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Effective magnetic moment of neutrinos in strong magnetic fields

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In this paper we compute the effective magnetic moment of neutrinos propagating in dense high magnetized medium. Taking typical values of magnetic field and densities of astrophysical objects (such as the cores of supernovae and neutron stars) we obtain an effective type of dipole magnetic moment in agreement with astrophysical and cosmological bounds.

Keywords: Strong magnetic fields; neutrinos; magnetic moment.

En el artículo se calcula el momento magnético efectivo de los neutrinos que se propagan en un medio denso y altamente magnetizado. Para ello se tomaron valores típicos del campo magnético y densidades presentes en objetos astrofísicos, tales como núcleos de supernovas y estrellas neutrónicas. Bajo estas condiciones se obtiene un valor para el momento magnético efectivo que se corresponde con las cotas astrofísicas y cosmológicas aceptadas en la literatura.

Descriptores: Campos magnéticos intensos; neutrinos; momento magnético.

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The study of the effects of intense magnetic fields on various astrophysical phenomena has gained a considerable importance nowadays. Large magnetic fields of the order $B_m \sim 10^{12} - 10^{14}$ Gauss are being associated with the surfaces of supernovae [1] and neutron stars [3,4]. There are no reasons to avoid the possibility of finding magnetic fields inside the neutron stars or cores of supernovae which may rise up to 10^{20} Gauss [5].

On the other hand there are strong experimental evidences that something happens with neutrinos that leave the Sun and arrive to the Earth [6,7]. The most accepted theory to explaining this fact is related to neutrinos oscillations [8] but it is possible to consider other effects that could give similar consequences.

Moreover, it is also known that this strong magnetic field quantize the motion of electrons in the interior of neutron stars [5].

In the Ref. 2 the dispersion curves for neutrinos propagating in such extremely dense plasmas in the strong magnetic fields of order $B \leqslant m_W^2/e$ have already been calculated. Moreover we have shown that these neutrinos, while propagating through a very dense electroweak plasma in the presence of a very strong magnetic field ($eB \leq m_W^2$), acquire a large effective mass m_{eff}.

It is well known that the electromagnetic and dynamical properties of particles (electric charge, mass etc.) [9,10] are modified in a magnetized medium. For that reason, we expect that the propagation of neutrinos in a magnetized medium has strong influence on its dipole magnetic moment in the same way that it leads to the appearance of an effective mass.

On the other hand a nonvanishing neutrino magnetic moment μ_ν could have important consequences in astrophysics and particle physics. Interactions with the magnetic field B cause an L-R helicity flip where left-handed electron neutrinos precess into right-handed ones, which are sterile with respect to the weak interaction. This fact can have effects on the solar-neutrino puzzle [11,12], the neutrino emission from supernovae, and the big-bang nucleosynthesis [13].

At this point we want to emphasize that one can talk about two types of magnetic moment related to neutrinos: i) the transition magnetic moment connecting different flavors of neutrinos $\nu_i \rightarrow \nu_j$ ii) direct magnetic moment meaning the flip of left-handed neutrinos to their right-handed counterpart $\nu_{eL} \rightarrow \nu_{eR}$. In our work we only consider the second possibility.

In this paper we focus our attention on getting an effective dipole magnetic moment of neutrinos by starting from an electron Dirac neutrino by means of a dispersion relation and by the energy modified due to the presence of medium and magnetic field, and we check its value by using typical values of magnetic fields and densities of neutron stars and core of supernovae.

Our results are interestingly in agreement with most of the bounds obtained from a wide range of different approaches from scattering of neutrinos with charged fermions in the background, (for instance $\nu_L e^- \rightarrow \nu_R e^-$ [14]) to effective electromagnetic form factor or vertex function in presence of the medium [15,16].

In order to gain understanding we rewrite briefly the main expressions and calculations obtained in the Ref. 2.

We work in the framework of imaginary time formalism and replace the vacuum propagators with the corresponding background corrected propagators in presence of strong magnetic fields. All the Feynman rules of the vacuum theory remain the same, otherwise.

The effect of the medium is taken into account through the solution of the dispersion equation and the presence of the magnetic field is considered via propagators of electrons and W-bosons.

The electron propagator in the configuration space with the background corrections turns out to be [9]

$$G^e(x,x') = -\frac{1}{(2\pi^2)\beta}\sum_{n'=0}^{\infty}\sum_{p_4}\int\frac{dp_3dp_2}{(p_4^{*^2}+p_3^2+m_e^2+2eBn')}$$
$$\times\{(ip_4^*\gamma_4+ip_3\gamma_3-m_e)(\sigma_+\psi_{n'}(\xi)\psi_{n'}(\xi')+\sigma_-\psi_{n'-1}(\xi)\psi_{n'-1}(\xi'))$$
$$+\frac{1}{2}\sqrt{2eBn'}[\gamma_+\psi_{n'}(\xi)\psi_{n'-1}(\xi')-\gamma_-\psi_{n'-1}(\xi)\psi_{n'}(\xi')]\}\exp[ip_4(x_4-x_4')+ip_3(x_3-x_3')+ip_2(x_2-x_2^{'})], \quad (1)$$

where $\xi = \sqrt{eB}(x_1+x_o)$, $\xi' = \sqrt{eB}(x_1'+x_o)$, $x_o = \frac{p_2}{eB}$, $\sigma^{\pm} = 1/2[1\pm\sigma_3]$, $\gamma_{\pm} = 1/2[\gamma_1 \pm i\gamma_2]$, $\sigma_3 = i/2[\gamma_1,\gamma_2]$, $p_\lambda^* = p_\lambda - i\mu\delta_{4\lambda}$, n' is the electron Landau quantum number and $\psi_{n'}(\xi)$ are Hermite functions for $A_\mu = (0,Bx,0,0)$. The W-boson propagator in the magnetic field has the form

$$D_{\mu\nu}^W(x,x^{'}) = \frac{1}{(2\pi)^2\beta}\sum_{p_4}\sum_{n}\int dp_2dp_3[\frac{R^-+R^+}{2}\Psi_{\mu\nu}^1+R^0\Psi_{\mu\nu}^2+i\frac{(R^--R^+)}{2}\Psi_{\mu\nu}^3)]$$
$$\times\psi_n(\xi)\psi_n(\xi')\text{exp}[ip_4(x_4-x_4')+ip_3(x_3-x_3')+ip_2(x_2-x_2')]. \quad (2)$$

where n is the W Landau quantum number,

$$R^{\pm} = [p_4^{*^2}+E_n^{W_2}\pm 2eB]^{-1}, \quad R^0 = [p_4^{*2}+E_n^{W_2}]^{-1},$$

with

$$E_n^{W_2} = M_W^2+p_3^2+2eB(n+1/2); \quad \Psi_{\mu\nu}^1 = \frac{1}{B^2}G_{\mu\nu}^{02},$$

$$\Psi_{\mu\nu}^2 = \delta_{\mu\nu}-\frac{1}{B^2}G_{\mu\nu}^{02} \quad \text{and} \quad \Psi_{\mu\nu}^3 = \frac{1}{B}G_{\mu\nu}^0$$

($G_{\mu\nu}^{02}$ is the field tensor of the $SU(2)\times U(1)$ electromagnetic external field). Concerning the gauge fixing term, we are taking $D_{\mu\nu}^W$ in a transverse gauge which is expected to guarantee the gauge independence of the neutrino spectrum. The charged current contribution to the self-energy of neutrino is

$$\Sigma^C(x,x') = -i\frac{g^2}{(2\pi)^3}\gamma_\mu G^e(x,x')D_{\mu\nu}^W(x-x')\gamma_\nu; \quad (3)$$

the solution of this equation can be calculated from the inverse propagator of the neutrino in the momentum space given by

$$\Sigma^C(k) = \frac{g^2eB}{(2\pi)^2}$$
$$\times\left(\sum_{p_4}\int dp_3G_e^o(p_3+k_3,p_4+k_4)\Sigma_{\alpha\beta}\right)P_L, \quad (4)$$

where P_L is the usual left projection operator. By solving

$$\det(-i\gamma_\mu k_\mu+\Sigma^C) = 0, \quad (5)$$

in the special case of strong magnetic fields, which means that the Landau ground state for electrons and W-bosons are dominant, we can get a not trivial expression for the energy of neutrino quasiparticles as a function of the magnetic field and the electron density N_e [2], giving

$$\omega = \frac{g^2N_e}{m_W^2-e|B|}, \quad (6)$$

where N_e is related to the chemical potential ν_e and magnetic field B by $\mu_e = N_e/(2\pi)^2eB$ [2]. The expression (6) is valid whenever $\sqrt{M_W^2-eB} >> \mu_e$ and for a wide range of values of B and T.

In Fig. 1 we plot the behavior of ω respect to B. Let us remark that this quantity increase very fast when $eB \to m_W^2$. The value of this field is 10^{24} Gauss. For fields under 10^{23} Gauss ω is practically constant and its values is given by (8) as we can see below.

For $eB << m_W^2$ we can expand Eq. (6) and get the expression

$$\Delta\omega = \frac{g^2N_e}{m_W^2}+\frac{g^2N_eeB}{m_W^4}. \quad (7)$$

The first term of Eq. (7) represents an effective mass dependent on the electron density, for magnetic fields $B < B_c$ (where $Bc = m_W^2/e$) which is of order 250 eV for $N_e = 10^{39}$ cm^{-3}. The second term contains the dipole moment contribution [17], *i.e.* the effective dipole moment is

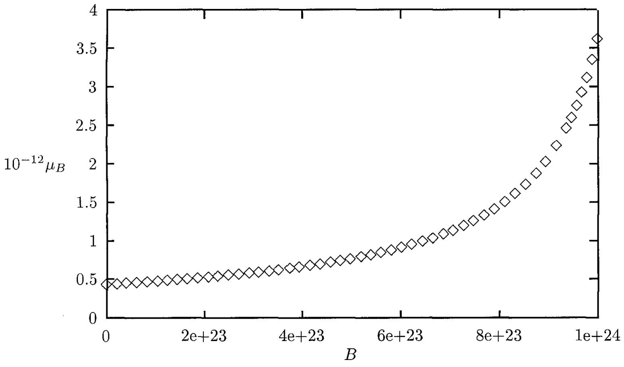

FIGURE 1. Behavior of the energy ω as a function of B.

$$\mu_{eff} = \frac{g^2 N_e e}{m_W^4}. \tag{8}$$

When we evaluate it for the same density, typical in cores of supernovae, we get $\mu_{eff} \approx 0.36 \times 10^{-12}\mu_B$. This value of the effective magnetic dipole moment of neutrinos is in agreement with astrophysical bounds [14,15].

Bounds of this order for the magnetic moment provides the possibility of direct transition of dipole moment which even could be a solution of the puzzle of SNP because the sunspot may give evidences of magnetic fields closer to 10^5 Gauss , this might produce spin-flip transitions of neutrinos in spite of $\mu_\nu \approx O(10^{-12}\mu_B)$ [12].

Our study at this level cannot be applied to the Sun because the relation between electron density and magnetic field inside it implies that we must consider Landau level occupation of above the ground state.

Let us remark that the assumption of considering only the first Landau level for electron and W-bosons is compatible with magnetic fields $B \geq 10^{17}$ Gauss. There are claims that such large magnetic fields can exist in the cores of the supernovae or neutron stars [5].

To conclude, we have shown that the solution of the dispersion equation in a magnetized dense plasma, gives information about the effective magnetic dipole moment which could allow conversion of ν_L into ν_R. This effect is extremely smaller than the mass-density effect, $m_{eff} \approx 250$ eV and for $B \approx 10^{18}$ Gauss, $\mu_{eff}B \approx 0.35 \times 10^{-15}$ eV.

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