

Radiative corrections to the weak-magnetic dipole moment of leptons in the unbroken minimal supersymmetric standard model

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We compute, within the context of the Minimal Supersymmetric Standard Model, the one loop radiative corrections, to the weak-magnetic dipole moment of leptons. We prove that this weak-magnetic dipole moment vanishes in the limit of exact supersymmetry, in analogy to the vanishing of the magnetic moment.

Keywords: Supersymmetry; standard model; weak-magnetic moment.

En el contexto del modelo estándar mínimo supersimétrico calculamos las correcciones radiativas, a un lazo, del momento débil magnético de leptones. Demostramos que este momento se anula en el límite de supersimetría exacta, en analogía con el resultado para el momento magnético.

Descriptores: Supersimetría; modelo estándar; momento débil magnético.

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1. Introduction

Supersymmetry is widely considered the most attractive candidate of new physics beyond the standard model. It provides a high degree of symmetry in the description of nature and solves some fundamental problems of grand unified theories. However, it is not manifestly realized in the particle spectrum of the real world, which means that it is broken at low energies.

In the unbroken limit of the supersymmetric version of the standard model, bosonic and fermionic partners have equal masses and couple with the same strengths, and so their corresponding perturbation theory series cancel exactly. An example of this is that the magnetic moment of a spin 1/2 matter field vanishes [1]. Alternatively, a general argument based on the concept of superfields shows that it is not possible to construct a supersymmetric operator for the magnetic moment $\sim \bar{\psi} \sigma_{\mu\nu} \psi q^\nu$, where ψ is the spinor field, q^ν is the photon four-vector, and $\sigma_{\mu\nu} = (i/2)(\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu)$ with γ_μ Dirac matrices. This implies that the anomalous magnetic moment, arising from radiative corrections, vanishes too. In other words, graphs involving standard model particles cancel against the corresponding diagrams with the superpartners in the loop. A verification of this result, as a consistency check, was also performed in Ref. 1 and in some subsequent reexaminations of this issue. Analogously, the coupling between fermions and the neutral Z boson allows us to define a weak-magnetic dipole moment. In the same token, an argument based on superfields, consistent essentially in replacing the photon field by the neutral weak Z-field, shows that this weak-magnetic moment vanishes. The explicit verification of this result is the concern of this work: we compute, within the context of exact susy, the radiative corrections to one loop of the lepton weak-magnetic dipole moment, and show the ex-

actly cancelation among the different contributions. We believe this is interesting by itself and also as an illustration of the calculations involved.

Radiative corrections to the weak-magnetic moment of leptons, in the Standard Model $SU(2) \times U(1)$, have been considered in Ref. 2, where a formal presentation of Feynman amplitudes in the unitary gauge is presented, and in Ref. 3, where an explicit numerical calculation is done. In the present work, following Ref. 4, we compute the Feynman amplitudes in the t'Hooft-Feynman gauge for the several contributing supermultiplets:

- (a) photon(γ)-photino($\tilde{\gamma}$),
- (b) W boson (W^\pm)-wino (\tilde{w}^\pm) - charged Higgs (H^\pm),
- (c) Z boson (Z) - zino (\tilde{z}) - neutral Higgs (H^0),
- (d) Higgs (h_i) - Higgsino(\tilde{h}), and leptons (ℓ) - scalar leptons ($\tilde{\ell}_i$).

A distinctive feature of this computation is that the contributions depicted in Figs. 4 and 6 below are not present in the radiative corrections to the anomalous magnetic moment. An evaluation of the different contributions in the broken case, taking into account bounds on several parameters arising from the recently reported measurement of the muon's anomalous magnetic moment [5], will be given elsewhere [6].

To proceed, we define the anomalous weak-magnetic moment of lepton ψ of mass m as the coefficient of $\bar{\psi}(i\sigma_{\mu\nu}q^\nu)\psi\varepsilon^\mu(q)$ in the coupling

$$\frac{iga_\ell}{4m \cos \theta_W} \bar{\psi}(i\sigma_{\mu\nu}q^\nu)\psi\varepsilon^\mu(q), \quad (1)$$

where g is the $SU(2)$ weak coupling, and $a_\ell = g_{\ell L} + g_{\ell R}$, with $g_{\ell L,R} = T_{3L,R} - Q \sin^2 \theta_W$, $T_{3L,R}$ being the lepton

weak isospin third component and Q its electric charge. Using $e = g \sin \theta_W$, we observe that the weak magnetic moment of a charged lepton is approximately 5% smaller than its magnetic dipole moment ($e/2m$).

The Feynman rules are described in a variety of monographs and papers; here, we follow Ref. 7, from which we also take notation. For definiteness we consider the case of a charged lepton, but the case for a neutral one can be deduced after some changes. In the next sections we give our results.

2. $(\gamma) - (\tilde{\gamma})$ Contribution

Figures 1(a)-(b) shows the contributing diagrams from the supermultiplet $(\gamma) - (\tilde{\gamma})$. For photon exchange, Fig.1(a), we obtain for the anomalous weak-magnetic moment, after Feynman parametrization,

$$\Delta\mu^{(\gamma)}(q^2) = -8g^2 \sin^2 \theta_W m^2 i \times \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4 k}{(2\pi)^4} \frac{(1-x-y)(x+y)}{[k^2 - f(\gamma)]^3}, \quad (2)$$

where $f(\gamma) = m^2(x+y)^2 - q^2 xy$, and $q^2 = M_Z^2$. For photino exchange, Fig.1(b), we obtain for the anomalous weak-magnetic moment,

$$\Delta\mu^{(\tilde{\gamma})}(q^2) = -\frac{8g^2 \sin^2 \theta_W}{a_\ell} m i \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4 k}{(2\pi)^4} \times \left\{ \frac{(\frac{1}{2} - \sin^2 \theta_W)(1-x-y)[m(x+y) + M_{\tilde{\gamma}}]}{[k^2 - f_{\tilde{\gamma}}^L]^3} - \frac{\sin^2 \theta_W (1-x-y)[m(x+y) + M_{\tilde{\gamma}}]}{[k^2 - f_{\tilde{\gamma}}^R]^3} \right\}, \quad (3)$$

where

$$f_{\tilde{\gamma}}^{L,R} = -m^2(1-x-y)(x+y) + M_{L,R}^2(x+y)^2 + M_{\tilde{\gamma}}^2(1-x-y) - q^2 xy,$$

with $M_{L,R}$ the mass of the scalar lepton in the loop, and $M_{\tilde{\gamma}}$ the mass of the photino. To obtain Eq.(3) we have assumed that the sleptons are mass eigenstates. Furthermore, in the limit of exact supersymmetry (susy) $M_L = M_R = m$, $M_{\tilde{\gamma}} = 0$, $f_{\tilde{\gamma}}^L = f_{\tilde{\gamma}}^R = f(\gamma)$, and Eq.(3) reduces to

$$\Delta\mu^{(\tilde{\gamma})}(q^2) = 8g^2 \sin^2 \theta_W m^2 i \times \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4 k}{(2\pi)^4} \frac{(1-x-y)(x+y)}{[k^2 - f(\gamma)]^3}$$

cancelling the contribution from Eq.(2).

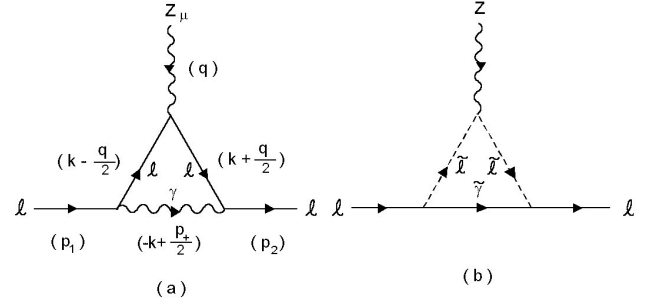


FIGURE 1. Diagrams corresponding to (a) photon and (b) photino exchange. Particles momenta are denoted in parenthesis.

3. $(h_1^0) - (h_2^0) - (\tilde{h})$ Contribution

The diagrams are shown in Fig.2(a)-(b). The contribution from the scalar Higgs particle h_1^0 is given by

$$\Delta\mu^{(h_1^0)}(q^2) = -2g^2 \frac{\cos^2 \alpha}{\cos^2 \beta} \frac{m^2}{M_W^2} m^2 i \times \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4 k}{(2\pi)^4} \frac{(2-x-y)(x+y)}{[k^2 - f(h_1^0)]^3}, \quad (4)$$

and the one from the pseudoscalar h_2^0 is

$$\Delta\mu^{(h_2^0)}(q^2) = 2g^2 \tan^2 \beta \frac{m^2}{M_W^2} m^2 i \times \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4 k}{(2\pi)^4} \frac{(x+y)^2}{[k^2 - f(h_2^0)]^3}, \quad (5)$$

where $f(h_j^0) = m^2(x+y)^2 + M_{h_j^0}^2(1-x-y) - q^2 xy$, with $M_{h_j^0}$ the mass of the neutral Higgs particle, $\tan \beta$ is the ratio of the two Higgs vacuum expectation values and α is the mixing angle that arises in the process of diagonalizing the 2×2 neutral scalar Higgs mass matrix. For the neutral Higgsino exchange in Fig.2(c), we obtain

$$\Delta\mu^{(\tilde{h})}(q^2) = 4g^2 \frac{m^2}{M_W^2} m^2 i \times \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4 k}{(2\pi)^4} \frac{(1-x-y)(x+y)}{[k^2 - f(\tilde{h})]^3}, \quad (6)$$

where we have assumed the same mass for the left and right sleptons and added their contributions, and

$$f(\tilde{h}) = -m^2(1-x-y)(x+y) + M_{L,R}^2(x+y)^2 + M_{\tilde{h}}^2(1-x-y) - q^2 xy,$$

with $M_{\tilde{h}}$ the mass of the higgsino. Again, in the limit of exact susy we have $M_L = M_R = m$, $M_{h_1^0} = M_{h_2^0} = M_{\tilde{h}} = M_h$, $\tan \beta = 1$, $\cos^2 \alpha = \cos^2 \beta$, and the sum of Eqs.(4) and (5) cancels Eq.(6).

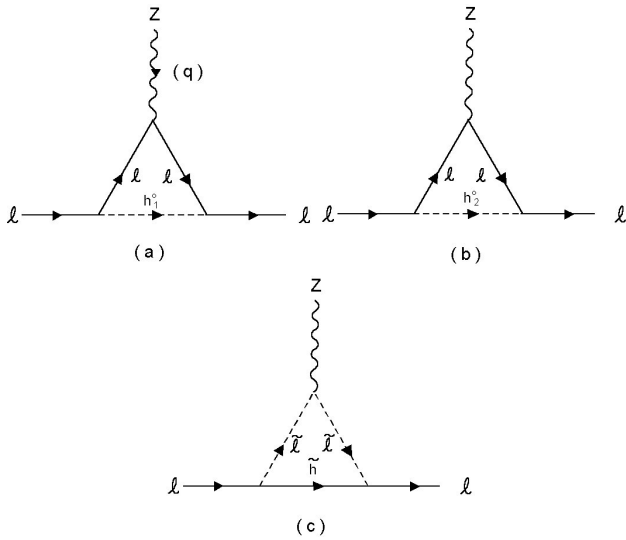


FIGURE 2. Set of diagrams for Higgs particle exchange (a) h_1^0 , (b) h_2^0 , and higgsino exchange (c) \tilde{h} .

4. $(W^\pm) - (\tilde{w}^\pm) - (H^\pm)$ Contribution

In this case we have two sets of diagrams, shown in Fig.3(a)-(f) and Fig.4(a)-(d). The contribution from Fig.3(a)

is given by

$$\Delta\mu^{(\nu)}(q^2) = \frac{2g^2 \cos^2 \theta_W}{a_\ell} m^2 i \times \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4 k}{(2\pi)^4} \frac{(1+2x+2y)(x+y)}{[k^2 - f(\nu)]^3} \quad (7)$$

where

$$f(\nu) = -m^2(1-x-y)(x+y) + M_W^2(x+y)^2 + m_\nu^2(1-x-y) - q^2 xy,$$

with M_W and m_ν the mass of the W-boson and the neutrino, respectively. For the sum of the contributions from Figs.3(b)-(c) we obtain

$$\Delta\mu^{(\nu)}(q^2) = \frac{2g^2 \sin^2 \theta_W}{a_\ell} m^2 i \times \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4 k}{(2\pi)^4} \frac{(x+y)}{[k^2 - f(\nu)]^3} \quad (8)$$

The contribution coming from Fig.3(d) is

$$\Delta\mu^{(\nu)}(q^2) = \frac{g^2 \cos 2\theta_W}{a_\ell} \frac{m^2}{M_W^2} i \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4 k}{(2\pi)^4} \frac{[(m^2 + m_\nu^2)(x+y) - 2m_\nu^2](1-x-y)}{[k^2 - f(\nu)]^3} \quad (9)$$

and from Fig. 3(e) is

$$\Delta\mu^{(\nu)}(q^2) = -\frac{g^2 \cos 2\theta_W}{a_\ell} \frac{m^2}{M_W^2} i \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4 k}{(2\pi)^4} \frac{[(m^2 + m_\nu^2)(x+y) - (\tan^2 \beta + \cot^2 \beta)m_\nu^2](1-x-y)}{[k^2 - g(\nu)]^3}, \quad (10)$$

where

$$g(\nu) = -m^2(1-x-y)(x+y) + M_H^2(x+y)^2 + m_\nu^2(1-x-y) - q^2 xy,$$

M_H being the mass of the charged Higgs boson in the loop.

The contribution from the wino, and the wino-higgsino mixing, in Fig.3(f) is

$$\Delta\mu^{(\tilde{\nu})}(q^2) = \frac{2g^2}{a_\ell} m^2 i \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4 k}{(2\pi)^4} \frac{(x+y)[2(1-x-y) \cos^2 \theta_W - (1+2 \cos^2 \theta_W) \frac{M_{\tilde{w}}}{M_W}]}{[k^2 - f(\tilde{\nu})]^3}, \quad (11)$$

with

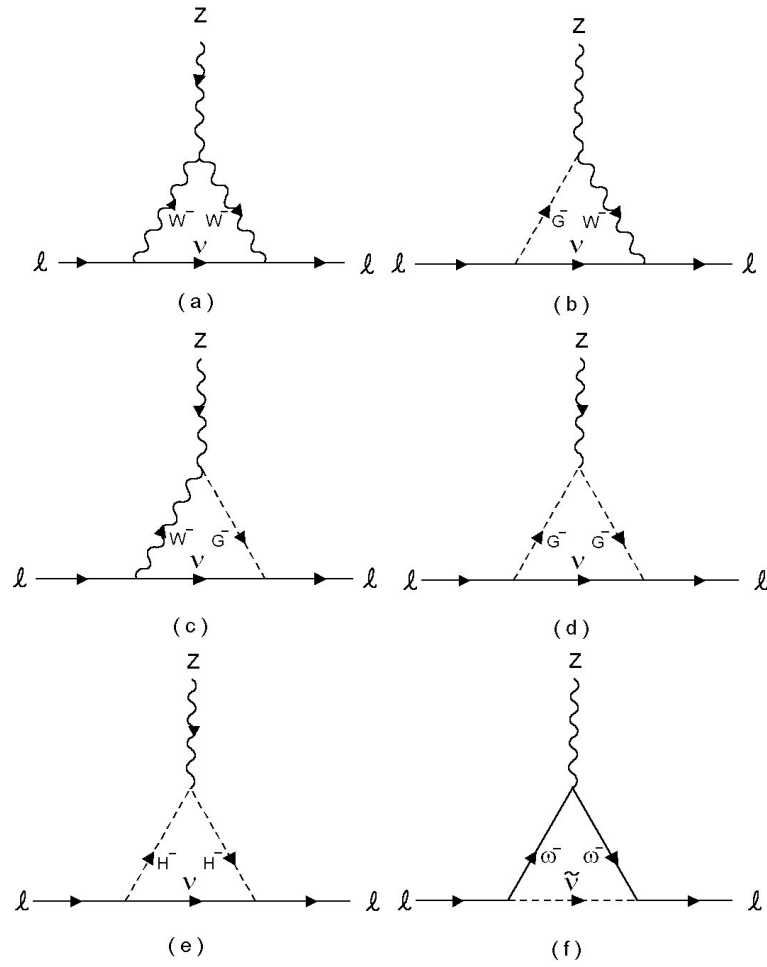
$$f(\tilde{\nu}) = -m^2(1-x-y)(x+y) + M_{\tilde{w}}^2(x+y)^2 + m_{\tilde{\nu}}^2(1-x-y) - q^2 xy.$$

In the limit of exact susy the sum of Eqs. (7) and (8) cancels Eq. (11), and Eqs. (9) and (10) cancel each other.

Now we consider the contributions coming from the set of graphs in Fig. 4, which are absent in the case of the mag-

netic moment. The contribution from Fig.4(a) is

$$\Delta\mu^{(W)}(q^2) = -\frac{4g^2(a_\nu - b_\nu)}{a_\ell} m^2 i \times \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4 k}{(2\pi)^4} \frac{(1-x-y)(2-x-y)}{[k^2 - f(W)]^3}. \quad (12)$$


 FIGURE 3. Neutrino (ν) exchange (a)-(e), and sneutrino ($\tilde{\nu}$) exchange (f).

Here

$$f_{(W)} = -m^2(1-x-y)(x+y) + M_W^2(x+y)^2 + m_\nu^2(x+y) - q^2xy,$$

and a_ν and b_ν are the neutrino Z-boson couplings. From Fig.4(b) we obtain

$$\Delta\mu^{(G)}(q^2) = -\frac{4g^2}{a_\ell} \frac{m^2}{M_W^2} i \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4k}{(2\pi)^4} \frac{(x+y)\{[m^2(a_\nu - b_\nu) + m_\nu^2(a_\nu + b_\nu)](1-x-y) - 4m_\nu^2 a_\nu\}}{[k^2 - f_{(W)}]^3} \quad (13)$$

and from Fig. 4(c)

$$\Delta\mu^{(H)}(q^2) = \frac{4g^2 a_\nu}{a_\ell} \frac{m^2}{M_W^2} i \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4k}{(2\pi)^4} \frac{(x+y)\{[m^2 \tan^2 \beta + m_\nu^2 \cot^2 \beta](1-x-y) - 2m_\nu^2\}}{[k^2 - f_{(H)}]^3}, \quad (14)$$

where

$$f_{(H)} = -m^2(1-x-y)(x+y) + M_H^2(x+y)^2 + m_\nu^2(x+y) - q^2xy.$$

The contribution from Fig.4(d) is

$$\Delta\mu^{(\tilde{\omega})}(q^2) = \frac{2g^2}{a_\ell} m^2 i \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4k}{(2\pi)^4} \frac{(1-x-y) \left(\frac{2M_{\tilde{\omega}}}{M_W} - x - y \right)}{[k^2 - f_{(\tilde{\omega})}]^3}; \quad (15)$$

here

$$f(\tilde{w}) = -m^2(1-x-y)(x+y) + M_{\tilde{w}}^2(1-x-y) + m_\nu^2(x+y) - q^2xy.$$

The term linear in wino mass arise from wino-higgsino mixing, a consequence of explicit soft susy breaking. When taking the susy limit we observe that Eq.(12) cancels Eq.(15), while Eq.(13) cancels Eq.(14).

5. $(Z) - (\tilde{z}) - (H^0)$ Contribution

The diagrams from this supermultiplet are depicted in Fig.5(a) - (d) and Fig.6(a) - (d). The results are given by

$$\Delta\mu^{(Z)}(q^2) = \frac{4g^2}{\cos^2\theta_W} m^2 i \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4k}{(2\pi)^4} \times \frac{(1-x-y) [(a_\ell^2 + 3b_\ell^2)(x+y) - 8b_\ell^2]}{[k^2 - f(Z)]^3} \quad (16)$$

from Fig.5(a), where

$$f(Z) = m^2(x+y)^2 + M_Z^2(1-x-y) - q^2xy.$$

Diagrams in Fig.5(b) and (c) leads to

$$\Delta\mu^{(G)}(q^2) = 2g^2 \frac{m^2}{M_W^2} m^2 i \times \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4k}{(2\pi)^4} \frac{(x+y)^2}{[k^2 - f(Z)]^3} \quad (17)$$

and

$$\Delta\mu^{(H)}(q^2) = 2g^2 \frac{m^2}{M_W^2} \frac{\sin^2\alpha}{\cos^2\beta} m^2 i \times \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4k}{(2\pi)^4} \frac{(2-x-y)(x+y)}{[k^2 - f(H)]^3}. \quad (18)$$

here $f(H) = m^2(x+y)^2 + M_H^2(1-x-y) - q^2xy$. Finally, for the zino contribution we obtain

$$\Delta\mu^{(\tilde{z})}(q^2) = -\frac{2g^2}{\cos^2\theta_W} m^2 i \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4k}{(2\pi)^4} \times \frac{(1-x-y) \left[2(a_\ell^2 + 3b_\ell^2)(x+y) - \frac{M_{\tilde{z}}}{M_Z} \right]}{[k^2 - f(\tilde{z})]^3}, \quad (19)$$

where $f(\tilde{z}) = -m^2(1-x-y)(x+y) + M_{\tilde{z}}^2(1-x-y) + M_L^2(x+y) - q^2xy$, and the term linear in $M_{\tilde{z}}$ comes from zino-higgsino mixing. As can be seen, in the limit of exact susy, the sum of Eqs.(16)-(18) cancels the one in Eq.(19).

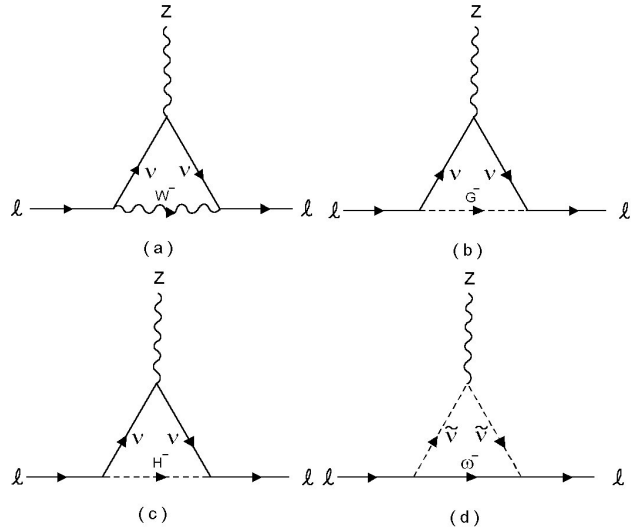


FIGURE 4. W-boson (W) exchange (a)-(c), and wino (\tilde{w}) exchange (d).

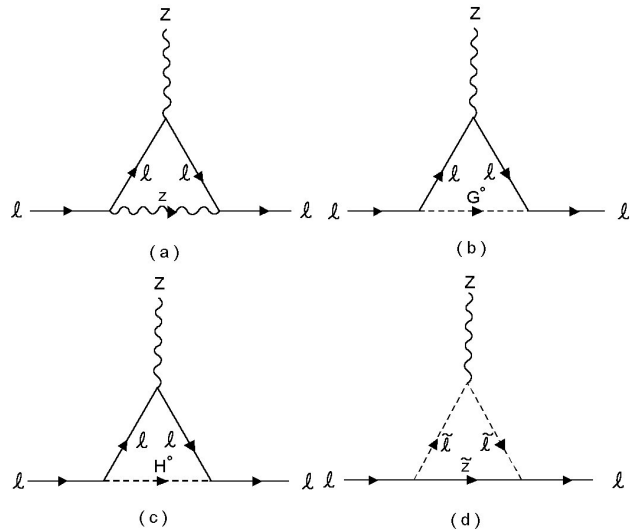


FIGURE 5. Z-boson exchange (a)-(b), Higgs (H) exchange (c), and zino (\tilde{z}) exchange (d).

The next set of graphs is depicted in Fig. 6, with the following contributions. From Fig.6(a),

$$\Delta\mu^{(h,Z)}(q^2) = \frac{g^2}{\cos^2\theta_W} \frac{\sin\alpha \sin(\beta-\alpha)}{\cos\beta} m^2 i \times \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4k}{(2\pi)^4} \frac{x}{[k^2 - f_h]^3}, \quad (20)$$

with $f_h = (1-x-y)^2 m^2 + xM_{h_1^0}^2 + yM_Z^2 - xyq^2$. From Fig.6(b) we obtain

$$\Delta\mu^{(Z,h)}(q^2) = \frac{g^2}{\cos^2\theta_W} \frac{\sin\alpha \sin(\beta-\alpha)}{\cos\beta} m^2 i \times \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4k}{(2\pi)^4} \frac{y}{[k^2 - f'_h]^3} \quad (21)$$

with $f'_h = (1-x-y)^2 m^2 + yM_{h_1^0}^2 + xM_Z^2 - xyq^2$. The contributions from the higgs H are quite similar to these two previous:

$$\Delta\mu^{(H,Z)}(q^2) = \frac{g^2}{\cos^2\theta_W} \frac{\cos\alpha \cos(\beta-\alpha)}{\cos\beta} m^2 i \times \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4k}{(2\pi)^4} \frac{x}{[k^2 - f_H]^3} \quad (22)$$

where $f_H = (1-x-y)^2 m^2 + yM_Z^2 + xM_H^2 - xyq^2$, and

$$\Delta\mu^{(Z,H)}(q^2) = \frac{g^2}{\cos^2\theta_W} \frac{\cos\alpha \cos(\beta-\alpha)}{\cos\beta} m^2 i \times \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4k}{(2\pi)^4} \frac{y}{[k^2 - f'_H]^3} \quad (23)$$

where $f'_H = (1-x-y)^2 m^2 + xM_Z^2 + yM_H^2 - xyq^2$.

Finally, the contributions from Figs.6(c) are

$$\Delta\mu^{(\tilde{h},\tilde{z})}(q^2) = \frac{g^2}{\cos^2\theta_W} m^2 \left(\frac{M_{\tilde{z}}}{M_Z} \right) i \times \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4k}{(2\pi)^4} \frac{x}{[k^2 - \tilde{f}_{1L}]^3} \quad (24)$$

where

$$\tilde{f}_{1L} = -(1-x-y)(x+y)m^2 + (1-x-y)M_L^2 + xM_{\tilde{h}}^2 + yM_{\tilde{z}}^2 - xyq^2,$$

and From Fig.6(d)

$$\Delta\mu^{(\tilde{z},\tilde{h})}(q^2) = \frac{g^2}{\cos^2\theta_W} m^2 \left(\frac{M_{\tilde{z}}}{M_Z} \right) i \times \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4k}{(2\pi)^4} \frac{y}{[k^2 - \tilde{f}_{2L}]^3} \quad (25)$$

with

$$\tilde{f}_{2L} = -(1-x-y)(x+y)m^2 + (1-x-y)M_L^2 + xM_{\tilde{z}}^2 + yM_{\tilde{h}}^2 - xyq^2.$$

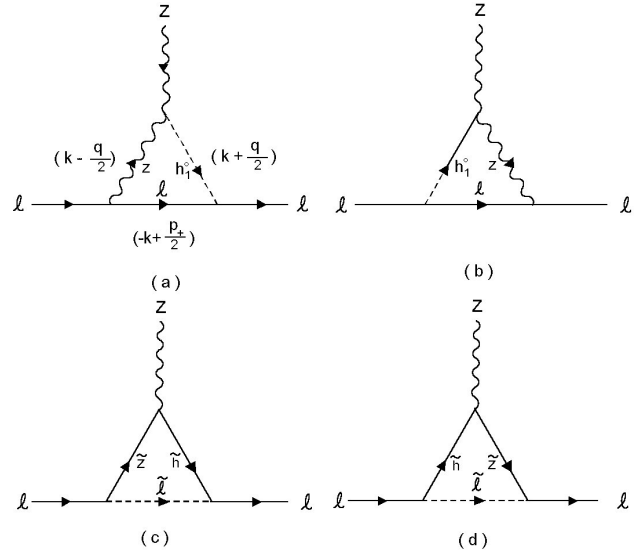


FIGURE 6. Lepton (ℓ) exchange (a)-(b), and slepton ($\tilde{\ell}$) exchange (c)-(d).

In the limit of exact susy $\cos(\beta-\alpha) = 0$, then Eqs. (20) and (21) vanish. Also $M_{\tilde{h}} = M_{h_1^0}$, $M_{\tilde{z}} = M_Z$, $M_L = m$, and $\sin\alpha = -\cos\beta$, then the sum of Eqs.(22) and (23) cancels the sum of Eqs.(24) and (25).

6. Conclusions

We have computed the weak-magnetic dipole moment of leptons, in the minimal supersymmetric standard model, and we showed that in the unbroken limit the various contributions cancel within each multiplet, without performing the integrals. This is the analogous result for the magnetic case ($q^2 = 0$), which only include diagrams in Figs. 1-3 and 5.

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