were A, the fact that the wavepacket jumped at $t'_1 < t'_2$ means nothing but an effect to be expected when superluminal-subluminal rates of separation are involved. For an observer moving to the right at velocity V, these effects are exchanged. (From the point of view of relativistic quantum field theory, the polarizations of photons arriving at A, B, as being spacelike separated, considered as local observables in quantum electrodynamics, are not dependent upon the order in which the measurements have been performed).

4. The shift of frequencies

Let us write the coordinates x'_1, x'_2 corresponding to the location of the detectors at t'_1, t'_2. We have

$$x'_1 = \frac{-L\sqrt{1-\beta^2}}{1+\frac{V}{c}} = ct'_1 \ , \tag{9}$$

$$x'_2 = \frac{L\sqrt{1-\beta^2}}{1-\frac{V}{c}} = ct'_2 \ . \tag{10}$$

Thus $|x'_1| \neq |x'_2|$. These quantities are respectively the distances traveled by the left and right photons along the x' axis, according to the observers in K'.

If we name $R_{l,r} = L/\lambda_{l,r}$, the quotient of the length of the arm by the wavelengths $\lambda_{l,r}$ of the left and right photons, *i.e.*, the number of wavelengths contained in L at left and right, then by naming $\lambda'_{l,r}$ the wavelengths registered by the detectors A, B as observed from the K' system, we expect that $x'_1 = -R_l\lambda'_l$, and similarly, $x'_2 = R_r\lambda'_r$. We get thus

$$\lambda'_{l,r} = \frac{\lambda_{l,r}\sqrt{1-\beta^2}}{1 \pm \frac{V}{c}} \ . \tag{11}$$

Here we get the paradoxical result that the corresponding frequencies of the green (left) and red (right) photons are shifted to the violet and to the infrared, respectively. This is due to the fact that although the source is moving to the right in the K' system, the detectors are *also* moving to the right, the rates of separation between the light and the detectors being observed respectively as $c+V$ and $c-V$. We note also that in K' it is needed either a moving observer, or a set of observers with synchronized clocks in K' to follow the motion of the detectors in K and to register the results of the measurements.

To conclude, let us now turn our attention to investigate the conditions under which the pair of photons can be observed in K' in an equivalent way as in K.

If the polarizations of the pair of photons emitted by the source in K are measured with polarizers + detectors *at rest in* K', the entangled wavefunction is now

$$\psi = |v(x',\tau') > |h(-x',\tau') > \\ +|h(x',\tau') > |v(-x',\tau') > \ . \quad (12)$$

Let us find the right and left photon wavelengths as measured by fixed detectors in K'. As emitted by a source moving *to the right*, their wavelengths are Doppler shifted to the values

$$\lambda"_{l,r} = \frac{\lambda_{l,r}\sqrt{1-\beta^2}}{1 \mp \frac{V}{c}} \ . \quad (13)$$

Thus, the situation becomes the opposite than before: if measured by detectors at rest, the left photon is shifted to the red and the right photon to the violet.

Now let us consider the event of arrival of the left photon to A in the frame K and the simultaneous event of arrival of the right photon, as observed in the frame K'. (Obviously, as A is moving to the right with regard to K', we consider the measurement of the arrival of the left photon to A, observed from K', as equivalent to the one made by a polarizer + detector $A"$ at rest in K', which would coince with A at the space-time point (x'_1, t'_1) , being $x'_1/\lambda"_l = -(c-V)R_l/(c+V)$). The right photon arrival must be measured with a detector B' fixed in K', at time t'_1, and at a distance $x_2" = -x'_1$ from the source, the length of the wavetrain in units $\lambda"_r$ observed in K' being

$$\frac{x_2"}{\lambda"_r} = R_r \ , \quad (14)$$

which is the same length of the wavetrain, in units λ_r, observed by K at the detector B. We conclude that if the left photon is measured at is arrival to A by both observers, the simultaneous reduction of the right photon in the frames K, K' would be detected in a different and independent way respectively by detectors B, B'. The detected photons would have obviously different quantum numbers (respectively, momenta $p = \hbar/\lambda_r$ and $p' = \hbar/\lambda"_r$). But we have assumed that the process of measurement does not change the photon, which is not true, since the right photon in B' is either absorbed or its momentum changed by the measurement, and the detector at B would measure either no photon or a photon with momentum $p" \neq p$.

5. Conclusions

We have seen how the relative superluminal velocity concept is useful in understanding the conflict between simultaneous quantum measurements and relative simultaneity, arising when correlated EPR pairs are observed in frames at rest K, and moving K': the relative rates of separation photon-detectors are different for both observers.

We have seen also that the simultaneous event to a measurement of the left photon in A, for the photon moving to the right (*i.e.*, simultaneous reduction of the wavepacket at the right), tested by detectors at rest with regard to different frames, occurs independently for each frame K,K' and *the* R *action on the right photon in one frame disturbs in general the* R *action on the same photon in any other frame* (see Aharonov and Albert [11]).

From all our discussion, we have an illustrative example of the statement made in [11] concerning the fact that in relativistic quantum field theories, the quantum states themselves make sense only within a given frame, (*e.g.*, the entangled wavefunctions (5) and (12) *are not* connected by a Lorentz transformation, and cannot be, since they represent non-local, and in consequence, non-covariant objects) and that covariance resides in the experimental probabilities (obtained from the results of the R action). In this way we may understand the present coexistence of special relativity and quantum mechanical concepts.

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