Polarization rotation of electromagnetic waves by two abelian gauge fields coupled to torsion

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By considering the coupling of two abelian gauge fields to pseudoscalar torsion we demonstrate that three different rotated polarized waves result for propagation over cosmological distances in a torsion field with constant gradient. It is also demonstrated that one of the waves is twice as intense as the other two providing us with a signature to identify the above coupling.

Keywords: Gauge fields; torsion; polarization rotation.

Considerando el acoplamiento de dos campos de norma abelianos con la torsión seudoescalar, demonstramos que tres distintas ondas giradas y polarizadas parecen propagarse sobre distancias cosmológicas en un campo de torsión con gradiente constante. También pretendemos demonstrar que una de estas ondas tiene una intensidad doble con respecto a las otras dos, lo que nos proporciona la oportunidad de identificar dicho acoplamiento.

Descriptores: Campo de norma; torsión; polarización rotación.

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1. Introduction

In recent years studies on polarization rotation of EM waves over cosmological distances has pointed to the cosmos as a laboratory to test for the anisotropy of space and violations of Lorentz invariance [1, 2]. Actually two decades ago Birch found a correlation between polarization angles and the source location angle relative to a fixed cosmological axis [3]. This finding was later confirmed by Kendall and Young [4]. Such a result suggests that either the universe was rotating or that there was an intrinsic anisotropy of phenomenon Field *et al.* [5] introduced a Lorentz violating term into the Lagrangian of electromagnetism and studied what effect this would have on the polarization rotation of EM waves propagating over cosmological distances. Actually, Faraday rotation [6] of the plane of polarization has been known for quite some time and is a rotation of the polarization due to a magnetic field aligned with the direction that the wave propagates. The rotation due to the Faraday effect is proportional to λ^2 and has to be subtracted from any experimental result that seeks to measure the rotation angle due to violation of Lorentz invariance or other effects. In addition to polarization rotation generated by violations of Lorentz invariance, two decades ago DiSabbata and Gasperini [7] showed how pseudoscalar generated torsion will rotate the plane of polarization of EM waves propagating over cosmological distances. Such an effect may provide us with signatures to identify torsion as an inseparable component of the gravitational field [8–10]. The particular form of torsion we study in this paper is that of a pseudoscalar field generating torsion that couples to electromagnetism in a gauge invariant manner [11, 12]. Previous studies of torsion have suggested various experimental signatures to identify it, they include Zeeman splitting induced by a torsion potential [13], Stern Gerlach separation of spinning particles induced by torsion [14] as well as perturbations generated by torsion on a spin polarized gyroscope [15]. Carrroll and Field [16] have also suggested that pseudoscalar torsion will give rise to interaction of Fermions and gauge bosons in addition to the usual electroweak interactions.

In a previous note [17] we have shown that if pseudoscalar torsion has a constant spatial gradient it will rotate the plane of polarization of EM waves clockwise (looking down the axis of propagation). In what follows we study what effect two abelian gauge fields coupled to torsion will have on the plane of polarization of EM waves. A second abelian gauge field was long ago discussed by Lee and Yang as a long range field coupled to baryon number [18]. Okun more recently suggested that both baryon and lepton charge couple to abelian gauge fields [19], also the same authors suggest that a second photon might exist that couples to muons and not electrons [20]. In a cosmological setting Axenides and Brandenberger [21] consider the possibility of a second photon (paraphoton) to understand anomalies in the cosmic background radiation. Such ideas were also proposed by Davidson and Peskin [22] and Holdom [23] wherein a second photon was the consequence of GUT theories. The other motivation for suggesting a second abelian gauge field comes from studies of the ”fifth force” which gives rise to a Yukawa type addition to the usual Newtonian potential [24, 25]. If the second abelian gauge field exists in nature it should couple to torsion in the same way that electromagnetism does. In what follows we consider the interaction of two abelian gauge fields with torsion. When we consider right and left polarized components of a plane wave we find that there are four possible rotations of the plane of polarization of EM waves. Thus, multiple rotations of the plane of polarization over cosmological distances will not only be a signature to identify torsion but also a signature to identify other abelian long range fields in nature.

2. Polarization rotation of two abelian gauge fields

We begin by writing the Lagrangian of two abelian gauge fields coupled to torsion as [11,12]

$$L=\frac{C^4}{16\pi G}\left(R+\frac{3}{2}\varphi_{,\alpha}\varphi^{,\alpha}\right)\sqrt{-g}-\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu}\sqrt{-g}$$
$$-\frac{1}{16\pi}B_{\mu\nu}B^{\mu\nu}\sqrt{-g}-\frac{\alpha_1}{16\pi}\left(\frac{\varepsilon^{\mu\nu\alpha\beta}A_\mu F_{\nu\alpha}\varphi_{,\beta}}{\sqrt{-g}}\right)\sqrt{-g}$$
$$-\frac{\alpha_2}{16\pi}\left(\frac{\varepsilon^{\mu\nu\alpha\beta}B_\mu B_{\nu\alpha}\varphi_{,\beta}}{\sqrt{-g}}\right)\sqrt{-g}-C_1F_{\mu\nu}B^{\mu\nu}\sqrt{-g} \quad (1)$$

here

$$F_{\mu\nu}=\frac{\partial A_\mu}{\partial x^\nu}-\frac{\partial A_\nu}{\partial x^\mu},\quad B_{\mu\nu}=\frac{\partial B_\mu}{\partial x^\nu}-\frac{\partial B_\nu}{\partial x^\mu},$$

α_1, α_2 are the fine structure constants for the electromagnetic (A_μ) and second Abelian gauge field (B_μ) respectively and $C_1F_{\mu\nu}B^{\mu\nu}$ represents a kinetic mixing term between electromagnetism and the second Abelian gauge field. Also, (ϕ = pseudoscalar torsion potential) and

$$\partial_\mu\varphi=\frac{1}{3!}\sqrt{-g}\varepsilon_{\mu mjk}T^{mjk},\ T^m_{jk}=\tau^m_{jk}-\tau^m_{kj}$$

represents torsion and τ^m_{jk} is the affine connection. When Eq. (1) is varied with respect to $A\mu$, $B\mu$ and ϕ we obtain the following field equations (in a flat background),

$$\frac{\partial}{\partial x^\nu}\left(\sqrt{-g}F^{\mu\nu}\right)=$$
$$\frac{\alpha_1}{2}\varepsilon^{\mu\nu\omega\alpha}F_{\nu\omega}\varphi_{,\alpha}-8\pi C_1\frac{\partial}{\partial x^\nu}\left(\sqrt{-g}B^{\mu\nu}\right),\quad (2)$$

$$\frac{\partial}{\partial x^\nu}\left(\sqrt{-g}B^{\mu\nu}\right)=$$
$$\frac{\alpha_2}{2}\varepsilon^{\mu\nu\omega\alpha}B_{\nu\omega}\varphi_{,\alpha}-8\pi C_1\frac{\partial}{\partial x^\nu}\left(\sqrt{-g}F^{\mu\nu}\right),\quad (3)$$

$$\Box\varphi=\frac{G\alpha_1}{6C^4}\frac{\varepsilon^{\alpha\beta\gamma\delta}F_{\alpha\beta}F_{\gamma\delta}}{\sqrt{-g}}+\frac{G\alpha_2}{6C^4}\frac{\varepsilon^{\alpha\beta\gamma\delta}B_{\alpha\beta}B_{\gamma\delta}}{\sqrt{-g}}.\quad (4)$$

We now consider the following field configuration propagating down the x axis:

$$F_{12}=B_{1z},\ F_{24}=E_{1y},\ F_{34}=E_{1z},\ F_{13}=-B_{1y},$$
$$B_{12}=B_{2z},\ B_{24}=E_{2y},\ B_{34}=E_{2z},\ B_{13}=-B_{2y}.$$

For right circularly polarized light we have

$$E_{y1}=E_{01}\cos\left(\omega t-kx\right),$$
$$B_{z1}=\frac{kC}{\omega}E_{01}\cos\left(\omega t-kx\right),$$
$$E_{z1}=E_{01}\sin\left(\omega t-kx\right),$$
$$B_{y1}=-\frac{kC}{\omega}E_{01}\sin\left(\omega t-kx\right),\quad (5)$$

and similar components for $B_{\mu\nu}$, in terms of E_{y2}, B_{z2}, E_{z2}, B_{y2}. Substituting Eq. (5) into Eq. (2) and Eq. (3) we obtain (for μ =2)

$$-\frac{k^2C}{\omega}E_{01}+\frac{E_{01}}{C}\omega E_{01}=\alpha_1E_{01}\varphi_{,1}$$
$$+8\pi C_1\frac{k^2C}{\omega}E_{02}-\frac{8\pi C_1}{C}\omega E_{02},\quad (6)$$

$$-\frac{k^2C}{\omega}E_{02}+\frac{E_{02}}{C}\omega E_{02}=\alpha_2E_{02}\varphi_{,1}$$
$$+\frac{8\pi C_1k^2C}{\omega}E_{01}-\frac{8\pi C_1}{C}\omega E_{01},\quad (7)$$

or

$$E_{01}\left(-\frac{k^2C}{\omega}+\frac{\omega}{C}-\alpha_1\varphi_{,1}\right)$$
$$+E_{02}\left(-8\pi C_1\frac{k^2C}{\omega}+\frac{8\pi C_1}{C}\omega\right)=0\quad (8)$$

$$E_{01}\left(\frac{-8\pi Ck^2C_1}{\omega}+\frac{8\pi C_1\omega}{C}\right)$$
$$+E_{02}\left(-\frac{k^2C}{\omega}+\frac{\omega}{C}-\alpha_2\varphi_{,1}\right)=0\quad (9)$$

(here we assume that $\varphi_{,1}$ = constant) or the spatial gradient of torsion is constant. We also assume $\varphi_{,4}=0$ so that torsion is time independent. In order that Eq. (8) and Eq. (9) possess a solution the determinant of the coefficients must be zero, calling

$$x=\frac{-k^2C}{\omega}+\frac{\omega}{C},$$

the solutions for x are

$$x=A_1\pm A_2 \text{ where } A_1=\frac{\left(\alpha_1+\alpha_2\right)\varphi_{,1}}{2\left(1-\left(8\pi C_1\right)^2\right)},$$
$$=B\pm.$$
$$A_2=\frac{\sqrt{(\alpha_1\varphi_{,1}+\alpha_2\varphi_{,1})^2-4\alpha_1\alpha_2\left(\varphi_{,1}\right)^2\left(1-(8\pi C_1)^2\right)}}{2\left(1-\left(8\pi C_1\right)^2\right)}.\quad (10)$$

The equation for ω is

$$\omega^2-CB_\pm\omega-k^2C^2=0.$$

To first order in α_1, α_2 we find

$$\omega_{R^+}=kC+\frac{C}{2}\left(A_1+A_2\right),\quad (11)$$
$$\omega_{R-}=kC+\frac{C}{2}\left(A_1-A_2\right).\quad (12)$$

For left circularly polarized light we have

$$\begin{aligned} E_{y1} &= E_{01}\cos(\omega t - kx)\,, \\ B_{y1} &= \frac{E_{01}kC}{\omega}\sin(\omega t - kx)\,, \\ E_{z1} &= -E_{01}\sin(\omega t - kx)\,, \\ B_{z1} &= \frac{E_{01}kC}{\omega}\cos(\omega t - kx)\,, \end{aligned} \tag{13}$$

with similar expressions for B$\mu\nu$ in terms of E_{y2}, B_{y2}, E_{z2} B_{z2}. Substituting Eq. (13) with similar expressions for B$\mu\nu$ into the μ = 2 components of Eq. (2) and Eq. (3) we obtain

$$\begin{aligned} &E_{01}\left(\frac{-k^2C}{\omega} + \frac{\omega}{C} + \alpha_1\varphi_{.1}\right) \\ &\qquad + E_{02}\left(-8\pi C_1\frac{k^2C}{\omega} + \frac{8\pi C_1\omega}{C}\right) = 0, \\ &E_{01}\left(\frac{-8\pi C_1 k^2 C}{\omega} + \frac{8\pi C_1\omega}{C}\right) \\ &\qquad + E_{02}\left(\frac{-k^2C}{\omega} + \frac{\omega}{C} + \alpha_2\varphi_{,1}\right) = 0. \end{aligned} \tag{14}$$

Again defining

$$x = \frac{-k^2C}{\omega} + \frac{\omega}{C}$$

we find for the determinant of the coeffecients in Eq. (14) to be zero

$$x = -A_1 \pm A_2. \tag{15}$$

Again solving for ω to first order in α_1, α_2 we obtain

$$\omega_{+L} = kC + \frac{C}{2}\left(-A_1 + A_2\right), \tag{16}$$

$$\omega_{-L} = kC + \frac{C}{2}\left(-A_1 - A_2\right), \tag{17}$$

where A_1, A_2 are defined in Eq. (10). For the rotation of the plane of polarization of an initially plane polarized EM wave we have [26]

$$\Delta = \frac{kL}{2}\left(\frac{1}{n_R} - \frac{1}{n_L}\right), \tag{18}$$

where if

$$\frac{1}{n_R} > \frac{1}{n_L}$$

the plane is rotated clockwise looking down the axis of propagation. For the following combinations of Eq. (11), Eq. (12), Eq. (16) and Eq. (17) we have using

$$\frac{C}{n} = \frac{\omega}{k}$$

$$\begin{aligned} &n_{R+}, n_{L+}, \\ &\Delta = \frac{kL}{2}\left(\frac{1}{2k}\left(A_1 + A_2\right) + \frac{1}{2k}\left(A_1 - A_2\right)\right) \\ &\Delta = \frac{L}{2}A_1, \end{aligned} \tag{19}$$

similarly for the other combinations of right and left polarized light we have

$$\begin{aligned} &n_{R-}, n_{L+}, \\ &\Delta = \frac{kL}{2}\left(\frac{A_1 - A_2}{2k} + \frac{1}{2k}\left(A_1 - A_2\right)\right) \\ &\Delta = \frac{L}{2}\left(A_1 - A_2\right), \end{aligned} \tag{20}$$

also for

$$\begin{aligned} &n_{R+}, n_{L-}, \\ &\Delta = \frac{L}{2}\left(A_1 + A_2\right), \end{aligned} \tag{21}$$

and

$$\begin{aligned} &n_{R-}, n_{L-} = \\ &\Delta = \frac{LA_1}{2}. \end{aligned} \tag{22}$$

Thus the combination R_-, L_- and R_+, L_+ have the same rotation angle while the combination R_+, L_- and R_-, L_+ have different rotation angles. Thus there will be three clockwise rotated polarized waves produced by the constant gradient torsion over cosmological distances and one of the rotated waves should be twice as intense as the other two. Also the measurement of the three rotation angles would provide us with information on the second Abelian fine structure constant and the mixing constant between F$\mu\nu$, B$\mu\nu$. If we use the limits on $\Delta\exp$ given by Carroll *et al.* [5] we have $\Delta < 0.1$ radians for a source with Z =0.9. Converting this to distance we have

$$\approx (0.9)(10^{18})(10^{10}) \cong 0.9 \times 10^{28}\ \text{cm}.$$

From Eq. (19) with $\alpha_1 \cong 1/137$,

$$0.1 \approx \frac{1}{137}\varphi_{,1}\,10^{28}, \quad \varphi_{,1} \cong 10^{-27}\ \text{cm}^{-1}.$$

Thus the gradient of the pseudoscalar torsion potential cannot be greater than this to be consistent with observation.

3. Conclusion

The additional last term in Eq. (1) resembles that of an axion coupling to electromagnetism [27] as well as a coupling where $\varphi_{,\mu}$ = constant which intrinsically violates both Lorentz invariance and parity [5]. The additional term in Eq. (1) also resembles a term added by Ni [28] when he discussed a violation of Schiff's conjecture demonstrating that the weak equivalence principle and Einstein equivalence principle may not be obeyed at the same time. Actually, from an observational point of view, Ni's theory, a theory with an axion, a theory with a constant vector Pμ violating Lorentz invariance and parity and the present theory of torsion are all equivalent for $\varphi,_1$ = const. In Ref. 26 it was pointed out that geomagnetic data set a value of $\varphi,_1$ to be $\varphi,_1 < 10^{-12}$ cm^{-1} whereas the data of Ref. 5 set a limit of $\varphi,_1 < 10^{-27}$ cm^{-1}. Though conflicting conclusions were drawn from the authors in Refs. 1 and 26 regarding the presence of the term of the above type in the Lagrangian of electromagnetism, its presence if reaffirmed would open up profound questions on its physical origin. Also, if the gradient of a scalar field couples to two Abelian gauge fields as discussed in this paper, the signatures of three different rotated plane polarized waves generated by such couplings would be a convincing demonstration that such a coupling exists in nature independent of its physical origin.

As pointed out in Refs. 18 and 19 a second abelian gauge field would exist if in fact Baryon and Lepton number are gauge charges generating long range forces in natrue. Actually due to the chiral anomaly the combination B-L would remain non-anomalous under quantum effects and this combination is also suggested by GUT. theory [23]. In fact any extension of the standard model that retains an additional unbroken U(1) factor in addition to electromagnetism would generate a second abelian gauge field as represented in Eq. (1). Of interest also is the fact that the kinetic mixing term in Eq. (1) would be generated by quantum corrections while keeping both abelian gauge fields massless [29]. More recently Glashow [30] has proposed a second photon that mixes with the ordinary photon in a Lorentz non-invariant manner, the content of such a proposal is to study how photon-velocity oscillations affect radiation from cosmological sources.

The particular lagrangian in Eq. (1) with $\varphi,_1$ = const. has profound implications concerning the structure of the universe since as mentioned in Ref. 5 it breaks the most sacred of all symmetries (Lorentz symmetry) by picking out a preferred frame thus forcing us to rethink the entire foundation upon which special and general relativity are built. If $\varphi,_1$ = const. is only an approximate relation these syummetries can be retained and the present study if confirmed may prod us to ask if the pseudoscalar torsion potential does in fact point to a breakdown of pure Einstein symmetric gravity emerging from either a more fundamental torsion theory based on gauge gravity [31] or perhaps string theory [32] where Eq. (1) would be the low-energy effective action generated by string theory at a higher scale. Thus the results of our investigation could be used as a basis of studying other abelian gauge fields in nature, a breakdown of Lorentz symmetry or a search for a more ultimate theory of gravity including torsion possibly emerging from gauge gravity or the superstring.

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