Hydrodynamic model for 2D degenerate free-electron gas for arbitrary frequencies

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Following Halevi's procedure for 3D degenerate free-electron gas (3D-DEG), we investigate the response function in the hydrodynamic model (HM) for 2D-DEG confined in low dimensional systems when collisions are included. For small wavevectors we found from the two-dimensional Boltzmann-Mermin model a useful expression for the HM complex stiffness parameter of the nonlocal dielectric function β, which is $\beta^2 = [((3\omega/4) + i(\nu/2))\,/(\omega + i\nu)]v_F^2$, where ω and ν are the circular and collisional frequencies and v_F is the Fermi velocity.

Keywords: Theories and models of many-electron systems; optical properties of low dimensional materials; theory of electronic transport; scuttering mechanisms.

Siguiendo el procedimiento de Halevi para un gas libre degenerado de electrones en 3D (3D-GED), investigamos la función de respuesta en el modelo hidrodinámico (MH) de un 2D-GED confinado en sistemas de baja dimensionalidad cuando las colisiones son incluidas. Utilizando el modelo bidimensional de Boltzmann-Mermin, encontramos en el MH para vectores de onda pequeños una expresión útil para el parámetro de rigidez complejo de la función dieléctrica no local β, la cual es $\beta^2 = [((3\omega/4) + i(\nu/2))\,/(\omega + i\nu)]v_F^2$, donde ω y ν son las frecuencias circular y de colisión y v_F es la velocidad de Fermi.

Descriptores: Teoría y modelos de sistemas de muchos electrones; propiedades ópticas de materiales de baja dimensionalidad; teoría de transporte electrónico; mecanismos de dispersión.

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Since its inception [1], the hydrodynamic model (HM), also known as the warm-plasma approximation, has proved to be very useful in describing electrical, transport and optical properties of conductors and plasmas. In particular, it can yield information on diverse physical properties of conducting systems, such as the nonlocality or spatial dispersion of electromagnetic response of conduction electron gas, the existence of charge fluctuations (bulk plasmons), coupling between plasmons and transverse waves at inhomogeneities [2] including derivation of boundary conditions at interfaces of local and nonlocal conductors [3]. The main advantage of the HM approach lies in the simplicity of accounting for nonlocality or spatial dispersion, which often leads to analytical results in the wavevector dependence of the dielectric function [4]. On the other hand, advances in manufacturing techniques have given experimental access to systems of reduced dimensionality such as two-dimensional (2D) electron gases, one-dimensional quantum wires and quantum dots. Recent examples of applications of the HM in low-dimensional systems are studies of electron flow in high-mobility wires [5], magnetic field dependence of 2D static shielding [6], scattering of plasmons in quasi-2D electron gas containing a fixed-point charge [7], nonlocality effects of a bulk semiconductor plasma in interaction with quantum-well bound states [8] and superlattice plasmons [9], negative differential conductance in a voltage-biased GaAs superlattice [10], and synchronization and chaos in miniband semiconductor superlattices [11].

The purpose of this paper is to obtain, within the HM, an analytical expression for the stiffness parameter of a 2D degenerate free-electron gas (2D-DEG) valid for the full range of frequencies. To perform this task for 2D-DEG we follow the same procedure used for the 3D-DEG case [12].

The HM can be employed for long range oscilations or small wave vectors and it is derived from the continuity equation and Newton's second law for a charge carrier of effective mass m, charge q, and average velocity $\mathbf{v}$ experiencing electric an magnetic fields $\mathbf{E}$ and $\mathbf{B}$, a phenomenological damping force proportional to the collision frequency ν, and a pressure force proportional to the pressure gradient;

$$m\frac{d\mathbf{v}}{dt} = q\left(\mathbf{E}+\frac{\mathbf{v}}{c}\times\mathbf{B}\right) - m\nu\mathbf{v} - \frac{\nabla p}{n}, \tag{1}$$

where c is the speed of light in vacuum. The last term characterizes the HM and describes the inhomogeneity of the carrier density n. $\mathbf{E}$ and $\mathbf{B}$ can include both internal and external contributions. On the other hand the continuity equation is given by

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{v}) = 0, \tag{2}$$

and the Poisson equation is

$$\nabla\cdot\mathbf{E} = -4\pi q n. \tag{3}$$

We proceed to linearize these model equations. Thus, denoting by "0" equilibrium quantities (which are homogeneous for bulk electron gas) and by "1" small out-of-equilibrium quantities (which are inhomogeneous),

$$\nabla p = \nabla p^{(1)} = \frac{\partial p^{(1)}}{\partial n^{(1)}} \nabla n^{(1)}. \quad (4)$$

The relevant parameter β (usually called the stiffness constant of the gas or plasma since it yields information of the compressibility of the system) is defined as

$$\beta^2 = \frac{1}{m} \frac{\partial p^{(1)}}{\partial n^{(1)}}, \quad (5)$$

to yield after linearization for the HM dielectric function

$$\epsilon_H = 1 + \frac{4\pi i \sigma_H}{\omega}, \quad (6)$$

in which the HM conductivity σ_H is given by

$$\sigma_H \propto \frac{1}{\omega(\omega + i\nu) - \beta^2 q^2}, \quad (7)$$

where the Lorentz force has been neglected in the absence of an external magnetic field since it is of second order.

To obtain these expressions it is assumed in the HM that *all* particles move with the same average velocity for a given value of the field, as shown in Eq. (1). It is clear that a better physical description requires a more complete statistical information about particle velocities, as contained in more sophisticated models such as random-phase approximation (RPA) or Boltzmann model. Furthermore, the HM, as applied to the degenerate free-electron gas, has a serious shortcoming, namely, it is valid only for frequencies ω that are either very small or very large in comparison to the collisional frequency ν. For $\omega \ll \nu$ collisions dominate, and thus a conduction electron possesses three degrees of freedom. In the opposite limit, $\omega \gg \nu$, the influence of collisions is negligible and the particle motion is essentially limited to the direction of the electric field. Recently, Halevi [12] extended the HM of the 3D degenerate free-electron gas (3D-DEG) by generalizing its application for arbitrary values of ω/ν. His approach was based on a straightforward comparison of the HM dielectric function with the Boltzmann equation including the Mermin correction [13]. This correction is necessary in order to conserve local carrier density when collisions are taken into account within a relaxation time approximation, that is, ω cannot be merely replaced by $\omega + i\nu$. Within the same spirit, here we follow Halevi's procedure to explore the 2D-DEG case. Namely, we write here the expresions of the dielectric functions; the HM model and the Boltzmann-Mermin model and compare them for small values of the wave vector. In this way we are able to fit the important parameter β^2 in the HM to the Boltzmann-Mermin results for all frequencies.

For 2D-DEG, the Boltzmann dielectric function with the Mermin correction (Boltzmann-Mermin model) ε_{BM} is simpler than the corresponding RPA, and is written in terms of the Boltzmann longitudinal dielectric function ε_B as [13]

$$\varepsilon_{BM}(\mathbf{q},\omega) - 1 = \frac{\varpi\left[\epsilon_B(\mathbf{q},\varpi) - 1\right]}{\omega + i\nu\left[\epsilon_B(\mathbf{q},\varpi) - 1\right]/\left[\epsilon_B(\mathbf{q},0) - 1\right]}, \quad (8)$$

where wavevector $q = |\mathbf{q}|$ is 2D, $\varpi = \omega + i\nu$, and ε_B is given by

$$\varepsilon_B(\mathbf{q},\omega) - 1 = \frac{4\pi}{\omega} e^2 \int \frac{(\mathbf{q}.\mathbf{v})^2/q^2}{\omega - \mathbf{q}.\mathbf{v}} \frac{df^0}{dE} d\mathbf{v}. \quad (9)$$

At zero-temperature, the Fermi distribution is a Heaviside step function; $f^0(v) = \theta(v_F - |\mathbf{v}|)$, being $\mathbf{v}$ the 2D velocity and v_F the Fermi velocity.

The normal modes of the system are obtained by requiring $\varepsilon_{BM}(\mathbf{q},\omega) = 0$. Since the HM is a good model for small wavevectors, we will proceed to compare the Boltzmann-Mermin model [Eq. (8)] for small $q = |\mathbf{q}|$ with the corresponding HM dielectric function ϵ_H in which σ_H for small q is given by

$$\varepsilon_H(\mathbf{q},\omega) - 1 \propto \sigma_H \propto \frac{1}{\omega\varpi}\left(1 + \frac{\beta^2 q^2}{\omega\varpi}\right), \quad (10)$$

We now assume that carriers are confined in 2D to obtain

$$\varepsilon_B(\mathbf{q},\omega) - 1 = \frac{4\pi v_F e^2}{m}\left[\frac{1}{\omega^2 - q^2 v_F^2}\right]. \quad (11)$$

To find $\varepsilon_{BM}(\mathbf{q},\omega)$ for small q, we need, on one hand, to evaluate $\varepsilon_B(\mathbf{q},\omega)$ at $\omega = 0$ and, on the other hand, to expand $\varepsilon_B(\mathbf{q},\omega)$ in powers of qv_F/ω, to yield

$$\varepsilon_{BM}(\mathbf{q},\omega) - 1 \propto -\frac{1}{\omega\varpi}\left[1 + \frac{3}{4}\frac{q^2 v_F^2}{\varpi^2}\right]\left(1 + i\frac{\nu}{\omega}\frac{v_F^2 q^2}{2\varpi^2}\right).$$

Neglecting terms of order $\frac{q^4 v_F^4}{\varpi^4}$ we finally get

$$\varepsilon_{BM}(\mathbf{q},\omega) - 1 \propto -\frac{1}{\omega\varpi}\left[1 + \left(\frac{3}{4} + i\frac{\nu}{2\omega}\right)\frac{q^2 v_F^2}{\varpi^2}\right],$$

which can be compared with the expansion of the hydrodynamic dielectric function ϵ_H given by Eq. (10). This comparison yields $[\beta^2 = (3\omega/4 + i\nu/2)/(\omega + i\nu)]v_F^2$ for 2D-DEG. For 3D-DEG the corresponding value obtained by Halevi is [12] $[\beta^2 = (3\omega/5 + i\nu/3)/(\omega + i\nu)]v_F^2$.

In order to interpret this result and check its consistency with the well-known low- and high-frequency limits, let us consider an adiabatic process where the pressure p is proportional to n^κ with $\kappa = (f+2)/f$ as the adiabatic constant, and where f is the number of degrees of freedom, to yield

$$\frac{\partial p^{(1)}}{\partial n^{(1)}} = \kappa \frac{p^{(0)}}{n^{(0)}}.$$

Regardless of the dimensionality, a free electron gas has $u = (f/2)k_B T$ and $p^{(0)} = n k_B T$, where u, k_B, and T are, respectively, the internal energy, the Boltzmann constant, and the temperature. For 2D-DEG it follows that $p^{(0)} = n^{(0)} u$ where $u = (1/2)\epsilon_F$ and $\epsilon_F = (1/2)mv_F^2$. In addition one obtains $p^{(0)}/n^{(0)} = (1/2)\epsilon_F = (1/4)mv_F^2$ and since $\beta^2 = (1/m)(\partial p^{(1)}/\partial n^{(1)})$ then $\beta^2 = \frac{1}{4}\kappa v_F^2$. For very low frequencies ($\omega/\nu \to 0$), the randomness of the collisions permits kinetic motion in the two available dimensions, that is, $f = 2$ yielding $\beta^2 = (v_F^2)/2$. For very high frequencies ($\omega/\nu \to \infty$), the motion is deterministic and one-dimensional, with the velocity parallel to the direction of the oscillating electric field; $f = 1$ to yield $\beta^2 = (3v_F^2)/4$.

We found, as it happens in the 3D-DEG case, that β^2 is a complex expression, which in turn implies a complex adiabatic constant $\kappa(\omega)$ meaning that the pressure fluctuations are not in phase with the density fluctuations, as explained by Halevi [12] for 3D-DEG. Our generalized β^2 contains information about a wider range of frequencies than in the usual HM model. It is important to mention that our results for small wavevectors are simpler than those coming from more sophisticated models such as Boltzmann-Mermin or RPA models.

Figures 1 and 2 compare our 2D results for the real and imaginary parts of β^2 with those of 3D. We notice that the real part of β^2 is always larger in 2D than in 3D for all frequencies, which reflects that a 2D-DEG is always "stiffer" than its 3D counterpart due to the fact that the pressure gets distributed in less degrees of freedom in a low-dimensional system. For very large frequencies ($\omega/\nu \to \infty$) the oscillations in both cases are essentially one dimensional, as it was discussed above. In the case of the imaginary part of β^2 the differences between the 2D and 3D is less pronounced.

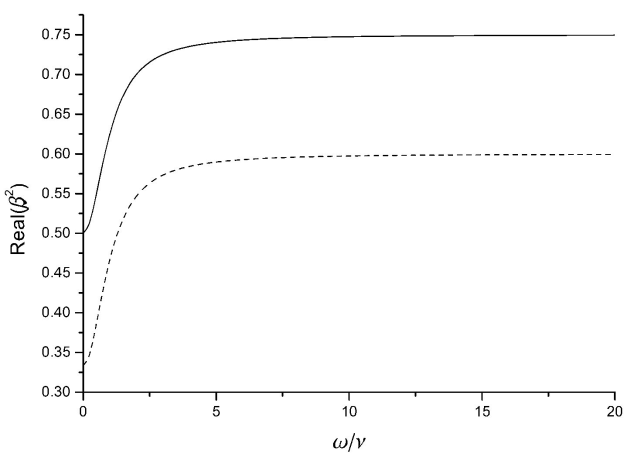

FIGURE 1. Comparison of the real part β^2 as a function of ω/ν for 2D (solid) and 3D (dashed) systems.

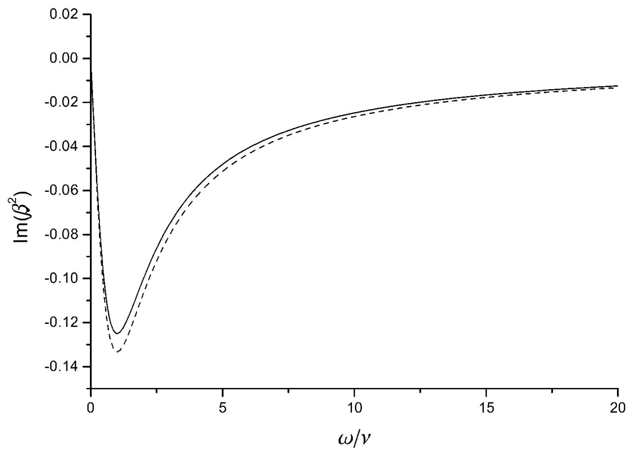

FIGURE 2. Same as Fig. 1, but for the imaginary part of β^2.

As a possible extension of this work it could be interesting to investigate to which extent external magnetic fields could be included in this procedure for both 2D and 3D-DEG, since the presence of such fields would yield a dielectric tensor, rather than a scalar dielectric function. Besides, applications of our results for any frequencies to the calculation of bulk plasma modes, one can think of calculating physical properties of conducting heterostructures (quantum wells, superlattices, etc) in which each homogeneous part of the system is caracterized by the parameter β, which in turn depends on v_F.

In summary, our results for the HM of the 2D-DEG can be applied to arbitrary ratios ω/ν and are consistent with the well-known low- and high-frequency limits. Our procedure is very simple and was similar to Halevi's for 3D-DEG, namely, we determine the HM parameter β^2 for arbitrary ω by comparison of the HM with the more sophisticated Boltzmann-Mermin model for small wavevectors. We hope that our efforts can stimulate further experimental and theoretical work on the study of small-wavevector excitations in low-dimensional systems.

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