Otto and Diesel engine models with cyclic variability

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Typically, in an internal combustion engine, thousands of cycles are performed in a minute. In this sequence of cycles many physical and chemical quantities change from cycle to cycle. For example, the combustion heat changes due to residual gases, imperfect combustion and other reasons. In this work, we present two finite-time thermodynamics models for both an Otto and a Diesel cycle, in which the cyclic variability is studied as occurring in the heat capacities of the working fluid. The fluctuations considered are of the uncorrelated type (uniform and gaussian) and one correlated case (logistic map distribution). We find that in the correlated fluctuations case, the power output and the efficiency of both cycles reach bigger fluctuations than in the uncorrelated cases. This result can provide insights over the performance of internal combustion engines.

Keywords: internal combustion engine; cyclic variability; fluctuations

En máquinas de combustión interna, típicamente, miles de ciclos son realizados en un minuto. En esta secuencia de ciclos algunas cantidades físicas y químicas cambian de ciclo a ciclo. Por ejemplo, el calor de combustión cambia debido a gases residuales, a combustión imperfecta y a otras razones. En este trabajo presentamos dos modelos a tiempo finito para los ciclos Otto y Diesel, en los cuales se estudia la variabilidad cíclica como si estuviera ocurriendo en las capacidades caloríficas de la sustancia de trabajo. Consideramos tanto fluctuaciones descorrelacionadas (uniforme y gaussiana) como correlacionadas (distribución tipo mapa logístico). Encontramos que en el caso de las fluctuaciones correlacionadas, la potencia y la eficiencia de ambos ciclos alcanzan mayores fluctuaciones que en los casos descorrelacionados. Este resultado puede ayudar a comprender mejor el funcionamiento de máquinas de combustión interna.

Descriptores: máquinas de combustión interna, variabilidad cíclica, fluctuaciones

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1. Introduction

In 1996, Badescu and Andresen [1] proposed that finite-time thermodynamics (FTT) can be complemented with some probabilistic concepts allowing a more accurate description of the performance indicators of a power system. These authors studied a continuous flow tube reactor which supplies heat to an engine from a chemical reaction with linear kinetics. In general, typical FTT-models of thermal cycles are worked in steady state and only one cycle is taken as representative of all the other cycles pertaining to a sequence of them. In a typical internal combustion engine, several thousands of cycles are performed in a minute and there exist theoretical and experimental reasons to expect important variations from one cycle to the next [2, 3]. These variations can be found for example in the combustion heat of an Otto cycle [3, 4]. In a recent paper, Daw *et al.* [3] proposed a discrete engine model that explains how both stochastic and deterministic features can be observed in spark-ignited internal combustion engines. These authors present a model which reproduces the experimental observations of the cyclic variability of the combustion heat in a four-stroke, spark-ignition Otto cycle. Recently, we have reported an irreversible Otto cycle model including chemical reactions [5]. In that model, we analyzed the performance of an Otto engine (see Fig. 1) taking into account power losses due to a kind of lumped friction and we also consider the combustion reaction at the end of the adiabatic compression. In other work [6], we took the concept of fluctuant combustion heat proposed by Daw *et al.* [3] as the input of our irreversible Otto cycle model, and then we analyze the behavior of performance outputs, such as the power (P) and the efficiency (η) of the Otto cycle model. In that work, we found that the size of the fluctuations in P and η around their mean values can be driven by the size of the combustion heat fluctuations and also by thermodynamic properties of the states of the working fluid. In the present work, we study two thermal cycles, namely the Otto and the Diesel cycles, under heat fluctuations, but using an alternative approach, where the fluctuations are taken as occurring in the heat capacity of the working fluid. In our approach, we take two previous FTT- models for both the Otto [7] and the Diesel [8] cycles. The paper is organized as follows: in Sec. 2, we present a brief resume of our previous thermal cycle models; in Sec. 3 we discuss our fluctuant models of both cycles and finally in Sec. 4 we present the conclusions.

2. Otto and Diesel FTT-models

In Fig. 1 we depict the Otto cycle pressure-volume diagram of the processes followed by the working fluid consisting of n moles of air. In Ref. 7 it was considered that both the "absorbed" heat Q_{eff} and the rejected heat Q_{out} occur at finite times given by

$$t_{1V} = K_1 \left(T_3 - T_2\right), \quad \text{and} \quad t_{2V} = K_2 \left(T_4 - T_1\right), \tag{1}$$

respectively, where K_1 and K_2 are constants linked to the mean variation rate of the temperatures. In this approach the adiabatic processes were taken as approximately instantaneous, as it is common in FTT-models [9]. In this way, the cycle's period is given by,

$$\tau = t_{1V} + t_{2V} = K_1 \left(T_3 - T_2\right) + K_2 \left(T_4 - T_1\right). \tag{2}$$

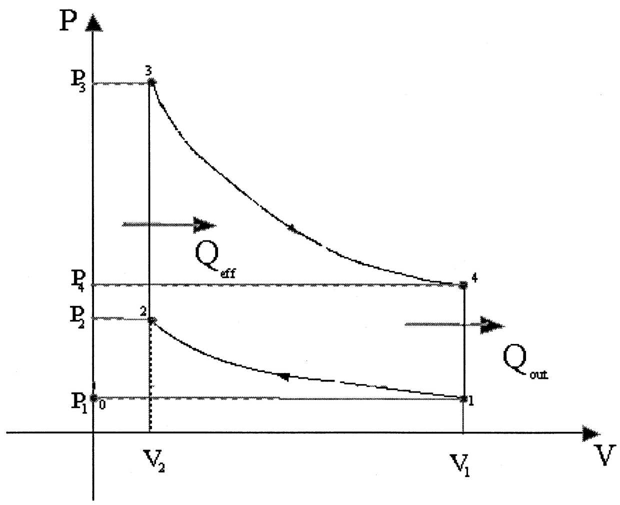

FIGURE 1 Pressure-volume diagram of an ideal Otto cycle.

In Ref. 7 the cycle's power output without losses was taken as

$$\mathrm{P_R} = \frac{W_{\text{TOT}}}{\tau} = \frac{C_{V_1} - C_{V_2} r^{1-\gamma}}{K_1 - K_2 r^{1-\gamma}}, \tag{3}$$

where C_{V_1} and C_{V_2} are the constant-volume heat capacities of air in both isochoric processes $2 \to 3$ and $4 \to 1$, and $\gamma = C_{P_1}/C_{V_1} = C_{P_2}/C_{V_2}$, being C_{P_1} and C_{P_2} the constant-pressure heat capacities of the working fluid during the processes $2 \to 3$ and $4 \to 1$ respectively, $r = V_1/V_2$ is the so-called compression ratio. When in the model, losses due to friction, turbulence in the working fluid, heat leaks etc, are added, all of them lumped in only a friction-like term, we have [7]

$$\mathrm{P}_\mu = -\mu\nu^2 = -b\left(r-1\right)^2, \tag{4}$$

where μ is a lumped friction coefficient that embraces all the global losses, ν is the piston speed and $b = \mu x_2^2/\Delta t_{12}^2$, being x_2 the piston position at minimum volume V_2 and $\Delta t_{12} = \tau/2$ the time spent in the power stroke. Thus, the Otto model effective power output is given by

$$\mathrm{P} = \mathrm{P_R} - \mathrm{P}_\mu = \frac{C_{V_1} - C_{V_2} r^{1-\gamma}}{K_1 - K_2 r^{1-\gamma}} - b\left(r-1\right)^2, \tag{5}$$

and the cycle's efficiency by

$$\eta = \frac{\mathrm{P}}{Q_{\text{eff}}/\tau} = 1 - \frac{C_{V_2}}{C_{V_1}} r^{1-\gamma} - \frac{b\left(r-1\right)^2}{C_{V_1}}\left(K_1 - K_2 r^{1-\gamma}\right). \tag{6}$$

Equation (6) reduces to $\eta_0 = 1 - r^{1-\gamma}$ (the ideal Otto efficiency) for $C_{V_1} = C_{V_2}$ and $\mu = b = 0$ and Eq. (5) reduces to $\mathrm{P} = 0$ for the ideal reversible (infinite-time) case.

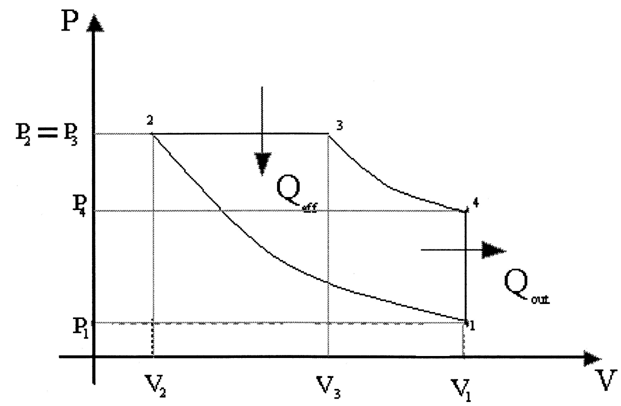

FIGURE 2 Pressure-volume diagram of an ideal Diesel cycle.

In Fig. 2, we depict a pressure-volume diagram of a Diesel cycle. In the FTT-approach of this cycle followed in Ref. 8 mainly based in the ideas of Ref. 7, the following expressions were obtained, for the cycle's period,

$$\tau = K_1 \left(T_3 - T_2\right) + K_2 \left(T_4 - T_1\right), \tag{7}$$

and for the lumped friction losses

$$\mathrm{P}_\mu = -b\left(r_C - 1\right)^2, \tag{8}$$

where r_C is the compression ratio $r_C = V_1/V_2$ and $b = \mu x_2^2/\Delta t_{12}^2$. The effective power output and the cycle's efficiency are given by [8]

$$\mathrm{P} = \frac{C_P\left(r_C - r_E\right)\left(r_E r_C\right)^{\gamma-1} - C_V\left(r_C^\gamma - r_E^\gamma\right)}{K_1\left(r_C - r_E\right)\left(r_E r_C\right)^{\gamma-1} - K_2\left(r_C^\gamma - r_E^\gamma\right)} - b\left(r_C - 1\right)^2, \tag{9}$$

and

$$\eta = 1 - \frac{r_E^\gamma - r_C^\gamma}{\gamma\left(r_E - r_C\right)\left(r_E r_C\right)^{\gamma-1}} - \frac{b\left(r_C - 1\right)^2\left[K_1\left(r_E - r_C\right)\left(r_E r_C\right)^{\gamma-1} + K_2\left(r_E^\gamma - r_C^\gamma\right)\right]}{C_P\left(r_E - r_C\right)\left(r_E r_C\right)^{\gamma-1}}, \tag{10}$$

where C_P is the constant-pressure heat capacity and $r_E = V_1/V_3$ is the expansion ratio. Eq. (10) immediately reduces to the ideal Diesel efficiency [10] when $\mu = b = 0$ and Eq. (9) reduces to $\mathrm{P} = 0$ for the ideal reversible case.

As it was remarked in Refs. 7 and 8, Eqs. (5), (6), (9) and (10) have a reasonable behavior when they are compared with real power and efficiency curves for actual Otto and Diesel engines. In fact, the obtained values for $\mathrm{P}_{\max}$ and $\eta_{\max}$ in both cases are close to the real $\mathrm{P}_{\max}$ and $\eta_{\max}$ values. In addition, these equations lead to loop-shaped curves for P versus η plots as it is common in many real engines [11].

3. Otto and Diesel cycles with cyclic variability

Lukanin [2] and Heywood [12] remark that in a sequence of thermal cycles pertaining to an internal combustion engine, one finds cycle to cycle variations in several quantities, such as the chemical composition of the working fluid, the combustion heat, etc.. Among the phenomena that cause cyclic variability (CV), the following ones can be mentioned: there exists in the process of admission and recharge of the fuel an overlap in the opening times of the intake and exhaust valves producing that not all of the gases in the cylinder are expelled during the exhaust process. This residual fraction includes combustion products and some unreacted fuel and air. Thus, the reactive mixture is different in each cycle of a sequence of them, and therefore, the combustion heat also changes from cycle to cycle [3, 6]. Small changes in the atmospheric pressure and in the chemical composition of the air can also produce minor changes in the combustion heat. In fact, all of the corner states in Figs. 1 and 2 can fluctuate from cycle to cycle due to variations in the combustion heat and in addition, due to friction losses, adiabatic imperfections, and turbulence in the working fluid, among many other reasons [2, 12, 13]. In the Daw *et al.* model [3] only the net effect of the mentioned CV-causes is taken into account through stochastic fluctuations in one or more key parameters as the injected fuel-air ratio for example. From the central-limit theorem these authors assume that the noisy parametric inputs are gaussian distributed because they arise from the combined contribution of many high-dimensional processes. They consider a primary deterministic element arising from the retained fuel and air from one engine cycle to the next. Retained residual gases influence succeeding cycles because of a strong nonlinear dependence of combustion rate on the time of the spark [3]. Daw *et al.* mainly work with the analysis of a parameter called the equivalence ratio ϕ_0 defined by the quotient between the mass of fuel and mass of air for the i-th cycle. Finally, they reduce a high-dimensional problem to one with only one degree of freedom expressed by a chaotic deterministic map of the form $Q_{\text{eff}}[i+1] = f(Q_{\text{eff}}[i], \lambda_j)$; $j = 1, 2, \ldots$, where Q_{eff} is the heat released in each combustion event, and λ_j are parameters such as the equivalence ratio and others [3]. In ref. [6], we study the role of the thermodynamic properties of an Otto engine model over the fluctuations of the combustion heat, expressed through outputs such as the power P and the efficiency η. In the present paper we extend the study to a Diesel engine model and we assume that the heat fluctuations can be represented by fluctuations in the heat capacities of the working fluid along the branches of the thermal cycles.

3.1. The Otto cycle case

In this case, we assume that the intake mixture (gas r) is composed by methane and air according to the following expression:

$$\text{gas r} = (1-\alpha)\,\mathrm{CH_4} + 2\left[1+\frac{\alpha}{9.546}\right](\mathrm{O_2}+3.773\mathrm{N_2}), \quad (11)$$

where $\alpha \in [0, 0.1]$ determines the proportions of methane and air in the intake mixture, that is, it defines a mixture poor or rich in fuel contents. The molar number n_{r} of gas r is determined by the initial state of the mixture taken as ideal gas:

$$n_{\mathbf{r}} = \frac{P_1 V_1}{\widetilde{R} T_1}; \quad (12)$$

if we take $T_1 = 350\mathrm{K}$, $P_1 = 1.03 \times 10^5\mathrm{Pa}$, and $V_1 = 4 \times 10^{-4}\mathrm{m}^3$, which are typical values of P, V, and T [14] for an initial state in an Otto cycle, with $\widetilde{R} = 8.31451$ J mol^{-1}K^{-1} the universal gas constant, we have, $n_{\mathbf{r}} = 0.01415$ mol. When the combustion reaction occurs, gas r [Eq. (11)] is converted in gas p according to

$$\begin{aligned} \text{gas r} &= (1-\alpha)\,\mathrm{CH_4} + 2\left[1+\frac{\alpha}{9.546}\right](\mathrm{O_2}+3.773\mathrm{N_2}) \\ &\xrightarrow{\uparrow Q_{\text{eff}}} (1-\alpha)\,\mathrm{CO_2} + 2(1-\alpha)\,\mathrm{H_2O} + (2.2095\alpha)\,\mathrm{O_2} \\ &\quad + (7.546+0.7904\alpha)\,\mathrm{N_2} = \text{gas p}, \quad (13) \end{aligned}$$

where Q_{eff} is the combustion heat. When $\alpha = 0$, we have the stechiometric case, and when $0 < \alpha \leq 0.1$, we have a poor-fuel mixture. In fact, α can vary in the interval $[0, 1]$, but we are only interested in small fluctuations around the stechiometric case. In Fig. 1, gas r undergoes the process $1 \to 2$, then the combustion reaction occurs in some place of the process $2 \to 3$, then gas p undergoes the processes $3 \to 4$ and $4 \to 1$. The constant-pressure heat capacities of gases r and p, $C_{P_{\mathrm{r}}}$ and $C_{P_{\mathrm{p}}}$, respectively, in the appropriate temperature intervals are calculated by means of the polynomials reported by Heywood [12] for some α values. Then, we obtain the constant- volume heat capacities $C_{V_{\mathrm{r}}}$ and $C_{V_{\mathrm{p}}}$ by means of the Mayer relation

$$C_{V_{\mathrm{r}}} = C_{P_{\mathrm{r}}} - n_{\mathrm{r}}\widetilde{R}$$

and

$$C_{V_{\mathrm{p}}} = C_{P_{\mathrm{p}}} - n_{\mathrm{p}}\widetilde{R} \quad (14)$$

From Eq. (13), we observe that $n_{\mathrm{p}} = n_{\mathrm{r}}$. Thus, we have all of data to calculate the heat capacities of gases r and p for several α values, which we present in Table I.

TABLE I. Values of heat capacities C_P and C_V for several values of α

α	$C_{P_{\mathbf{r}}}$ (J/K)	$C_{P_{\mathbf{P}}}$ (J/K)	$C_{V_{\mathbf{r}}}$ (J/K)	$C_{V_{\mathbf{P}}}$ (J/K)
0.00	0.4712502	0.5988764	0.3535998	0.4812260
0.02	0.4706447	0.5974308	0.3529943	0.4797804
0.04	0.4700389	0.5959877	0.3523885	0.4783373
0.06	0.4694360	0.5945464	0.3517856	0.4768960
0.08	0.4688305	0.5931025	0.3511801	0.4754521
0.10	0.4682470	0.5916577	0.3505743	0.4740073

In Fig. 3, we depict the linear dependence resulting between C_{V_p} and C_{V_r} respectively, for values taken from Table I. The linear fit gives

$$C_{V_\text{p}} = -0.362478 + 2.38604 C_{V_\text{r}} \,. \tag{15}$$

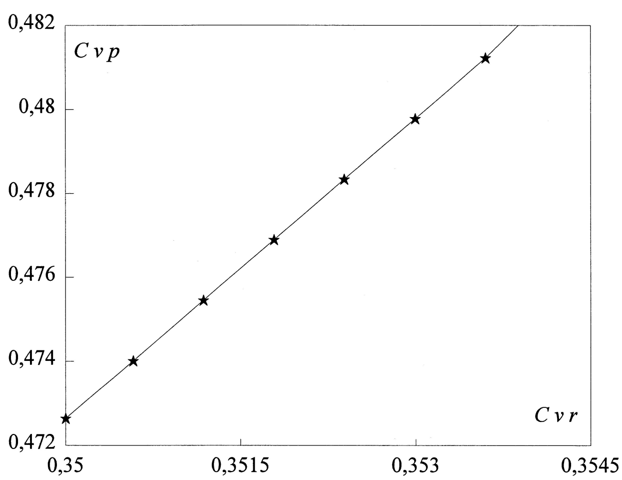

FIGURE 3 Linear behavior of the heat capacities for several values of α.

If in Eqs. (5) and (6) for the power output and the efficiency of the Otto cycle model, we take $C_{V_1} = C_{V_\text{r}}$ and $C_{V_2} = C_{V_\text{p}} = 2.38604 C_{V_\mathbf{r}} - 0.362478$, then we obtain both C_{V_1} and C_{V_2} in terms of C_{V_r}. The adiabatic exponent γ is taken with the same value for both adiabatic branches $1 \rightarrow 2$ and $3 \rightarrow 4$,

$$\gamma = \frac{C_{P_\text{r}}}{C_{V_\text{r}}} = 1 + \frac{n_\mathbf{r} \widetilde{R}}{C_{V_\text{r}}} \,, \tag{16}$$

the additional parameters appearing in Eqs. (5) and (6) are taken with the same values used in Ref. 7, which are: $r = 8$, $K_1 = 8.128 \times 10^{-6}\text{sK}^{-1}$, $K_2 = 18.67 \times 10^{-6}\text{sK}^{-1}$ and $b = 32.5\text{W}$. Thus, P and η can be expressed in terms of C_{V_r} only, then, Eqs. (5) and (6) become

$$\text{P}\left(C_{V_\text{r}}\right) = \frac{C_{V_\text{r}} - \left(2.38604 C_{V_\text{r}} - 0.362478\right) r^{\frac{n\tilde{R}}{C_{V_\text{r}}}}}{K_1 + K_2 r^{\frac{n\tilde{R}}{C_{V_\text{r}}}}} - b\left(r-1\right)^2 , \tag{17}$$

and

$$\eta\left(C_{V_\text{r}}\right) = 1 - \frac{\left(2.38604 C_{V_\text{r}} - 0.362478\right) r^{\frac{n\tilde{R}}{C_{V_\text{r}}}}}{C_{V_\text{r}}} - \frac{b\left(r-1\right)^2}{C_{V_\text{r}}}\left(K_1 + K_2 r^{\frac{n\tilde{R}}{C_{V_\text{r}}}}\right) . \tag{18}$$

As we said before, our approach to the cyclic variability will be through fluctuations in the heat capacity C_{V_r}. With this objective, we propose C_{V_r} fluctuating around $C_{V_{\text{r}_0}} = C_{V_\text{r}}(\alpha = 0) = 0.3535998\,\text{JK}^{-1}$ [15] according to, $C_{V_\text{r}} = (1+\varepsilon) C_{V_{\text{r}_0}}$, where ε is a fluctuating parameter such that $\varepsilon \in [-0.1, 0.1]$, the reference value for $C_{V_{\text{r}_0}}$ corresponds to a methane-air mixture. The ε fluctuations will be considered as uncorrelated noises of the gaussian and uniform type and we will also study a correlated nonlinear case given by a logistic map in the chaotic region.

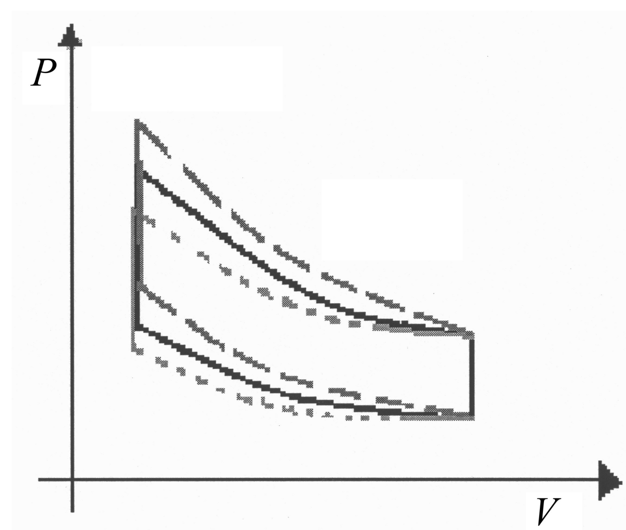

FIGURE 4 Schematic variations of an Otto cycle between the extreme values of the parameter ε.

The variation interval of C_{V_r} is $0.9 C_{V_{\text{r}_0}} \leq C_{V_\text{r}} \leq 1.1 C_{V_{\text{r}_0}}$, thus, the Otto cycle in a $P-V$ diagram will change between two extreme configurations as can be seen schematically in Fig.4. In this manner the pressure in state 3 will fluctuate from cycle-to-cycle, such as occurs in pressure diagrams of real engines [12] and in other fluctuating engine models [3, 6]. For example, in Fig. 5 we depict the pressure traces arising of our previous fluctuating Otto cycle model [6]. We calculate the P and η fluctuations for a 3000 cycles sequence. In Figs. 6a, 6b and 6c we show the power output

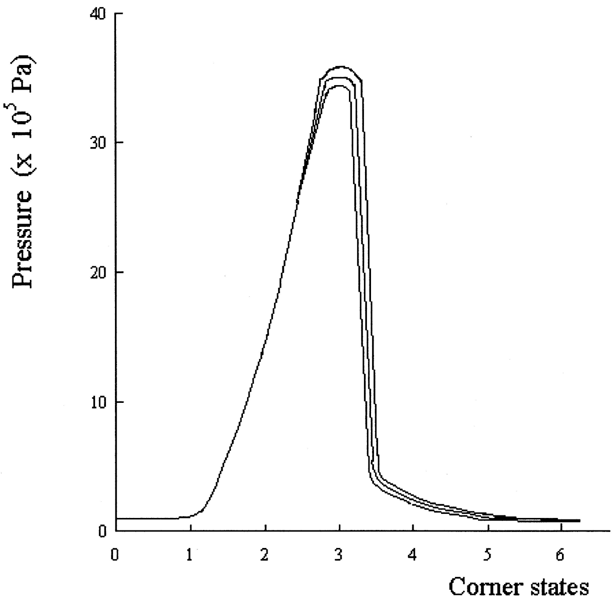

FIGURE 5 Pressure traces in an internal combustion engine (Otto case) with cyclic variability.

time series for three cases of fluctuations in the heat capacity C_{V_r} in Eq. (17): In Fig. 6a an uncorrelated uniform distribution; in Fig.6b an uncorrelated gaussian distribution and

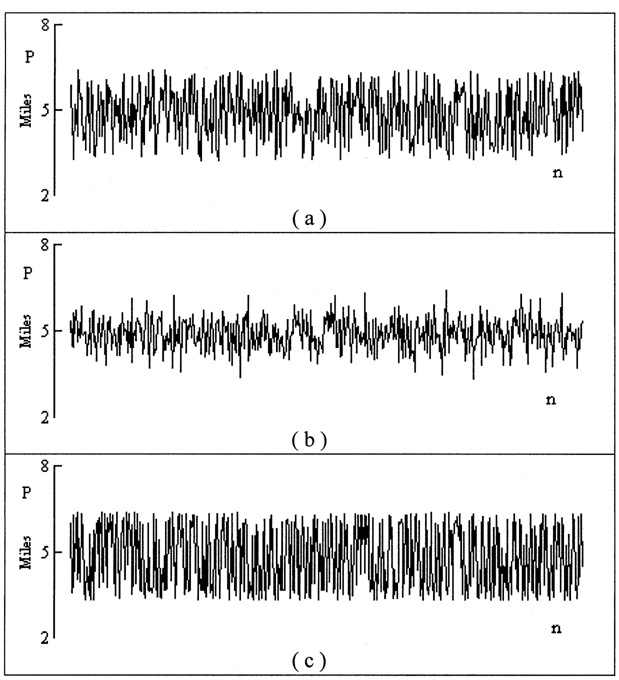

FIGURE 6 Power output fluctuations of the Otto cycle model: a) Uniform; b) Gaussian; c) Logistic

in Fig. 6c, a correlated logistic map distribution given by $\varepsilon(\mathrm{n}) = 3.8\varepsilon(\mathrm{n}-1)[1-\varepsilon(\mathrm{n}-1)]$, that is, with the parameter $\lambda = 3.8$ corresponding to the chaotic region [16]. In Table II, we show the characteristic parameters of the fluctuant power output of the cycle model. As it can be seen in Table II, the mean power output for the three cases is almost the same and they correspond to realistic values of P [2], the uncorrelated gaussian noise has the smallest standard deviation $\sigma_{\mathrm{P}} = 550.61\mathrm{W}$ and also the smallest relative fluctuation $\sigma_{\mathrm{P}}/\overline{\mathrm{P}} = 0.1133$, on the other hand the correlated logistic map leads to the most noisy case with both the greatest standard deviation and relative fluctuation $\left(\sigma_{\mathrm{P}} = 1096.29\mathrm{W}\right.$ and $\left.\sigma_{\mathrm{P}}/\overline{\mathrm{P}} = 0.2265\right)$, that is, the case that resembles the situation in which combustion residuals of the previous cycle affect the next cycle produces bigger fluctuations as it is expected. For the efficiency case, and using the same three noisy inputs (uniform, gaussian and logistic) in Eq. (18), we obtain similar results, as can be seen in Figs. 7a, 7b, 7c and in Table II. In the case of the Diesel engine

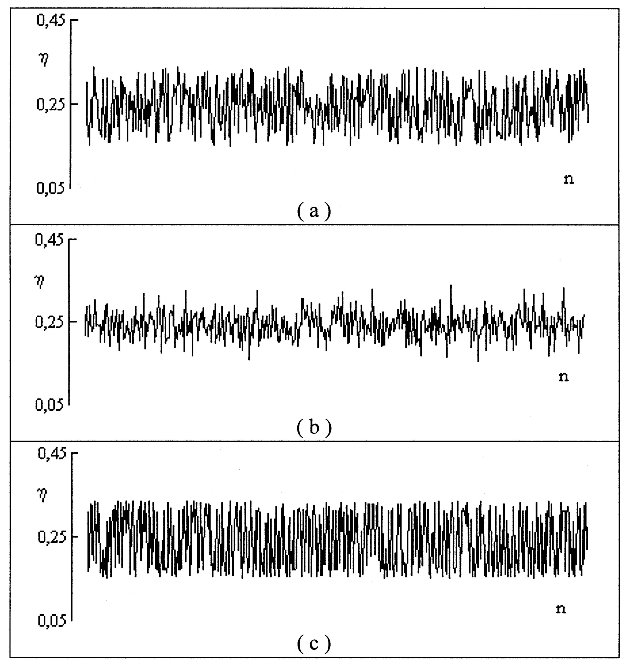

FIGURE 7 Efficiency fluctuations of the Otto cycle model: a) Uniform; b) Gaussian; c) Logistic

TABLE II. Characteristic parameters of a fluctuant Otto cycle

ε	$\overline{\mathrm{P}}$ (W)	σ_{P} (W)	$\sigma_{\mathrm{P}}/\overline{\mathrm{P}}$	$\overline{\eta}$	σ_η	$\sigma_\eta/\overline{\eta}$
uniform	4830.95	910.59	0.1884	0.2405	0.0537	0.2232
gaussian	4855.73	550.61	0.1133	0.2407	0.0325	0.135
logistic	4840.03	1096.29	0.2265	0.2419	0.0647	0.2677

model, we can use the same approach, that is, first we write Eqs. (9) and (10) for the power output and the efficiency in terms of C_V only. Taking $C_P = C_V + n\widetilde{R}$ and $\gamma = C_{P_r}/C_{V_r} = 1 + n_r\widetilde{R}/C_{V_r}$, for Eqs. (9) and (10), we have

$$\mathrm{P}\left(C_V\right)=\frac{\left(C_V+n\widetilde{R}\right)\left(r_C-r_E\right)\left(r_E r_C\right)^{\frac{n\widetilde{R}}{C_V}}-C_V\left[r_C^{1+\frac{n\widetilde{R}}{C_V}}-r_E^{1+\frac{n\widetilde{R}}{C_V}}\right]}{K_1\left(r_C-r_E\right)\left(r_E r_C\right)^{\frac{n\widetilde{R}}{C_V}}-K_2\left[r_C^{1+\frac{n\widetilde{R}}{C_V}}-r_E^{1+\frac{n\widetilde{R}}{C_V}}\right]}-b\left(r_C-1\right)^2 \tag{19}$$

and

$$\eta\left(C_V\right)=1-\frac{r_C^{1+\frac{n\widetilde{R}}{C_V}}-r_E^{1+\frac{n\widetilde{R}}{C_V}}}{\left[1+\frac{n\widetilde{R}}{C_V}\right]\left(r_C-r_E\right)\left(r_E r_C\right)^{\frac{n\widetilde{R}}{C_V}}} -\frac{b\left(r_C-1\right)\left[K_1\left(r_C-r_E\right)\left(r_E r_C\right)^{\frac{n\widetilde{R}}{C_V}}+K_2\left(r_C^{1+\frac{n\widetilde{R}}{C_V}}-r_E^{1+\frac{n\widetilde{R}}{C_V}}\right)\right]}{\left(C_V+n\widetilde{R}\right)\left(r_C-r_E\right)\left(r_E r_C\right)^{\frac{n\widetilde{R}}{C_V}}}. \tag{20}$$

As in the Otto case, we simulate the cyclic variability through the variations in the constant-volume heat capacity by means of $C_V = (1+\varepsilon)\,C_{V_0}$, where $C_{V_0} = 0.2988\mathrm{JK}^{-1}$ and $C_{P_0} = 0.41832\mathrm{JK}^{-1}$ [15], thus $n = (C_P - C_V)/\widetilde{R} = 0.0143748$ mol, in this case our reference value for C_{V_0} corresponds to air. The additional values for the parameters involved in Eqs. (19) and (20) were taken from Ref. 8, $K_1 = 8.12\times10^{-6}\mathrm{sK}^{-1}$, $K_2 = 18.67\times10^{-6}\mathrm{sK}^{-1}$, $b = 3\mathrm{W}$, $r_E = 1.35$ and $r_C = 12.5$. As in the Otto case the C_V interval of variation is $0.9C_{V_0} \leq C_V \leq 1.1C_{V_0}$, of course, this interval is somewhat arbitrary, but our objective is only a qualitative and comparative analysis of fluctuations. In Fig. 8, we depict a scheme of the fluctuant Diesel cycle. In Table III, we show some statistical parameters of the fluctuant Diesel cycle model, as in the Otto case, we observe that for the three fluctuant variations of C_V, the mean power output results almost the same. On the other hand, the logistic fluctuation leads to the biggest fluctuations in both the standard deviation σ_{P} and the relative fluctuation $\sigma_{\mathrm{P}}/\overline{\mathrm{P}}$. A similar result is obtained for the efficiency of the Diesel cycle (see Table III). It is remarkable that for similar fluctuant inputs the Diesel cycle is less noisy than the Otto cycle, yet in the case of the logistic fluctuations. In all the cases, we have calculated de Hurst exponent H of both the time series of power and efficiency, and we find $H \approx 10^{-3}$, that is, the series have an antipersistent and stable behavior [17].

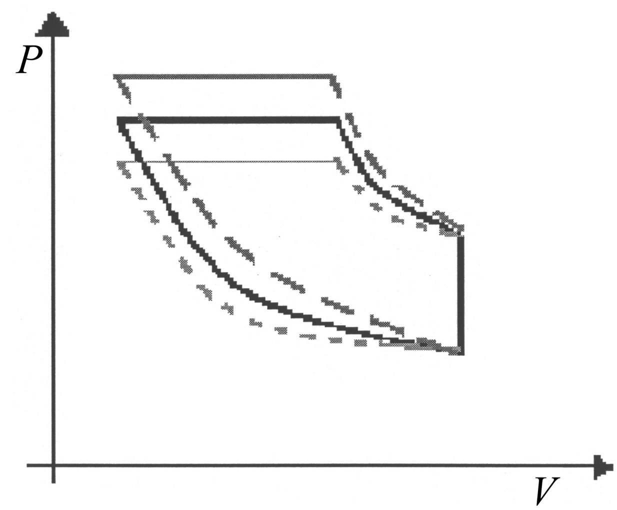

FIGURE 8 Schematic variations of a Diesel cycle between the extreme values of the parameter ε.

TABLE III. Characteristic parameters of a fluctuant Diesel cycle

ε	$\overline{\mathrm{P}}$ (W)	σ_{P} (W)	$\sigma_{\mathrm{P}}/\overline{\mathrm{P}}$	$\overline{\eta}$	σ_η	$\sigma_\eta/\overline{\eta}$
uniform	4800.41	32.9295	6.85×10^{-3}	0.2971	0.0133	0.0448
gaussian	4799.91	19.8738	4.14×10^{-3}	0.2970	0.008	0.0271
logistic	4801.72	39.6953	8.266×10^{-3}	0.2976	0.016	0.054

4. Conclusions

There exist many theoretical and experimental reasons for taking a sequence of thermal cycles no as an identical repetition of a representative steady-state cycle, but as a sequence of cycles changing in several thermodynamic quantities. In fact, in internal combustion engines, as the Otto and Diesel engines, the combustion heat changes from cycle to cycle due to imperfect combustion and residual gases inside the cylinder after each combustion event. Evidently, this cyclic variability in the combustion heat must produce changes in the performance of a cyclic sequence, for example, in both the power output and efficiency. Recently, Daw *et al.* [3] proposed an internal combustion engine model, in which, the combustion heat changes from cycle to cycle. Those authors obtained a reasonable reproduction of the experimental time series of the fluctuant combustion heat for a spark-ignited Otto engine. Starting from this model, we proposed another model [6] in which, the fluctuant combustion heat drives the thermodynamics of an Otto engine including a chemical combustion reaction, and dissipative losses. In that work [6], we find that power and efficiency are fluctuant quantities whose fluctuation sizes (standard deviation and relative fluctuation) can be driven through the managing of the thermodynamic state variables of the working fluid [6]. In the present work, we develop fluctuant models for both an Otto and a Diesel cycle, but assuming that the cyclic variability can be lumped through the fluctuations of the constant-volume heat capacity. In both cases we have obtained that the size of fluctuations is bigger in a logistic correlated noise than in two uncorrelated noises (uniform and gaussian), that is, in the case when the time series qualitatively resembles the situation in which combustion residuals are maintained from cycle to cycle. Our results also suggest that the Diesel cycle is a better "thermodynamic filter" of the fluctuations than the Otto cycle.

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