Heat transfer in asymmetric convective cooling and optimized entropy generation rate

G. Ibáñez*, S. Cuevas and M. López de Haro
Centro de Investigación en Energía, Universidad Nacional Autónoma de México,
Apartado Postal 34, Temixco, Mor. 62580, México,
**e-mail: gid@cie.unam.mx*

Recibido el 27 de noviembre de 2002; aceptado el 13 de enero de 2003

The steady viscous flow between two infinite parallel planes, is used to illustrate the possibility of minimizing the global entropy generation rate by cooling the external surfaces convectively in an asymmetric way. The flow is generated by both an axial pressure gradient and the uniform motion of the upper surface (generalized Couette flow). The temperature field is determined using boundary conditions of the third kind. The analytic expressions for the velocity and temperature fields of the fluid are used to calculate the global entropy generation rate explicitly. In dimensionless terms, this function depends on the dimensionless ratio of the two possible velocity scales (characterized by the magnitudes of the pressure gradient and the upper surface velocity), the dimensionless ambient temperature and the convective heat transfer coefficients (Biot numbers) of each surface which, in general, are not assumed to be the same. When the Biot numbers for each surface are equal, the entropy generation rate shows a monotonic increase. However, when the Biot numbers are different this function displays a minimum for specific cooling conditions. Besides, we calculate the local Nusselt number at the upper wall for minimum entropy generation conditions.

Keywords: Entropy generation minimization; optimization; heat transfer.

Se estudia el flujo de un fluido viscoso entre dos planos paralelos infinitos con el objetivo de ilustrar la posibilidad de minimizar la producción global de entropía a través del enfriamiento asimétrico por convección del sistema. El flujo se debe a un gradiente de presión axial y al movimiento uniforme del plano o pared superior del sistema (flujo de Couette generalizado). El campo de temperatura se determina usando condiciones de frontera del tercer tipo. Las expresiones analíticas de los campos de velocidad y temperatura del fluido se utilizan para calcular explícitamente la producción global de entropía del sistema. Esta función, expresada en forma adimensional, depende de la razón de las dos posibles escalas de velocidad (una caracterizada por la magnitud del gradiente de presión y la otra por la velocidad del plano superior), de la temperatura ambiente adimensional y de los coeficientes de transferencia de calor por convección de cada plano (números de Biot), los que, en general, se consideran distintos. Cuando los números de Biot de cada superficie son iguales, la producción global de entropía tiene un comportamiento monótono creciente; sin embargo, cuando los números de Biot son diferentes, esta función muestra un mínimo para condiciones de enfriamiento específicas. Además, se calculó el número local de Nusselt de la pared superior para condiciones de mínima disipación de energía.

Descriptores: Minimización de la producción de entropía; optimización; transferencia de calor.

PACS: 05.70.Ln; 44.27.+g; 47.27.Te

1. Introduction

The minimization of the entropy generation rate has proven to be a suitable method to optimize the operating conditions of a given process or device by reducing the intrinsic irreversibilities to a minimum, according to the physical constraints imposed on the system [1,2]. The method relies on the knowledge of the dynamical and thermal fields through which the entropy generation rate characterizes the energy dissipation in the system [3]. Once this function is known explicitly, it can be minimized with respect to the relevant physical parameters in order to find conditions that lead to minimum dissipation. In this paper, the entropy generation minimization method is applied to the analysis of the steady viscous flow between two infinite, parallel plane walls of finite thickness, that exchanges heat with the ambient following Newton's cooling law. The flow is generated by both an axial pressure gradient and the uniform motion of the upper surface (generalized Couette flow). This problem, while remaining sufficiently simple to allow an almost fully analytical treatment, is used to demonstrate the possibility of minimizing the global entropy generation rate by cooling the external surfaces convectively in an asymmetric way. We solve the heat transfer equation with thermal boundary conditions of the third kind, assuming that the heat transfer coefficients for each surface are in general different. From the analytic expressions for the velocity and temperature fields, the local and global entropy generation rates are determined. Extending a previous analysis on the asymmetric convective cooling [4], in the present work the local Nusselt number for minimum entropy generation conditions is explicitly found. Under these conditions an optimum value of the dimensionless ambient temperature that maximizes the heat transfer is derived. The asymptotic behavior of the optimum local Nusselt number for large ambient temperatures is also explicitly determined.

2. Generalized Couette flow

Let us consider the steady flow of a viscous fluid between two infinite parallel walls in the presence of a longitudinal pressure gradient, dp/dx, and where the upper plate is moving with velocity U. The upper plate is located at $y' = a/2$ and the lower plate is at $y' = -a/2$, y' denoting the transversal coordinate. The dimensionless velocity field, subjected to the dimensionless no-slip boundary conditions $u(y=-1/2)=0$ and $u(y=1/2)=1$, is

$$u = \frac{G}{8}(1-4y^2) + \frac{1}{2} + y, \tag{1}$$

where the velocity, u, has been normalized by the boundary value U, the dimensionless transversal coordinate, y, is normalized by a, and $G = -\left(a^2/\eta U\right) dp/dx$ is the dimensionless ratio of velocities, one characterized by the magnitude of the pressure gradient $\left((a^2/\eta)dp/dx\right)$ and the other by the magnitude of the upper surface velocity (U). Here, η is the dynamic viscosity of the fluid.

We note that the form of the velocity distribution depends on the magnitude of the dimensionless ratio of velocities. When $G = 0$, the fluid motion is only due to the motion of the upper wall, and the velocity field reduces to a linear shear flow $u = 1/2 + y$.

With the velocity field given by Eq.(1), we proceed to solve the energy balance equation considering viscous dissipation. The temperature field reaches a steady state because the surfaces of the slab are bathed by a fluid of fixed ambient temperature T_a with which the system exchanges heat following Newton's cooling law. In dimensionless terms, the heat transfer equation for this system reduces to

$$\frac{d^2\Theta}{dy^2} + \left(\frac{du}{dy}\right)^2 = 0, \tag{2}$$

where the dimensionless temperature is now given by $\Theta = k\left(T - T_a\right)/\eta U^2$, with T and k being the temperature and the thermal conductivity of the fluid, respectively. Evidently, the thermal behavior of the system, particularly heat flow irreversibilities, strongly depends on boundary conditions. Here the heat transfer equation is solved using boundary conditions of the third kind that indicate that the normal temperature gradient at any point in the boundary is assumed to be proportional to the difference between the temperature at the surface and the external ambient temperature. Hence, the amount of heat entering or leaving the system depends on the external temperature as well as on the convective heat transfer coefficient. Let us assume that the external fluid streams that wash each wall are in general different. Then, the convective heat transfer coefficients, although taken to be constant, do not have the same value on both walls. Therefore, Eq. (2) must satisfy the boundary conditions

$$\frac{d\Theta}{dy} + Bi_1\Theta = 0, \quad \text{at} \;\; y = \frac{1}{2}, \tag{3}$$

$$\frac{d\Theta}{dy} - Bi_2\Theta = 0, \quad \text{at} \;\; y = -\frac{1}{2}, \tag{4}$$

where the Biot numbers

$$Bi_1 = (h_{eff})_1 a/k \quad \text{and} \quad Bi_2 = (h_{eff})_2 a/k$$

are the dimensionless expressions of the effective convective heat transfer coefficients of the upper and lower walls, $(h_{eff})_1$ and $(h_{eff})_2$, respectively, that take into account the existence of walls of finite thickness. Here

$$(h_{eff})_j = \frac{1}{\dfrac{(\delta_w)_j}{(k_w)_j} + \dfrac{1}{(h_e)_j}}, \quad j = 1, 2\,, \tag{5}$$

where δ_w and k_w are the wall thickness and the wall thermal conductivity, respectively, while $(h_e)_1$ and $(h_e)_2$ are the external convective heat transfer coefficient of the upper and lower walls, respectively.

The solution for the temperature field is given in the form

$$\Theta_f(y, Bi_1, Bi_2) = -G^2\frac{y^4}{12} + G\frac{y^3}{3} - \frac{y^2}{2} + Cy + D, \tag{6}$$

where

$$C = -\frac{2G\left(3Bi_1 + 3Bi_2 + Bi_1Bi_2\right) + \left(Bi_1 - Bi_2\right)\left(G^2 + 12\right)}{24\left(Bi_1 + Bi_2 + Bi_1Bi_2\right)},$$

and

$$D = \frac{1}{192Bi_2}\left[96C\left(2 + Bi_2\right) + 24(4 + Bi_2) + 8G(6 + Bi_2) + G^2(8 + Bi_2)\right].$$

We now proceed to calculate the entropy generation rate using the previous velocity and temperature fields.

2.1. Entropy generation rate

In the flow of a monocomponent viscous fluid, the local entropy generation rate, $\dot{S}$, can be written explicitly in dimensionless terms as [1,3]

$$\dot{S} = \frac{1}{\left(\Theta + \Theta_a\right)^2}\left(\frac{d\Theta}{dy}\right)^2 + \frac{1}{\Theta + \Theta_a}\left(\frac{du}{dy}\right)^2, \tag{7}$$

where $\dot{S}$ is normalized by k/a^2 and the dimensionless ambient temperature is given by $\Theta_a = kT_a/\eta U^2$. In writing Eq. (7), we have taken into account irreversibilities caused by both viscous dissipation and heat flow. The global entropy generation rate per unit length in the axial direction, $\langle\dot{S}\rangle$, is obtained by integrating $\dot{S}$ from $y = -1/2$ to $y = 1/2$. The explicit result reads

$$\langle\dot{S}\rangle = \frac{Bi_1F_1}{F_1 + 24\Theta_a(Bi_1 + Bi_2 + Bi_1Bi_2)} + \frac{Bi_2F_2}{F_2 + 24\Theta_a(Bi_1 + Bi_2 + Bi_1Bi_2)}, \tag{8}$$

where

$$F_1 = (2 + Bi_2)(12 + G^2) - 4Bi_2G, \tag{9}$$

$$F_2 = (2 + Bi_1)(12 + G^2) - 4Bi_1G. \tag{10}$$

Notice that this quantity only depends on the dimensionless parameters Bi_1, Bi_2, G and Θ_a. Since the global entropy generation rate considers the whole dissipation produced by irreversibilities in the system, we can look for values of the parameters that minimize the function $\langle \dot{S} \rangle$. Let us first explore the behavior of $\langle \dot{S} \rangle$ when the Biot numbers of each surface are the same ($Bi_1 = Bi_2$). This corresponds to symmetric convective cooling. In Fig. 1 we show the global entropy generation rate as a function of the single Biot number for different values of the dimensionless ambient temperatures (Θ_a=5, 7 and 9) and for different dimensionless ratios of velocities ($G = 0$ and 2). For instance, $\Theta_a = 7$ is obtained using the physical properties of engine oil [5] at an ambient temperature $T_a = 20^oC$. As can be observed from Fig. 1, for this case the global entropy generation rate is always a monotonous increasing function of Bi and reaches, for given Θ_a and G, a limiting value as $Bi \to \infty$.

Let us now consider conditions of asymmetric convective cooling, namely, the case when the Biot numbers for each surface are different. In this case, it is possible to find an optimum Biot number for one of the surfaces which leads to a minimum global entropy generation rate provided the dimensionless ambient temperature, the dimensionless ratio of velocities and the Biot number of the other surface remain fixed. For instance, if we fix Θ_a, G and the lower surface Biot number, Bi_2, it is found that the value of the upper surface Biot number, Bi_1, that minimizes $\langle \dot{S} \rangle$ is given by

$$(Bi_1)_{opt} = \frac{1}{2\beta}\left(-\alpha + \sqrt{\alpha^2 - 4\Gamma\beta}\right), \qquad (11)$$

where

$$\begin{aligned}\alpha = {}& 4\{576(6+2G+3Bi_2) + 4G^2(12+48G+G^2) \\ & \times(18+5Bi_2) + G^5(8+G\,(2+Bi_2)) + 12\Theta_a \\ & \times\{(144+G^2)(4+10Bi_2+3Bi_2^2+3Bi_2^3) \\ & +8G[G(12+30Bi_2+7Bi_2^2)+(12+G^2) \\ & \times(2+3Bi_2)]\} + 288\Theta_a Bi_2\{(12+G^2) \\ & \times(2+3Bi_2)+4GBi_2\}\},\end{aligned}$$

$$\begin{aligned}\beta = {}& \left(1728+G^2\right)(2+Bi_2) + 4G\{(144+G^4)(4+Bi_2) \\ & +G(12+G^2)(26+5Bi_2) + 8G^2\,(12+Bi_2)\} + 24\Theta_a \\ & \times\{(144+G^4)(4+7Bi_2+2Bi_2^2) + 8G[G(12+19Bi_2 \\ & +2Bi_2^2) + (12+G^2)(2+3Bi_2)]\} + 576\Theta_a^2\{(12+G^2) \\ & \times(2+7Bi_2+6Bi_2^2+Bi_2^3) + 4G(Bi_2-Bi_2^3)\},\end{aligned}$$

and

$$\begin{aligned}\Gamma = {}& 4\,(2+Bi_2)\,(1728+432G^2+36G^4+G^6) - 16Bi_2 \\ & \times G(12+G^2)^2 + 24\Theta_a\{8Bi_2(12+G^2)^2 + Bi_2^3(-144 \\ & +96G-40G^2+8G^3-G^4)\} + 576\Theta_a^2 Bi_2^2 \\ & \times\{12+G^2(2-Bi_2)+4Bi_2 G\}.\end{aligned}$$

In Fig. 2 we display the optimum upper wall Biot number, $(Bi_1)_{opt}$, as a function of Bi_2 for different values of G and $\Theta_a = 7$.

It is found that as Bi_2 increases, $(Bi_1)_{opt}$ approaches a limiting value which explicitly reads

$$\lim_{Bi_2\to\infty}(Bi_1)_{opt} = \left(1 + \frac{1}{2\Theta_a} - \frac{G\,(4-G)}{24\Theta_a}\right)^{1/2}. \qquad (12)$$

It is also observed that for small values of Bi_2, $(Bi_1)_{opt}$ takes negative values which evidently have no physical meaning. In fact, for these negative values of $(Bi_1)_{opt}$ no minimum values of $\langle \dot{S} \rangle$ are found. For instance, when $G = 0$ minimum values of $\langle \dot{S} \rangle$ are observed to occur for a given Bi_1

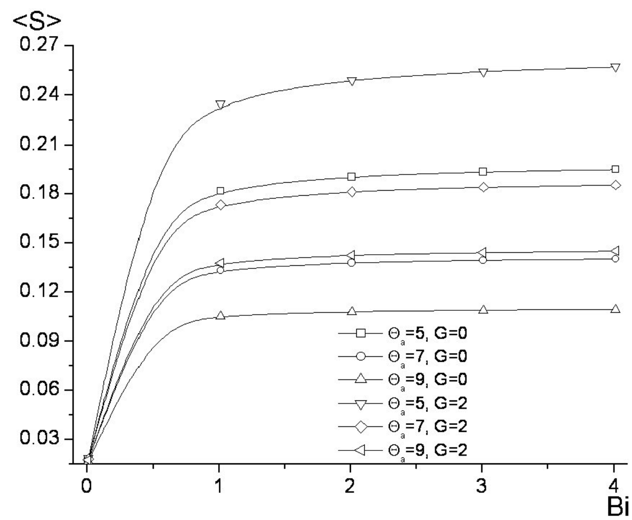

FIGURE 1. Global entropy generation rate as a function of the single Biot number for different ambient temperatures and $G = 0$ and 2 (symmetric convective cooling).

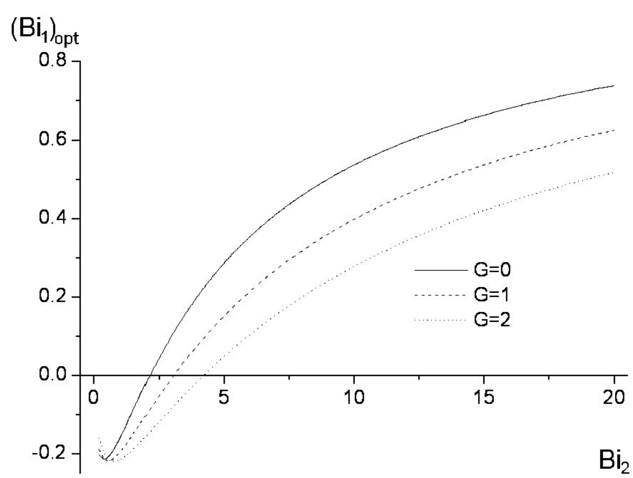

FIGURE 2. Optimum upper wall Biot number as a function of the lower wall Biot number for different dimensionless ratios of velocities. $\Theta_a = 7$.

provided $Bi_2 > 2$ and when $G = 2$ minimum values of $\langle \dot{S} \rangle$ are observed provided $Bi_2 > 4$. In Fig. 3 we have displayed $\langle \dot{S} \rangle$ as a function of Bi_1 for $\Theta_a = 7$, different values of Bi_2 $(20, 25$ and $30)$ and $G = 0$ and 2. For each curve, the function $\langle \dot{S} \rangle$ is normalized by its value at $Bi_1 = 0$. As Bi_2 increases the optimum value of Bi_1 also increases but it reaches the limit given by Eq. (12) as $Bi_2 \to \infty$. The foregoing results indicate, therefore, that the minimum dissipation can be reached by extracting heat in the system in an asymmetric way.

Let us now calculate the local Nusselt number at the upper wall, based on the internal convective heat transfer coefficient, h_i, namely [6],

$$h_i = -\frac{k}{T_w - T_b}\left(\frac{\partial T}{\partial y'}\right)_{y'=a/2}, \tag{13}$$

where T_b and T_w are the dimensional expressions of the bulk temperature (*i.e.* the cross-section averaged temperature of the stream) and the temperature at the wall, respectively. Hence, the local Nusselt number at the upper wall is given by

$$Nu = \frac{h_i a}{2k} = -\frac{\left(\dfrac{d\Theta}{dy}\right)_{y=1/2}}{2(\Theta(y=1/2)+\Theta_a-\Theta_b)} = \frac{Bi_1\Theta(y=1/2)}{2(\Theta(y=1/2)+\Theta_a-\Theta_b)}, \tag{14}$$

where the dimensionless bulk temperature is defined as

$$\Theta_b = \frac{\int\limits_{-1/2}^{1/2} u(\Theta+\Theta_a)dy}{\int\limits_{-1/2}^{1/2} u dy}.$$

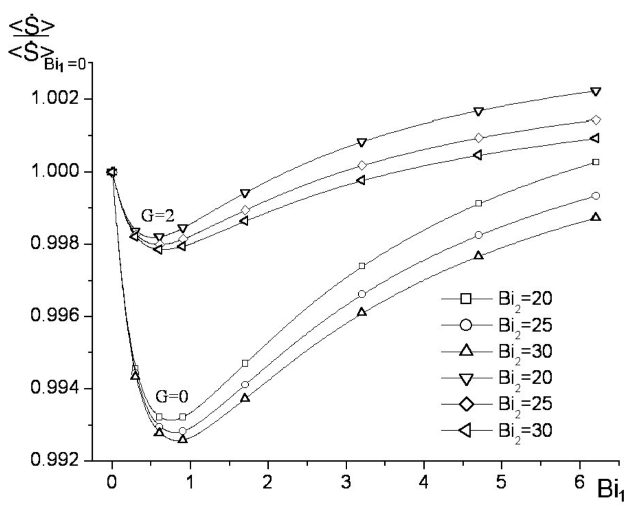

FIGURE 3. Normalized global entropy generation rate as a function of the upper wall Biot number for different lower wall Biot numbers and different dimensionless ratios of velocities. $\Theta_a = 7$.

Figure 4 shows the local Nusselt number evaluated at the optimum upper wall Biot number, $(Bi_1)_{opt}$, as a function of Θ_a for different lower wall Biot numbers and $G = 0$. This local Nusselt number for minimum entropy generation conditions displays a maximum value, namely, there is an optimum value of the dimensionless ambient temperature, $(\Theta_a)_{opt}$, where the heat transfer is maximum, once we fix G and Bi_2. Although we have not been able to derive the value of the maximum analytically, in Fig. 5 we present the numerically determined maximum Nusselt number, $(Nu)_{\max}$, evaluated at the optimum upper wall Biot number, as a function of Bi_2 for simple Couette flow $(G = 0)$. The maximum Nusselt number shows a monotonic increase with Bi_2.

From Fig. 4, it is also observed that Nu reaches a limiting value as $\Theta_a \to \infty$. In fact, this limiting value can

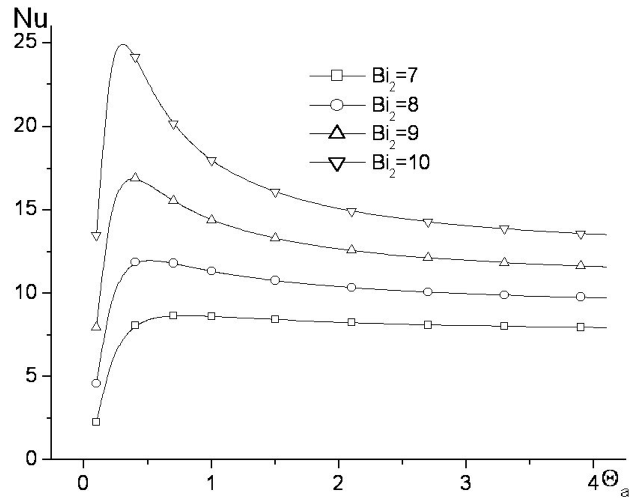

FIGURE 4. Local Nusselt number for minimum entropy generation conditions as a function of the dimensionless ambient temperature for different lower wall Biot number. $G = 0$.

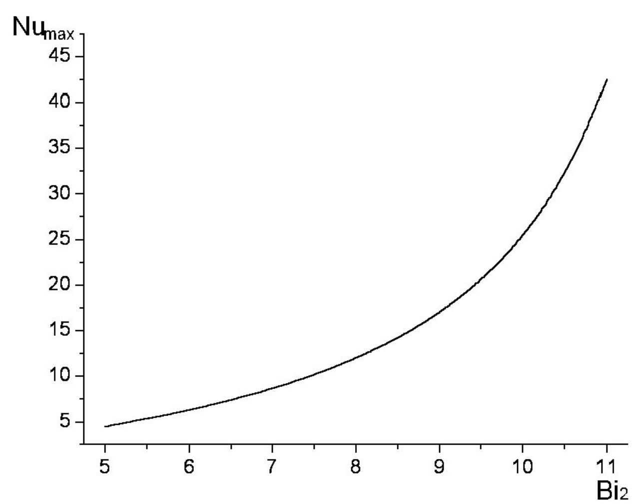

FIGURE 5. Maximum Nusselt number for minimum entropy generation conditions as a function of the lower wall Biot number. $G = 0$.

be determined analytically, namely,

$$\lim_{\Theta_a\to\infty}(Nu)_{opt} = \frac{\lim_{\Theta_a\to\infty}(Bi_1)_{opt}\,(6+G)\,F_1}{6\left[F_3-\lim_{\Theta_a\to\infty}(Bi_1)_{opt}\,(F_4+F_5)\right]}, \tag{15}$$

where

$$\lim_{\Theta_a\to\infty}(Bi_1)_{opt} = \frac{-Bi_2F_6}{(1+Bi_2)\,F_7},$$

$$F_3 = 0.0642Bi_2\,(3.5475{+}G)\left(8.7658{-}5.1031G{+}G^2\right),$$
$$F_4 = 0.0190Bi_2\,(5.8959{+}G)\left(17.8087{-}0.6459G{+}G^2\right),$$
$$F_5 = 0.1024\,(4.7391+G)\left(12.3662+2.7493G+G^2\right),$$
$$F_6 = \left(12+G^2\right)\left(2+Bi_2-Bi_2^2\right)-4Bi_2^2G\,(1+Bi_2)\,,$$
$$F_7 = \left(12+G^2\right)\left(2+5Bi_2+Bi_2^2\right)+4Bi_2G\left(1-Bi_2^2\right).$$

This limiting value of Nu depends on the lower wall Biot number and the dimensionless ratio of velocities.

Figure 6 shows the local Nusselt number evaluated at the optimum upper wall Biot number, $(Bi_1)_{opt}$, as a function of Bi_2 for different dimensionless ambient temperatures and $G=0$. It is of interest to note that at large values of Bi_2 a discontinuity of the curve $Nu=f(Bi_2)$ $(Nu{\to}\infty)$ is possible and in such circumstances the Nusselt number becomes negative. The negative values of Nu correspond to the temperature profiles with inflections, when the cross-section averaged temperature of the flow is greater than the wall temperature and hence the heat flux reverses.

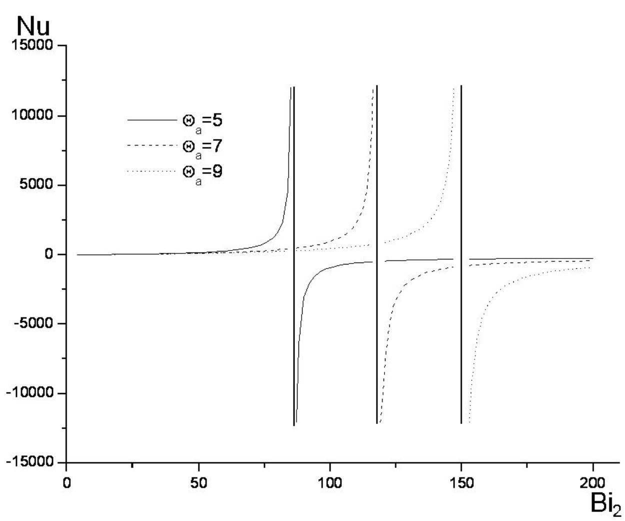

FIGURE 6. Local Nusselt number for minimum entropy generation conditions as a function of the lower wall Biot number for different dimensionless ambient temperature. $G=0$.

3. Concluding remarks

In this paper, the steady viscous flow between two infinite parallel planes, has been used to illustrate the possibility of minimizing the global entropy generation rate by cooling the external surfaces convectively in an asymmetric way. The flow is generated by both an axial pressure gradient and the uniform motion of the upper surface (generalized Couette flow).

The analytic expressions for the velocity and temperature fields were used to calculate the global entropy generation rate explicitly. In dimensionless terms, this function depends on the dimensionless ambient temperature, the dimensionless ratio of the two possible velocity scales (characterized by the magnitudes of the pressure gradient and the upper surface velocity) and the convective heat transfer coefficients (Biot numbers) of each wall.

The heat transfer problem was solved using thermal boundary conditions of the third kind, assuming different heat transfer coefficients for each wall. Under these conditions, the global entropy generation rate displayed a minimum for a given value of the upper wall Biot number when the lower wall Biot number, the dimensionless ambient temperature and the dimensionless ratio of velocities were kept fixed. Therefore, this determines the conditions that minimize the irreversibilities due to viscous friction and heat flow and, consequently, can be used to obtain optimal operation conditions that minimize the energy loss in the system.

On the other hand, the behavior of the local Nusselt number and its value for minimum entropy generation conditions was also explored. A maximum local Nusselt number was found for a given value of the dimensionless ambient temperature, namely, there is an optimum value of the dimensionless ambient temperature, $(\Theta_a)_{opt}$, for which both the heat transfer is maximum and the global entropy generation is minimum, when we fix the lower Biot number and the dimensionless ratio of velocities.

Acknowledgments

This research was supported by DGAPA-UNAM under project IN103100. G. Ibáñez thankfully acknowledges financial support from DGIA-UNAM.

1. A. Bejan, *Entropy Generation through Heat and Fluid Flow*, (Wiley, New York, 1994).
2. A. C. Baytas, *J. Non-Equilib. Thermodyn.* **22** (1997) 145.
3. S.R. De Groot and P. Mazur, *Non-Equilibrium Thermodynamics* (Dover, New York, 1984).
4. G. Ibáñez, S. Cuevas and M. López de Haro, *Int. J. Heat and Mass Transfer* **46** (2003) 1321.
5. J.P. Holman, *Heat Transfer*, Seventh ed. (Mc Graw-Hill, New York, 1990).
6. A. Bejan, *Heat Transfer* (Wiley, New York, 1993).