Quintessence-like dark matter in spiral galaxies

F. Siddhartha Guzmán
Max Planck Institut für Gravitationsphysik, Am Mühlenberg 1,
D-14476 Golm, Germany.

Tonatiuh Matos
Departamento de Física, Centro de Investigación y de Estudios Avanzados del IPN,
Apartado Postal 14-740, 07000 México D.F., MEXICO.

Darío Núñez and Erandy Ramírez
Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México,
Apartado Postal 70-543, 04510 México D.F., MEXICO.

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Through the geodesic analysis of a static and axially symmetric space time, we present conditions on the state equation of an isotropic perfect fluid $p = \omega d$, when it is considered as the dark matter in spiral galaxies. The main conclusion is that it can be an exotic fluid ($-1 < \omega < -1/3$) as it is found for Quintessence at cosmological scale.

Keywords: Quintessence; perfect fluid.

Por medio de un análisis de las geodesicas en un espacio axisimétrico y estático, obtenemos condiciones sobre la ecuación de estado para un fluido perfecto isotrópo, $p = \omega\, d$, cuando éste es considerado como la materia obscura en las galaxias espirales. La conclusión principal es que dicho fluido puede ser exótico, es decir, con $-1 < \omega < -\frac{1}{3}$, como se obtiene a escalas cosmológicas dentro del modelo de Quintaesencia.

Descriptores: Quinta-esencia; fluido perfecto.

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There is no doubt about the importance of the mystery concerning the nature of dark matter in the Universe and particularly in galaxies. The consequences of observations made on SNIa supernovae [1] have posed challenges to the available theoretical machinery, and models explaining such phenomena have arisen. Several of them propose exotic types of matter and therefore unusual equations of state, such as a Cosmological Constant, Cold Dark Matter models, Dilaton Fields [2] and Quintessence [3]. However, at the galactic level there are no models consistent with the cosmological ones which give some light in the understanding of dark matter's nature.

In order to be precise about the problem, let us recall the situation of the galactic dark matter, for which we confine ourselves to the observations made by Rubin *et al.* [4] who found that, for a few sample of spiral galaxies the interstellar gas and stars lying far away from the center (in the equatorial plane) of the corresponding galaxy behave in a very peculiar way: Their circular velocity seems to be independent of the radius starting from a certain distance to the galactic center, *i.e.* the rotation curve profile of a spiral galaxy is flat outside a central galactic region. It was then inferred a distribution $\sim 1/r^2$ of non luminous matter (dark matter) which should contribute to the flatness of the rotation curves. There exists certain controversy about the flatness of such curves [5], but in general it is accepted that rotation curves are flat up to the precision of the measurements, and that this behavior is observed in large samples of spiral galaxies [6].

The most accepted scenario for a spiral galaxy reads as follows: it is an object composed by a luminous disc whose density exponentially decays. At the same time, there is a dark halo whose density is distributed as $\sim 1/r^2$ [7]. With these assumptions, an explanation is found for the kinetic behavior of gas and stars composing a spiral galaxy, but many questions remain unsolved. This phenomenological model describes how the density should behave, but does not tell what is the nature of dark matter, how was such mixture between dark and luminous matter formed and under which laws? If the dark matter is baryonic such as MACHOS for instance, why does its density have a non exponential distribution as luminous matter density does? If it is non baryonic, what is it made of, or at least which is its equation of state? This last question is the one that occupies ourselves in the present work.

In this letter we proceed in the following way: We assume that a spiral galaxy lies on a background axysimmetric static space time which is characterized by the presence of a perfect fluid with an arbitrary equation of state, *i.e.*, $p = \omega d$ being ω a free function, and then we find conditions over ω that permit flat rotation curves of test particles. Other types of candidates to dark matter are discussed in Refs. 8 and 9.

First of all it must be clear that our treatment is valid only in the dark matter dominated region, i.e. where the rotation curves are flat, and we do not consider the galactic core region. We only consider the external region, and a complete model should be one where we match this description to another for the internal region. Some advances have been made in this direction, considering an oscilaton in the center [10], or with scalar fields non-minimally coupled to gravity [11, 12].

In order to explain the observed dynamics of particles in the luminous sector of the galaxy, the galaxies must be composed by almost 90% of dark matter, distributed in the halo. We can thus assume that luminous matter does not contribute in a very important way to the total energy density. That is, the curvature, the gravitational effects, are due mainly to the halo, so the luminous matter will be treated as a test fluid, that is, that it moves in geodesics of the spacetime curved by the dark matter, but the luminous matter itself does not produce changes in the curvature.

Finally, the exact symmetry of the halo is still unknown, but it is reasonable to suppose that the halo is symmetric with respect to the rotation axis of the galaxy, so we choose the space time to be axial symmetric.

The line-element of such space-time, given in the Papapetrou form is

$$ds^2 = -e^{2\psi}(dt + \omega\, d\varphi)^2 + e^{-2\psi}\left[e^{2\gamma}(d\rho^2 + dz^2) + \mu^2 d\varphi^2\right], \quad (1)$$

where ψ, ω, γ, and μ, are functions of (ρ, z).

On the other hand, the observed circular velocity of stars around the galactic center is of the order of 230 Km/s, that is $7 \times 10^{-4} c$, where c is the speed of light. If we suppose that this is also a measure of the rotation of the galactic halo, such rotation will be too small to generate curvature effects such as gravitational dragging. Hence, in the region of interest we will consider the space-time to be static as well.

Therefore, we start from the background described by the following line element:

$$ds^2 = -e^{2\psi} dt^2 + e^{-2\psi}[e^{2\gamma}(d\rho^2 + dz^2) + \mu^2 d\varphi^2], \quad (2)$$

which corresponds to an static axially symmetric space-time; and the coordinates ψ, γ, and μ , are functions of (ρ, z) .

We recall the reader that observations are made upon objects lying in the galactic equatorial plane *i.e.* $z = 0$, thus the Lagrangian for a test particle travelling on such slide of the space time described by (2) is

$$2\mathcal{L} = -e^{2\psi}\dot{t}^2 + e^{-2\psi}[e^{2\gamma}\,\dot{\rho}^2 + \mu^2\,\dot{\varphi}^2], \quad (3)$$

where dot means derivative with respect to the proper time τ of the test particle. Due to the symmetry of the metric, we have two conserved quantities associated with the energy and the angular momentum. Also, the Hamiltonian of the system is another constant. From this we can perform a dynamical analysis upon our system and obtain the canonical momentums associated with it. This will serve us to give an expression for the Hamiltonian using a Legendre transformation which will lead us to the following radial geodesic motion equation:

$$\dot{\rho}^2 - e^{2(\psi-\gamma)}\left[E^2 e^{-2\psi} - L^2\,\frac{e^{2\psi}}{\mu^2} - 1\right] = 0, \quad (4)$$

where E, and L, are constants associated with this geodesic motion along the equatorial plane.

We are interested in circular and stable motion of test particles, therefore the following conditions must be satisfied

i) $\dot{\rho} = 0$, for circular trajectories,

ii) $\dfrac{\partial V(\rho)}{\partial\,\rho} = 0$, extreme ones,

iii) $\left.\dfrac{\partial^2 V(\rho)}{\partial\,\rho^2}\right|_{extr} > 0$, and stable.

being

$$V(\rho) = -e^{2(\psi-\gamma)}\left[E^2 e^{-2\psi} - L^2\,e^{2\psi}/\mu^2 - 1\right].$$

Recalling that E and L are constants of motion for each circular orbit, it is straightforward to obtain expressions for the energy E, angular momentum L, angular velocity $\Omega{=}d\varphi/dt$ and the tangential velocity $v^{(\varphi)}{=}e^{-2\psi}\mu\Omega$ [13], corresponding to a circular, stable equatorial motion:

$$E = e^{\psi}\sqrt{\frac{\frac{\mu_{,\rho}}{\mu} - \psi_{,\rho}}{\frac{\mu_{,\rho}}{\mu} - 2\,\psi_{,\rho}}}, \quad (5)$$

$$L = \mu\, e^{-\psi}\sqrt{\frac{\psi_{,\rho}}{\frac{\mu_{,\rho}}{\mu} - 2\,\psi_{,\rho}}}, \quad (6)$$

$$\Omega = \frac{e^{2\psi}}{\mu}\sqrt{\frac{\psi_{,\rho}}{\frac{\mu_{,\rho}}{\mu} - \psi_{,\rho}}}, \quad (7)$$

$$v^{(\varphi)} = \sqrt{\frac{\psi_{,\rho}}{\frac{\mu_{,\rho}}{\mu} - \psi_{,\rho}}}, \quad (8)$$

and for the stability condition:

$$V_{,\rho\rho}|_{extr} = -\frac{2\,e^{2(\psi-\gamma)}}{\frac{\mu_{,\rho}}{\mu} - 2\,\psi_{,\rho}} \times \left(\frac{\mu_{,\rho}}{\mu}\,\psi_{,\rho\rho} - \frac{\mu_{,\rho\rho}}{\mu}\,\psi_{,\rho} + 4\,{\psi_{,\rho}}^3 - 6\frac{\mu_{,\rho}}{\mu}\,{\psi_{,\rho}}^2 + 3\left(\frac{\mu_{,\rho}}{\mu}\right)^2\psi_{,\rho}\right) > 0, \quad (9)$$

where a coma stands for partial derivative.

Notice that equation Eq.(8) allows us to impose a condition over the tangential velocity of our test particle. Actually, if we want the rotational curves of galaxies to be model as radii independent ones (which is what is observed!), we can use this expression Eq.(8), derived from a purely geometrical and dynamical analysis, to do it. Considering the tangential velocity as constant we can easily solve this equation and observe that the functions ψ and μ are related by

$$e^{\psi} = \left(\frac{\mu}{\mu_0}\right)^l, \qquad (10)$$

being $l = const$, we obtain a necessary and sufficient condition for the velocity ${v_c}^{(\varphi)}$ to be the same for two orbits at different radii, given $l = ({v_c}^{(\varphi)})^2 / \left(1 + ({v_c}^{(\varphi)})^2\right)$, and Eq. (9) tells us that this motion is stable. We call Eq. (10) together with such value of l the *flat curve condition*.

We now write the Einstein's equations

$$G_{\alpha\beta} = 8\pi T_{\alpha\beta}$$

for an arbitrary energy momentum tensor for the line element (2):

$$\mu(\psi_{\rho\rho} + \psi_{zz}) + \mu_\rho\,\psi_\rho + \mu_z\,\psi_z = 4\pi\mu$$
$$\times[e^{-2(\psi-\gamma)}\,(e^{-2\psi}T_{tt} + \frac{e^{2\psi}}{\mu^2}\,T_{\varphi\varphi}) + T_{\rho\rho} + T_{zz}], \quad (11)$$

$$\mu_{\rho\rho} + \mu_{zz} = 8\pi\,\mu\,[T_{\rho\rho} + T_{zz}], \quad (12)$$

$$\gamma_\rho\,\mu_\rho - \gamma_z\,\mu_z - \mu\,({\psi_\rho}^2 - {\psi_z}^2) + \mu_{zz} = 8\,\pi\,\mu\,T_{\rho\rho}, \quad (13)$$

$$\gamma_\rho\,\mu_z + \gamma_z\,\mu_\rho - 2\,\mu\,\psi_\rho\,\psi_z - \mu_{\rho z} = 8\,\pi\,\mu\,T_{\rho z}. \quad (14)$$

The *flat curve condition* (10) is introduced in order to have flat tangential curve velocities. This condition is valid on the equatorial plane. Nevertheless, the halo is expected to be almost spherically symmetric, that means that if we know the functional dependence of the gravitational potential on the equatorial plane, this dependence should be the same one in almost the rest of the halo. In that case it is reasonable to suppose that the *flat curve condition* (10) is valid in a region around the equatorial plane. Thus, in this region we substitute the relation (10) into the left hand side of Eq. (11) obtaining

$$\mu(\psi_{\rho\rho} + \psi_{zz}) + \mu_\rho\,\psi_\rho + \mu_z\,\psi_z$$
$$= \frac{({v_c}^{(\varphi)})^2}{\left(1 + ({v_c}^{(\varphi)})^2\right)}(\mu_{\rho\rho} + \mu_{zz}), \quad (15)$$

and introducing Eq. (15) along with Eq. (12), into Eq. (11), we get a constrain equation among the components of the stress energy tensor:

$$-\left(\frac{1 - ({v_c}^{(\varphi)})^2}{1 + ({v_c}^{(\varphi)})^2}\right)(T_{\rho\rho} + T_{zz})$$
$$= e^{-2(\psi-\gamma)}\left(e^{-2\psi}T_{tt} + \frac{e^{2\psi}}{\mu^2}\,T_{\varphi\varphi}\right). \quad (16)$$

Notice that this relation must be satisfied by any stress energy tensor which, within the approximation made in the analysis, curves the space time in such a way that the motion of test particles corresponds to the observed one.

Let us consider the case of a stress energy tensor corresponding to a perfect fluid,

$$T_{\mu\nu} = (d + p)\,u_\mu\,u_\nu + g_{\mu\nu}\,p,$$

with d the density of the fluid and p its pressure. In this case we are thinking on a "dark fluid", which is not seen but it is thought that it could be there affecting the geometry in the way needed in order to have the observed behavior in the tangential velocities of the luminous matter, as just mentioned. Considering the dark fluid as static, the four velocity of such dark fluid is given by $u^\alpha = (u^0, 0, 0, 0)$ which, for the line element (2) reads: $u^0 = E\,e^{-2\psi}$, thus $u_0 = -E$ and from $u^\mu u_\mu = -1$, we obtain that $E = e^{2\psi}$. Therefore, the stress energy tensor has the form

$$T_{tt} = e^{2\psi}d, \quad (17)$$

$$T_{\rho\rho} = T_{zz} = e^{-2(\psi-\gamma)}p, \quad (18)$$

$$T_{\varphi\varphi} = \mu^2\,e^{-2\psi}p. \quad (19)$$

Substituting these expressions into (16), we obtain that in the equatorial plane, in order to satisfy the observed behavior on the tangential velocities, the "dark fluid" has to fulfill the relation

$$-2\left(\frac{1 - ({v_c}^{(\varphi)})^2}{1 + ({v_c}^{(\varphi)})^2}\right)\,p = (d + p). \quad (20)$$

Let us see which are the permitted relations between pressure and density of the perfect fluid providing flat rotation curves, we thus obtain

$$p = -\frac{1 + ({v_c}^{(\varphi)})^2}{3 - ({v_c}^{(\varphi)})^2}d. \quad (21)$$

From this relation the ${v_c}^{(\varphi)}$ function can be identified with the square velocity dispersion of the dark particles, thus, the pressure will be negative. We are now in a convenient position to constrain the state equation. As the velocities of the gas and stars rotating in the flat region must be within $0 < v_c^{(\varphi)2} < 1$, (the observed ones are of the order of $v_c^{(\varphi)} \sim 10^{-3}$ [6]), relation (21) implies $-1 < \omega < -1/3$, being $p = \omega d$. This result coincides with the one obtained at cosmological scale for the respective equation of state in the Quintessence model [3, 8].

Therefore the analysis presented in this letter, gives support to the hypothesis that a Quintessence-like equation of state could be the solution for the dark matter problem at galactic scale. In both cases it turns out the need of an exotic equation of state, with $\omega = -0.33$ at a galactic scale and $\omega = -0.64$ for the cosmos [2].

Thus, it has been shown that galactic dark matter satisfying an exotic equation of state certainly can be used to explain the observed behavior on the rotational curves of spiral galaxies.

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