INVESTIGACIÓN REVISTA MEXICANA DE FÍSICA **48** (6) 534–538 DICIEMBRE 2002

# Pattern formation in oscillatory granular flows

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Recibido el 29 de mayo de 2002; aceptado el 17 de julio de 2002

An experimental device has been used to examine the behavior of granular materials under vertical, horizontal and combined bi-axial vibrations. Containers with square, rectangular and circular cross sections geometries were used, with glass spheres, millet and lentil as granular media. The oscillating frequencies varied from 0 to 35 Hz, with amplitudes between 0 and 5.5 mm. Flow visualization was made possible with high-speed video cameras providing frontal and lateral views of the granular motion. Four different phenomena: heaping, arching, small amplitude surface waves and large amplitude surface waves were found. The onset of these phenomena depends basically on two dimensionless parameters; one involving the amplitude and frequency of oscillations coupled with gravity and the second one representing the amount of material in the vibrating container. In the case of vertical vibrations, results agreed well with those of Wassgren *et al*. (1996), although some discrepancies were found. For horizontal vibrations, convective cycles appeared rotating in opposite directions as well as lateral waves with one half the frequency of excitation. For the case of combined vibrations, a superposition of effects between those of horizontal and vertical vibrations was found and analyzed. Finally, comparison was made between experiments and numerical predictions based on molecular dynamics. The numerical solution agreed extremely well with experimental observations, particularly for the new results of combined vibrations.

*Keywords:* Granular dynamics; granular flow; granular materials.

Se examina experimentalmente el comportamiento de materiales granulares sujetos a vibraciones horizontales, verticales y combinaciones de ambas. Se utilizaron geometrías de sección cuadrada, rectangular y circular, con esferas de vidrio, mijo y lentejas como medios granulados. Las frecuencias de oscilación cubrieron el intervalo entre 0 y 35 Hz y las amplitudes se variaron entre 0 y 5.5 mm. Cámaras de video de alta velocidad localizadas frontal y lateralmente permitieron analizar los patrones de flujo. Cuatro distintos fenómenos fueron observados: apilamiento, arqueo, ondas superficiales de pequeña amplitud y ondas superficiales de gran amplitud. La aparición de dichos patrones depende básicamente de dos parámetros adimensionales: el primero involucra la amplitud, la frecuencia de oscilación y la aceleración gravitacional, mientras que el segundo es representativo de la cantidad de material granular en el contenedor experimental. Para vibraciones verticales, los resultados concuerdan con los reportados por Wassgren, *et al.* (1996), aunque con ciertas discrepancias. En el caso de vibraciones horizontales se presentaron ciclos convectivos que giraban en direcciones opuestas y ondas laterales a una frecuencia equivalente a la mitad de la frecuencia de excitación. Para el caso de vibraciones combinadas, una superposición de efectos entre los de vibración horizontal y vertical apareció y fue analizada. Finalmente, se compararon los resultados experimentales con predicciones numéricas basadas en dinámica molecular con resultados aceptables.

*Descriptores:* Dinámica granular; flujo granular; materiales granulares.

PACS: 46.40.f; 62.40.+i; 81.05.Rm; 83.85.-c

## 1. Introduction

Experiments made with granular material under oscillatory vibrations have shown an extraordinary behavior [1-10]. Different phenomena can be observed and the reasons remain unexplained. Under vertical oscillations and low frequencies, the material inside a container flows with convective patterns and accumulates at one of the sides of the container forming a "heap". At higher frequencies, small amplitude surface waves can be observed moving along one side of the container. In contrast, large amplitude surface waves are present if the frequency is increased. Small amplitude waves only appear on the free surface of the material while the bottom remains flat. Within a range of intermediate frequencies between those of small and large amplitude surface waves, a fourth type of behavior, called arching, appears. Arching can be recognized because both the free surface and the bottom of the material exhibit a wave-shaped form. In arching, several convective flows may be present, depending on the number of layers of material and the amplitude of the vibration. In the present communication, different materials have been used to study these phenomena and to determine if any particular behavior is dependent on geometry, density or other material properties. As a result, the phenomenon does not seem to be affected by changes in material properties. Furthermore, geometrical changes of containers apparently do not affect the observed behavior.

Results of experiments made with vertical, horizontal and combined oscillations are shown. Numerical simulations

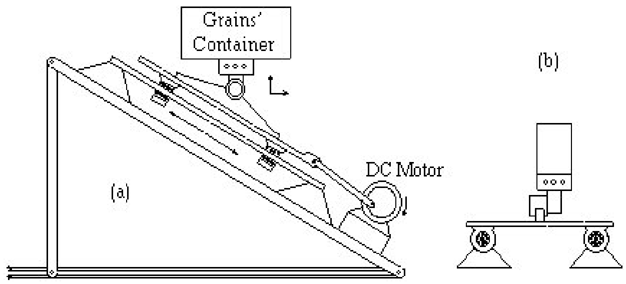


FIGURE 1. The experimental setup for studies of the vibration of grains. (a) Shows the side-view of the device at an inclination close to 30°; the cylindrical bearings on a precision shaft permit accurate displacements on this plane with the above inclination. The DC motor, via a crank-shaft, generates linear displacements with known frequency and amplitude. (b) the backend of the bearing-and-shaft assembly is shown on the horizontal position.

based upon molecular dynamics were also performed, and its results correlate extremely well with experiments.

## 2. Experiments

A simple mechanical device was designed and built to generate horizontal, vertical and combined bi-axial oscillations in containers of several dimensions. A square container with 12 cm by side and a height of 18 cm, two rectangular containers: 2 cm (d)$\times$ 20 cm (w)$\times$ 20 cm (h), and 3.6 cm $\times$ 20 cm $\times$ 20 cm; and a cylindrical container with 8 cm diameter and 18 cm in height were used in the experiments. The containers were made of transparent acrylic material in order to allow visualization with the aid of a high speed video camera (200 frames per second).

The granular media used in the experiments consisted of glass spheres of 1.28 mm in diameter, millet, an oval shaped grain of 2.25 mm average diameter and 3.25 mm average length, and lentil, a common grain with an average diameter of 3.5 mm.

The oscillating frequency was varied from 0 to 35 Hz with amplitudes between 0 and 5.5 mm. The lateral component of the vibrations was kept below 0.2% of the main vertical and horizontal components. A schematic diagram of the experimental apparatus is shown in Fig. 1. It consists of a stainless steel base which slides over cylindrical bearings on a high precision shaft guide. The motion is provided by a controlled speed DC motor with an accuracy of 2% via a crankshaft assembly. A digital frequency meter was used to monitor the motion and a strobe lamp was sometimes used to freeze the image. Two separate video cameras were used providing both frontal and lateral views of the granular motion. Finally, the slope of the container was monitored with an optical leveling device. The angle was controlled with a variable swivel support for the containers (Fig. 1).

In order to describe the vibrations and in accordance with other authors [1], a dimensionless acceleration parameter $\Gamma = \alpha\omega^2/g$ was used, where $a$ is the amplitude, $\omega = 2\pi F$, $F$ being the frequency of vibration, and $g$ is the gravity acceleration constant. A second dimensionless number $N = H/d$ represents the amount of layers of material in the container. Here $H$ is the height of material in the container, and $d$ is the diameter of the grain. For the present experiments, $N$was limited to values between $10 < N < 40$, and the dimensionless acceleration to $1 < \Gamma < 10$.

## 3. Numerical simulations

In addition to the experiments, a three dimensional numerical method based on molecular dynamics was used in order to compare the observed phenomena with numerical predictions. The numerical code is similar to the one suggested by Walton and Braun [11] and used extensively by the present authors [12], and will not be presented here in detail. The numerical predictions compare extremely well with the experimental observations as may be seen in Figs. 2 and 3, and Figs. 4 and 5, respectively, and is explained later.

## 4. Experimental results

### 4.1. Vertical vibration

When granular materials are subjected to vertical oscillations, four distinct phenomena appear which depend upon the value of $\Gamma$. In order to verify and extend the results reported by Wassgren, *et al.* [1], experiments were conducted with glass spheres in square, rectangular and cylindrical containers. Additionally, other materials such as millet and lentil were also studied.

In general, the results obtained for these experiments basically coincide with those reported by Wassgren, *et al.* [1]. In particular, most of the characteristic values of $\Gamma$, which correspond to the onset of the four phenomena, are practically the same for all materials, regardless of the geometry of the container. Nevertheless, some important differences were found between this work and the one by Wassgren, *et al*. These are described below.

This results show that the heap is not restricted to one preferred side of the container. It can also appear in the mid-section as described by Laroche, *et al.* [4]. Furthermore, for given values of amplitude, number of layers and frequency, the formation of the heap was observed on either side or at the mid-section of the container. Initially, it was believed that this apparently random behavior could be due to small components of vibration other than vertical [1]. Nevertheless, after repeated number of events in which the initial conditions of the experiment were varied, it became clear that the formation of the heap on different sides of the container was a consequence of small changes in initial conditions of stress in the material inside the container.

For low frequencies, Fig. 2 shows the existence of surface waves on the upper layers and arching at the bottom of the grains. For values of $N < 15$, arching always appears before large amplitude surface waves. For larger values of

$N$ and in all cases, large amplitude surface waves always appear before arching. For the case of glass spheres, arching appears for values of Γ in the range $5 < \Gamma < 6.5$. This value is slightly larger for other granular materials. Fig. 3 shows a numerical simulation for glass spheres, for $N > 15$ with a significant manifestation of surface waves and a less obvious presence of arching. The typical values for glass have been used for the elastic modulus, tangential friction, etc.; the restitution coefficient was set at $\varepsilon$= 0.8 [11,12] providing an inelastic collision mechanism between beads.

As the frequency is increased, the coexistence of surface waves and arching is not a stable one, even when large amplitude surface waves were observed superposed on arching.

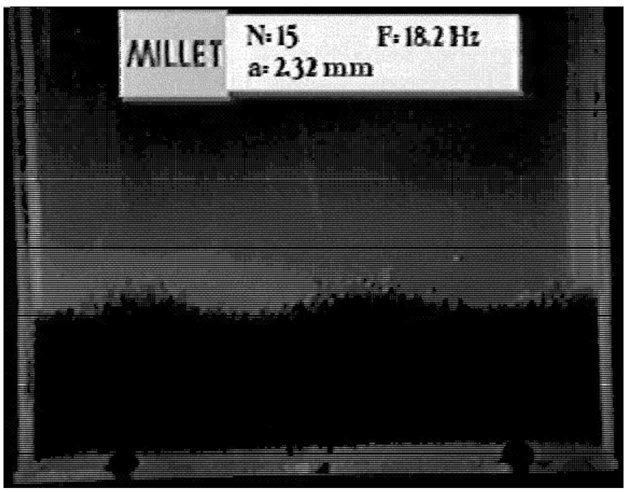


FIGURE 2. Experimental observation of small amplitude surface waves for vertical vibrations. The bottom of the container is delimited by a line below the base of two symmetrically located round-head screws. Arching is clearly observed at a Γ value of 3.09.

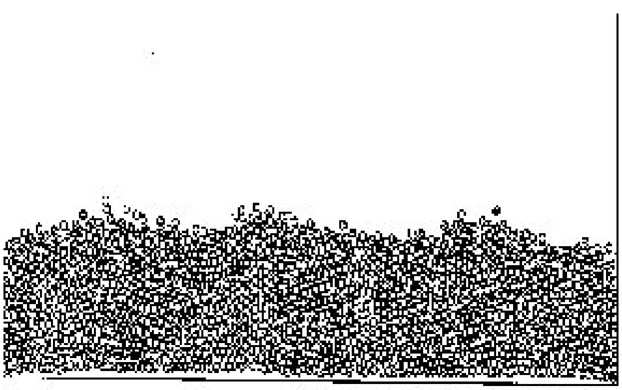

FIGURE 3. Numerical simulation of spheres subjected to vertical vibration. Small amplitude surface waves. A negligible arching is observed; material at the bottom separates from the container out of phase with the oscillation. Conditions $N > 15$, F= 30Hz, and a value of Γ=1.81.

In fact, the overlapping of these phenomena marked the values of Γ at which the transition from one regime to the other occurs.

### 4.2. Horizontal vibration

Experiments with horizontal vibrations were made using square, rectangular and cylindrical containers. Millet and glass spheres were used for these experiments. A repetitive pattern was observed in all cases, namely:

1. Initially, motion was restricted to the upper layers of particles for $0.5 < \Gamma < 0.77$. For smaller values of Γ, no motion was observed.

2. For $1.2 < \Gamma < 7$ particles flow from the center to the sides of the container parallel to the direction of vibration. In a frontal view, two convective flows ascending at the center and descending towards both sides of the container were clearly observed. The speed of the particles increases as they approximate the lateral walls (the walls perpendicular to the direction of vibration).

Using a strobe lamp, stationary waves were also detected along the lateral sides of the container (left and right diameter extremes in the case of the cylindrical container). Similarly to the results with small amplitude surface waves in vertical vibration, the frequency of lateral waves is one half the vibration frequency.

### 4.3. Combined vibrations

These experiments were made using the rectangular, square and cylindrical containers. The combination is achieved with an angle different than zero for the wedge-shaped support and with no inclination of the container base, as shown in Fig. 1. The material used was millet, with $N$=15 and two different angles for the movable base (30 and 60 degrees).

For these conditions a combination of effects was observed. For an inclination of 30° of the movable base, the dominant observed phenomena can be associated to the dynamics induced by horizontal vibrations. For inclinations higher than 60° the behavior is dominated by vertical vibrations. Nevertheless, the container's geometry appears to have certain influence on the flow patterns. A more detailed description is given below.

- 30° combined vibrations

#### *4.3.1. Rectangular Container.*

1. At the beginning of vibrations (for $\Gamma < 1$), the free surface of the material develops a gradual inclination which gives rise to continuous avalanches until it matches an inclination similar to that of the movable base; see Fig. 4.

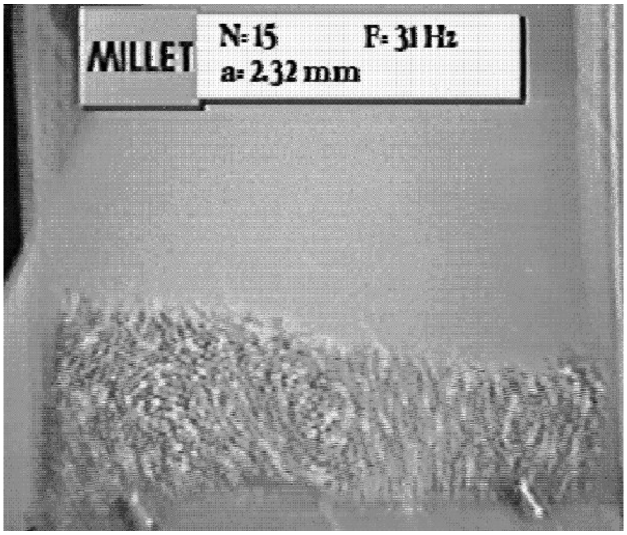


FIGURE 4. On the left side, a heap develops even for $\Gamma < 1$. Significant lateral waves and arching exist under 30° combined vibration. A single convection cell is observed for values greater than Γ=8.97.

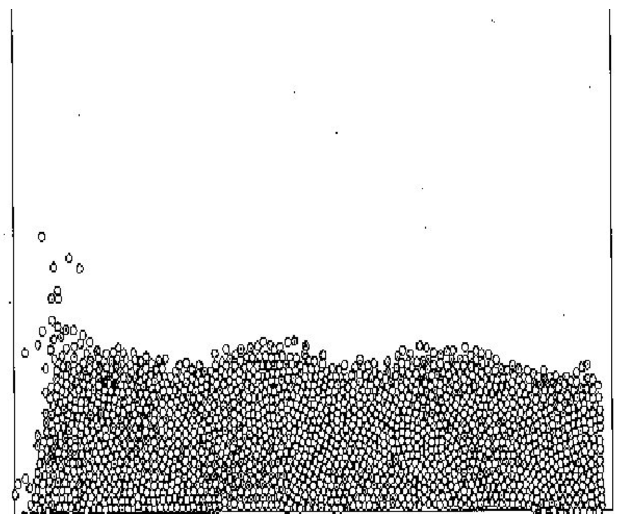

FIGURE 5. Numerical simulation of spheres under combined vibration at 30°, F=30Hz, $\Gamma = 1.81$, 3 mm glass beads: $\varepsilon$=0.8.

2. For Γ=2.26 and in a similar manner to heaping, two convective flows appear with a heap at the left side of the container. Initially, this convective motion of the particles is barely visible involving mainly those grains of the upper layers, and without perturbing the bottom particles of the container. As the frequency is increased, the left side convection cell is characterized by a higher speed and larger size, now affecting the whole mass of particles on that side of the container, with an ascending motion of the bottom particles at the central portion; particles descend close to the lateral walls. The second convection cell initially increases in size as the frequency increases with matched higher speeds. Eventually, it losses mass to the left side convection cell and disappears.

3. For values of frequency corresponding to Γ=6.8, now a single convective flow cell is observed, moving in a clock-wise direction. As shown in Figure 4, and predicted by the simulations of Fig. 5, it was possible to detect with the help of the strobe lamp lateral waves similar to the ones described for horizontal vibrations. At the bottom of the container, small waves similar to those observed with vertical vibrations also appeared, the size of the lateral waves being considerably larger than the bottom ones.

Considering that for the above experimental conditions the horizontal component of vibrations is larger than the vertical one, the former dominates the latter, as may be seen by the appearance of lateral waves as described above.

### *4.3.2. Cylindrical container*

Experiments made with the cylindrical container under identical conditions showed similar behavior to those described for the rectangular container; however, certain differences must be pointed out:

1. As Γ reaches values of 2.26, the convective motion of particles abruptly appears, spanning the whole container.

2. In contrast to the rectangular geometry, there are no regions within the container where particles remain motionless. This indicates that the geometry of the container clearly affects the behavior of particles in regions near corners or sharp-ends.

3. In the cylindrical geometry, the presence of lateral waves becomes more obvious than in the rectangular one.

4. At higher frequencies and corresponding to higher values of $\Gamma (\Gamma > 9.4)$, a single convective cell gives place to a large number of smaller cells; the latter are very stable, and visible throughout the whole container.

- 60° combined vibrations.

For values of $\Gamma < 1$, the behavior is similar to the one observed with $30^\circ$ vibrations. The pair of convective cells described for the $30^\circ$ combined vibrations were also observed for the case at hand. At this point, the main difference between the 30 and $60^\circ$ combined vibrations appears for values of $\Gamma > 7.4$, where instead of one single convective flow, arching appears superposed on the heap; this phenomenon is clearly visible on the shadowgraph in Fig. 6.

Figure 7 illustrates a frontal view of lateral and bottom waves for millet, plus the dynamics of the granular material is also shown: the slightly off-center portion of the flow cell presents a fountain of grains between the two large convection cells. Waves moving along the lateral sides and bottom of the container were also detected using the strobe lamp. Not surprisingly, lateral waves were smaller than those at the bottom, and in turn smaller than the lateral waves observed for $30^\circ$ combined vibration.

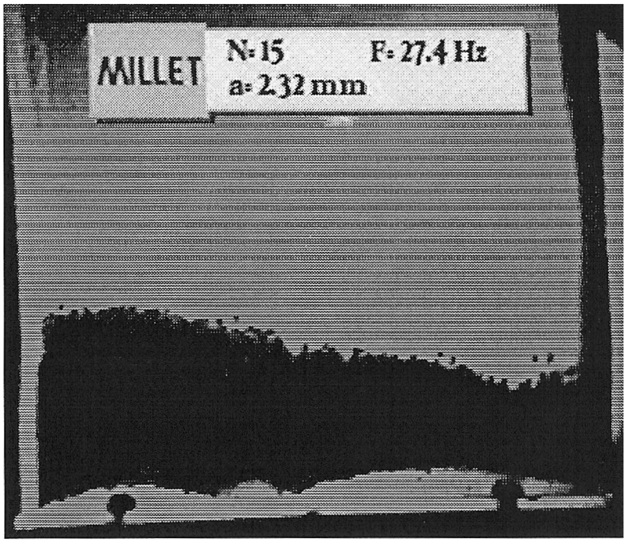


FIGURE 6. Combined oscillation at 60°; arching is observed with a wavelength of half the container's base width. Γ=7.01. The upper layers also present a heap and small lateral waves.

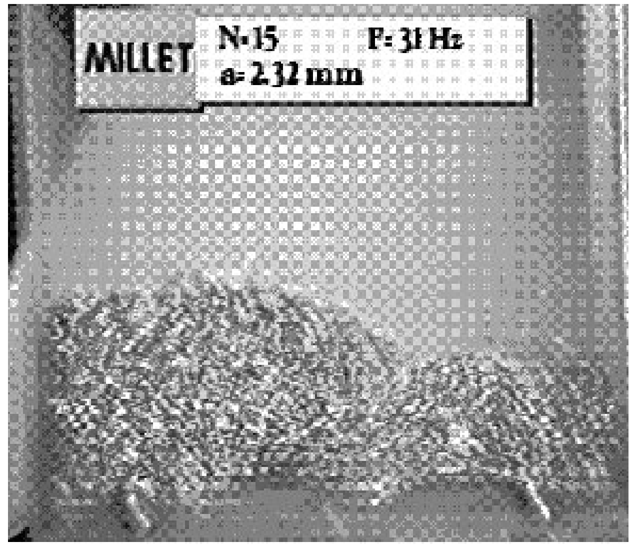


FIGURE 7. Combined vibrations at 60°, with Γ=8.97. The convection cells are clearly visible, with the central region displacing grains towards the upper layers.

## 5. Conclusions

An experimental device has been used to examine the behavior of granular materials under vertical, horizontal and combined vibrations. Four different phenomena: heaping, arching, small amplitude surface waves and large amplitude surface waves are reported. The onset depends basically on two dimensionless parameters: an acceleration number relating the amplitude and frequency of oscillations to gravity, and the second one representing the number of layers of material in the container. For the case of vertical vibrations, results agree well with those reported by other authors [1] although some discrepancies were found and analyzed. For the case of combined vibrations, a superposition of effects is observed and described in detail. Finally, a comparison was made between experiments and numerical predictions based on granular dynamics. The numerical predictions agree extremely well with the experimental observations, in particular for the results of combined vibrations here presented.

## Acknowledgements

The authors wish to acknowledge the assistance of the machine shop of the I.I.M. and in particular J. Camacho and M. Serrano, for the construction of the experimental device. G. F. Guevara provided assistance with the computer images. Support from grants IN302392 and IN503494 from PAPIIT, UNAM are also gratefully acknowledged. Many fruitful conversations with M. Hunt and C. Brennen and their groups at Caltech are also acknowledged.

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