Nonlinear size effects of hot electrons in semiconductor thin films

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Recibido el 26 de noviembre de 2002; aceptado el 2 de junio de 2003

Theory of nonlinear heat size effects is developed in semiconductor films in the presence of external d.c. electric field. It is supposed that this field is applied along the film surfaces. The electron temperature is introduced, and it is shown that it depends on the electric field and the film thickness. The main equations are obtained for calculation this temperature, and analysis is done for the case of the weak electron heating. The characteristic length of the problem is discussed. It is the electron cooling length measured on submicron scale. It is shown that the heat size effects arise in the case when this length is comparable or less of the film thickness.

Keywords: Semiconductors film; electron temperature; cooling length; size effects; nonlinear electric conductivity.

Se desarrolla una teoría de los efectos de grosor en semiconductores cuyo grosor es del orden de la longitud de difusión (longitud de enfriamiento), la cual normalmente es de dimensiones submicronicas. Se supone que el campo eléctrico estático se aplica a lo largo de la superficie. La temperatura de los electrones se introduce bajo la suposición de que se cumpla la condición de dispersión cuasielástica sobre los fonones acústicos. Se muestra que la temperatura depende de la intensidad del campo eléctrico y del grosor de la película. Se presentan las ecuaciones fundamentales para calcular esta temperatura y se hace el análisis para el caso en que se tiene un calentamiento débil. Por otro lado, se presenta la discusión sobre la conductividad eléctrica no lineal.

Descriptores: Películas semiconductoras delgadas; electrones calientes; longitud de enfriamiento; conductividad eléctrica no lineal; efectos de grosor.

PACS: 72.20.Ht; 73.50.Fq

1. Introduction

Microminiaturization of modern semiconductor devises leads to decreasing of dimensions of the semiconductor working regions taking part in the electric transport. In a number of cases these dimensions can be comparable with some characteristic parameters having the dimension of length. One of them is the energy diffusion length (cooling length), that appears under the quasielastic collisions between the carriers of charge and the scattering centers (for example the scattering on acoustic phonons) when d.c. electric field is applied to the semiconductor sample.

In this case the energy relaxation time τ_ε is much greater than the momentum relaxation time τ. Evaluations show that the ratio of these times are of the order $(\tau_\varepsilon)/(\tau) \approx 10^3$ at the room temperature. Presence of two relaxation times ("fast" and "slow") leads to appearance of the characteristic diffusion length $l_\varepsilon = V_T(\tau\tau_\varepsilon)^{1/2}$ [1] which has been called above by the cooling length, here V_T is the mean heat velocity of the carriers of charge. This length is considerably exceeds the mean free path, and the heat size effects are associated namely with it. In the simplest case the heat size effects are demonstrated by the coordinate dependence of the mean energy of carriers and in the nonlinearity of the current-voltage characteristics under decreasing of the film thickness. The existence of these effects firstly has been reported in the experimental paper [2].

The cooling lengths are different in different semiconductors but usually vary from 10^{-4} up to 10^{-2} cm [3]. They are one of the greatest in the kinetic phenomena of semiconductors, and fabrication of semiconductor films with these thickness does not the problem today. Just this fact determines the great theoretic and experimental interest to discussed effects. The present paper has the aim to study some aspects of them.

2. Electron heating and symmetric electron distribution function

Let us consider the isotropic, nondegenerate n-type semiconductor layer with the thickness $2a$ in the x-direction. At the same time this layer is supposed to be infinite in the y- and z- directions. We assume that the external, homogeneous d.c. electric field E_z is applied along the z-axis, and the sample surfaces $x = \mp a$ contact with the heat reservoirs having the constant temperature T_0 (the equilibrium temperature).

We will also assume that the carrier density is sufficiently high so the cooling length exceeds the Debye radius $r_D = \sqrt{(\kappa T_0)/(4\pi e^2 n)}$, where κ is the dielectric permittivity, e is the electron charge, and n is the electron concentration. We use the energetic units in this paper, so the Boltzmann constant is equal to unity. This supposing allows to neglect the charge redistribution in the presence of the electric field. In this case the electron density is equal to the equilibrium concentration in any point of the layer.

At the same time we are supposing similar to Ref. 4 that the characteristic time of the electron-electron collisions τ_{ee} is much less than the time τ_ε. For simplicity, we also assume that the inelastic scattering mechanisms such as impact ionization, scattering on the optic phonons, and so on do not occur.

The inequality $\tau_{ee} \ll \tau_\varepsilon$ formally permits the existence of arbitrary ratio between the times τ_{ee} and τ (from $\tau_{ee} \ll \tau$ to $\tau_{ee} \gg \tau$). Nevertheless, the following correlation $\tau_{ee} \geq \tau$ [4], really takes place in an unipolar semiconductor. Thus, the characteristic times of the problem satisfy the following conditions:

$$\tau \leq \tau_{ee} \ll \tau_\varepsilon. \tag{1}$$

From the physical point of view it does mean that being in the electric field, electrons first of all get the energy from that field and very fast redistribute it between themselves. After that the energy relaxation to phonons is beginning. In this case the electron subsystem is quasi independent from the phonon subsystem and can be characterized by the own thermodynamic electron temperature $T_e(x, E_z)$ [4], and the dependence on x appears due to the possible energy surface relaxation. That temperature can exceed the equilibrium temperature T_0, and by this reason these electrons are known as the hot electrons. Thus, the symmetric part of the electron distribution function in the d.c. electric field can be written in the following form [4]:

$$f_0(\varepsilon, x, E_z) = \frac{n}{[2\pi m T_e(x, E_z)]^{\frac{3}{2}}}\, e^{-\dfrac{\varepsilon}{T_e\,(x, E_z)}}. \tag{2}$$

Here ε is the electron energy, m is the electron effective mass.

It is well known that under quasielastic collisions the full nonequilibrium function $f(\varepsilon, \vec{r})$ can be represented by the following Eq. (5):

$$f(\varepsilon, \vec{r}) = f_0(\varepsilon, \vec{r}) + f_1(\varepsilon, \vec{r})\frac{\vec{p}}{|\vec{p}|}, \tag{3}$$

where $f_1(\varepsilon, \vec{r})$ is the asymmetric part of the electron distribution function, $\vec{p}$ is the momentum vector, $|\vec{p}| = \sqrt{2m\varepsilon}$, and $f_1 \ll f_0$.

It is important to note that the asymmetric part of the distribution function $f_1(\varepsilon, \vec{r})$can be easy obtained with the help of the function f_0 (see, for example, Ref. 5). In our case both f_0 and f_1 are depended on the electric field E_z also through the electron temperature $T_e(x, E_z)$.

So, finally the problem is reduced to the calculation of the electron temperature$T_e(x, E_z)$. The dependence of this temperature on coordinates just leads to appearing of the size effects. The dependence of the electron temperature T_e on the electric field E_z leads to the nonlinearity of the electric conductivity.

3. Main equations of theory

We will assume that the semiconductor sample dimensions along y- and z- directions essentially exceed the cooling length l_ε. In this case all values depend on the coordinate x only, and the problem is one-dimensional. Besides, we consider that the phonon subsystem is equilibrium, and is described by the temperature T_0. So, the electron temperature $T_e(x, E_z)$ can be found from the following heat balance Eq. (5):

$$\frac{dW_x(x, E_z)}{dx} + \frac{nT_0}{\tau_\varepsilon(T_e)}\left(\frac{T_e(x, E_z)}{T_0} - 1\right) = j_z E_z. \tag{4}$$

Here, $W_x(x, E_z) = -\chi_e(T_e)(dT_e/dx)$ is the electron thermal flux in the absence of the electric current along the x-axes ($j_x = 0$); $\chi_e(T_e)$ is the electron thermal conductivity at the same condition $j_x = 0$; $\tau_\varepsilon(T_e) = \tau_{o\varepsilon}(T_e/T_0)^{q-1}$; $j_z = \sigma(T_e)E_z$ is the electric current along z-axes; $\sigma(T_e)$ is the electric conductivity of the hot electrons. Equations for $\sigma(T_e)$ and $\chi_e(T_e)$ one can find in Ref. 7. The factor $\tau_{0\varepsilon}$ and exponent q are listed in Table I of Ref. 5.

Equation (4) has the simple physical sense. The change of the electron heat flux associates with the Joule heating (the right-hand side), and the energy transfer to the phonon subsystem (second term in the left-hand side).

This equation must be supplemented by the boundary conditions, determining the heat exchange at $x = \mp a$ surfaces . They can be written in the following form [5]:

$$W\,|_{x=\mp a} = \pm\eta_\pm\,(T_e - T_0)\,|_{x=\mp a}\ , \tag{5}$$

where $\eta_\pm$ are the electron surface thermal conductivities.

4. Nonlinear electric conductivity of thin films

Let us assume that the external field is weak enough (the correspondent criterion will be discussed later). By this reason the electron temperature $T_e(x, E_z)$ can be represented in the following form:

$$T_e(x, E_z) = T_0 + \Delta T(x, E_z), \tag{6}$$

where $\Delta T_e(x, E_z)$ is the small addition to the equilibrium temperature, $\Delta T_e(x, E_z) \ll T_0$.

In this case Eq.(4) can be rewritten in the following form:

$$-\frac{d^2\Delta T_e(x, E_z)}{dx^2} + k^2\Delta T_e(x, E_z) = \frac{\sigma_0}{\chi_0}E_z^2. \tag{7}$$

Here

$$k^2 = \frac{1}{l_\varepsilon^2} = \frac{1}{V_T^2\tau_0\tau_{\varepsilon 0}} = \frac{n}{\chi_0\tau_{\varepsilon 0}}$$

is the squire of the reciprocal cooling length; τ_0 is the momentum relaxation time of the energy equilibrium electrons; σ_0 and χ_0 are the electric and thermal conductivities at the equilibrium temperature, respectively.

The solution of Eq.(7) with the boundary conditions (5) is

$$\Delta T_e(x,E_z)=\frac{\sigma_0\tau_{\varepsilon 0}}{n}\left(1-c_1e^{kx}-c_2e^{-kx}\right)E_z^2, \quad (8)$$

where

$$c_{1,2}=\frac{\xi_+\left(1\pm\xi_-\right)e^{\pm ka}+\xi_-\left(1\mp\xi_+\right)e^{\mp ka}}{\left(1+\xi_+\right)\left(1+\xi_-\right)e^{2ka}-\left(1-\xi_+\right)\left(1-\xi_-\right)e^{-2ka}};$$

$$\xi_\pm=\frac{\eta_\pm}{\chi_0 a}.$$

The expression in the brackets in Eq.(8) does not exceeds unity within the limits of $0\le\xi_\pm<\infty$, *i.e.* under the variation of the heat boundary conditions from the adiabatic boundary conditions to the isothermal ones. So, the condition of the weak electron heating $\Delta T_e(x,E_z)\ll T_0$ can take place if

$$E_z\ll E_0=\sqrt{\frac{nT_0}{\sigma_0}}. \quad (9)$$

The characteristic electric field E_0 can be rewritten in the form $E_0=(T_0)/(el_\varepsilon)$. Thus, it is the field in which the electron increases its equilibrium energy twice much in the distance of the cooling length.

Setting the temperature $T_e(x,E_z)$ from Eq.(6) to equation for the electric current $j_z=\sigma(T_e)E_z$, and averaging it by the sample section we obtain that the average electric current

$$<j_z>=\frac{1}{2a}\int\limits_{-a}^{a}\Delta T\left(x,E_z\right)dx=\sigma_0\left(1+\beta E_z^2\right)E_z. \quad (10)$$

Here

$$\beta=\frac{1}{2a\sigma_0}\frac{d\sigma_0\left(T_e\right)}{dT_e}|_{T_e=T_0}\ \frac{1}{E_z^2}\int\limits_{-a}^{a}\Delta T_e(x)dx \quad (11)$$

is the coefficient of nonlinearity describing the carrier heating by the external electric field.

Taking into account Eq.(8) we can get that

$$\beta=\beta_\infty\left(1-\frac{thka}{ka}\gamma\right), \quad (12)$$

where

$$\beta_\infty=\frac{\tau_{\varepsilon 0}}{n}\frac{d\sigma_0(T_e)}{dT_e}|_{T_e=T} \quad (13)$$

is the coefficient of nonlinearity in the case when the electron temperature is homogeneous along the film thickness (infinitely long sample or the elastic scattering at the surfaces, $\xi_\pm=0$),

$$\gamma=\frac{2\xi_+\xi_-thka+\xi_++\xi_-}{2\left(1+\xi_+\xi_-\right)thka+\left(\xi_++\xi_-\right)\left(1+th^2ka\right)}. \quad (14)$$

The second term in Eq.(12) is accompanied with the cooling of the electron gas at the film boundaries due to the presence of inelastic surface scattering mechanisms.

It is followed from the definition β_∞ [see Eq. (13)] that $\beta>0$ if the electric conductivity increases with the electron temperature increasing under some electron scattering mechanisms. In this case the current-voltage characteristic is superlinear curve. In the opposite case, when the electric conductivity decreases with increasing the electron temperature, $\beta<0$, and the current-voltage characteristic is sublinear curve.

It is easy to see that the coefficient of nonlinearity β decreases with decreasing of the film's thickness, and tends to zero if this thickness is much less than the cooling lengthl_ε. This means that the deviation from the Ohm's law in the thin films have to be observed at the intensities of E_z being much greater than in the thick samples.

It is follows from Eqs. (6) and (8) that the surfaces $x=\mp a$ cool the hot electrons throughout the volume of the film if its thickness is less or is of the order of the cooling length. At the same time these surfaces are cooling the hot carriers only nearby the surfaces if $a\gg l_\varepsilon$.

5. Conclusion

It is analyzed the simplest size effect in thin films which comes to the inhomogeneous of the electron heating along the film's thickness in the presence of d.c. electric field directed along the film surfaces. This heating results in the deviation from the Ohm's law, and this deviation is different in the thick and thin semiconductor layers. The criterion of the thick and thin layer is defined by the ratio between the sample length and the cooling length. The observed size effects must be taken into account under the designing the submicron electric circuits.

Acknowledgements

This work has been partially supported by Consejo Nacional de Ciencia y Tecnologia(CONACYT), México.

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