Decay $t \to bWZ$ within the context of the left-right mirror model

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In this paper the left-right mirror model is applied to the decay $t \to bWZ$, according to the Feynman rules given by the model. We write the corresponding width in compact form in terms of the Standard Model width by assuming the contribution to the WZW vertex being of the same order of magnitude as that of the tZt and bZb vertices. The width has to be compared with recent experimental data in order to get preliminary values for the parameters of the model, since these quantities have not been measured yet. With the appropriate rules given by the model we can deal with other related decays and improve results.

Keywords: Left-right mirror; top quark.

En este artículo se aplica el modelo izquierdo-derecho especular al decaimiento $t \to bWZ$, siguiendo las reglas de Feynman proporcionadas por el modelo. Escribimos la anchura correspondiente en forma compacta en términos de la del Modelo Estándar suponiendo que la contribución al vértice WZW es del mismo orden de magnitud que la de los vértices tZt y bZb. La anchura debe de compararse con datos experimentales recientes para obtener valores preliminares de los parámetros del modelo, ya que estas cantidades no han sido medidas aún. Con las reglas apropiadas que da el modelo podemos tratar otros decaimientos del mismo tipo y mejorar resultados.

Descriptores: Modelo izquierdo-derecho especular; quark top.

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1. Introduction

In 1956 Lee and Yang [1] presented a theory where the coupling between a charged lepton and a neutrino contains both a vector (V) part and an axial (A) part, so that the fermion current has a $V - A$ structure. Thus the parity (P) is maximally violated. In the standard model (SM), the $V - A$ structure of electroweak interactions is built in by assuming that only the left-handed fermions transform nontrivially under the gauge group $SU(2)$, and only they couple to the weak gauge boson W. However, the inclusion of a second class of fermions coupling to the W boson with $V + A$ currents restores P. Such particles have been called mirror fermions. Technically, mirror fermions are introduced in specific models to cancel anomalies. Ordinary and mirror fermions are conjugated to each other with respect to the gauge group, and the anomaly cancellation is automatic. Mirror particles appear naturally in many extensions of the SM, such as the grand unified theories (GUT), extended supersymmetry $(2 \leq N \leq 8)$ [2], string theories, and Kaluza-Klein theories. The extended supersymmetric theories are vector like, implying the existence of mirror fermions. In a superstring-inspired unification, possibly connected to $N = 2$ supergravity, the standard fermions would have both mirror and supersymmetric partners [3].

The strong CP problem is associated with the suppression of the θ term that breaks P and CP symmetries of the QCD Lagrangian. Recently, models involving mirror fermions have discussed within a left-right (LR) symmetric context [4,5]. These models offer a possible solution to the strong CP problem. On the other hand, mirror fermions can manifest themselves through their mixing with the standard fermions generating, for instance, new sources of flavor changing neutral processes, which could lead to observable signatures and to constrain such models.

The left-right mirror model $(LRMM)$ assigns right-handed doublets and left-handed singlets of isospin within the context of the group $SU(2)_L \otimes SU(2)_R \otimes U(1)$. That is, the right-handed mirror fermions transform like doublets, and the left-handed like singlets, under $SU(2)_R$. This model gives results in agreement with experiments for the decays $Z \to \nu\bar{\nu}$, $Z \to \nu_M\bar{\nu}_M$, and $Z \to e\bar{e}$, and $b\bar{b}$, where the suffix M stands for "mirror". It also helps the problems of the deficit of atmospheric and solar neutrinos and of the dark matter [6,7]. In ref. [8] there were obtained the off-diagonal couplings of the model that produce flavor- changing neutral currents, which could be a sign of new physics. When quarks are considered, from the $LRMM$ one can obtain the associated rules in the charged sector, now applied to the rare decay $t \to bWZ$. This decay has a predicted branching ratio (BR) in Standard Model (SM) of order 10^{-6} [9], which is beyond the sensibility of Tevatron or even CERN Large Hadron Collider (LHC); in the general version with the incorporation of two Higgs doublets, the BR is $\approx 10^{-2}$ [10], which seems feasible to be detected at the LHC.

However, the decay $t \to bWZ$ has peculiar features because it occurs near the kinematical threshold $(M_t \sim M_W + M_b + M_Z)$. This fact has as a consequence that the W and Z finite-width affects the theoretical value of the corresponding width.

In this work we give an extended version of the decay $t \to bWZ$; in fact, in relation with a previous one [11], here we have discussed points such as connections with supersymmetric theories and other treatments, comparison with the event $t \to bW\gamma$, loop corrections, and so on.

2. Left-right mirror model

The first family of leptons and quarks is

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \quad e_R, \begin{pmatrix} \nu_e \\ e \end{pmatrix}_{MR}, \; e_{ML}$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \quad u_R, \; d_R, \begin{pmatrix} u \\ d \end{pmatrix}_{MR}, \; u_{ML}, \; d_{ML},$$

respectively. Similar expressions can be written for the other families.

The Lagrangian involving the interactions between fermions and gauge bosons is given by

$$L^{int} = \bar{\psi} i\gamma^\mu D_\mu \psi + \bar{\psi}_M i\gamma^\mu D_{M\mu}\psi_M \tag{1}$$

where D_μ and $D_{M\mu}$ are the ordinary and mirror covariant derivatives, respectively, written as:

$$iD_\mu = \frac{i\partial}{x^\mu} - \frac{1}{2} g \; \tau_i W_{i\mu} - \frac{1}{2} g' Y B_\mu \tag{2}$$

$$iD_{Mm} = \frac{i\partial}{x^\mu} - \frac{1}{2} g_M \; \tau_{Mi} W_{Mi\mu} - \frac{1}{2} g' Y B_\mu \tag{3}$$

Here $\tau_i, \; i = 1, 2, 3$, stands for the Pauli matrices. This Lagrangian can be decomposed in its neutral and charged parts. The first one is [4], with $g_M = g$:

$$L_{nc} = \frac{e}{sc} \sum_{a=L,R} \bar{\psi}\gamma^\mu U_a^+ [(c_\alpha - \frac{s^2 s_\alpha}{r}) T_{3a} - \frac{c^2 s_\alpha}{r} T_{M3} a$$

$$+ s^2 (\frac{s_\alpha}{r} - c_\alpha) Q] U_a \psi_a Z_\mu \tag{4}$$

where

$$s = \sin\theta_W, \quad c = \cos\theta_W, \quad s_\alpha = \sin\theta_\alpha,$$
$$c_\alpha = \cos\theta_\alpha, \quad r = c_\beta c_{\theta_W} \quad \text{and} \quad \theta_W$$

is the Weinberg angle; U_a is the unitary matrix containing the parameters of the model and relates the gauge eigenstates ψ_a^0 with the corresponding mass eigenstates ψ_a, *i.e.* [4,8]:

$$U_a = \begin{pmatrix} A_a & B_a \\ C_a & D_a \end{pmatrix} \tag{5}$$

and

$$\psi_a = (\psi_i, \psi_{Mi})_a^T, \quad i = L, R. \tag{6}$$

Further, α and β are the rotation angles between the $Z - Z'$, and $Z' - A$ gauge bosons, respectively. T_3 and T_{M3} are the respective diagonal generators of the $SU(2)_L$ and $SU(2)_R$ groups . By means of the definition of U we get for Lnc [12]:

$$\begin{aligned} L_{nc} = (\bar{\psi}\gamma^\mu A\psi + \bar{\psi}_M \gamma^\mu B \psi_M)_L Z_\mu \\ + (\bar{\psi}\gamma^\mu C\psi + \bar{\psi}_M \gamma^\mu D\psi_M)_R Z_\mu \\ + (\bar{\psi}\gamma^\mu K\psi_M + \bar{\psi}_M \gamma^\mu K^+ \psi)_L Z_\mu \\ + (\bar{\psi}\gamma^\mu L\psi_M + \bar{\psi}_M \gamma^\mu L^+ \psi_M)_R Z_\mu \end{aligned} \tag{7}$$

with

$$K = \frac{1}{2}\delta' A^+ B, \qquad L = \frac{1}{2}\varepsilon' C^+ D$$

$$\delta' = \frac{e}{sc}(c_\alpha - \frac{s^2 s_\alpha}{r})$$

$$\varepsilon' = -\frac{e}{sc}\frac{c^2 s_\alpha}{r}$$

where, as for the charged sector, it is found:

$$\begin{aligned} L_c = -\frac{g}{\sqrt{2}} [(\bar{\psi}^0_{uL}\gamma^\mu W^+_\mu \psi^0_{dL} + \bar{\psi}^0_{dL}\gamma^\mu W^-_\mu \psi^0_{uL} \\ + \bar{\psi}^0_{MuR}\gamma^\mu W^+_{M\mu}\psi^0_{MdR} + \bar{\psi}^0_{MdR}\gamma^\mu W^-_{M\mu}\psi^0_{MuR}) \end{aligned} \tag{8}$$

with

$$\bar{\psi}^0_{uL} = (\bar{u}^0, \bar{c}^0, \bar{t}^0)_L, \quad \bar{\psi}^0_{dL} = (\bar{d}^0, \bar{s}^0, \bar{b}^0)_L, \tag{9}$$

3. The transition amplitude

The tree-level diagrams corresponding to the decay $t \to bWZ$ are given in Fig. 1. The contributions of the $LRMM$ to the tZt vertex are [11]

$$\begin{aligned} \frac{g}{\cos\theta_W} \Bigg[-\frac{1}{2} \left(\cos\alpha - \frac{\sin^2\theta_W \sin\alpha}{r_{\theta_W}} \right) (A^+ A)_{ttL} \\ + \frac{2}{3}\sin^2\theta_W \left(\frac{\sin\alpha}{r_{\theta_W}} - \cos\alpha \right) \Bigg] \equiv A'_{ttL} \end{aligned} \tag{10}$$

$$\begin{aligned} -\frac{1}{2}\frac{g}{\cos\theta_W} \Bigg[\frac{\cos^2\theta_W \sin\alpha}{r_{\theta_W}} (C^+ C)_{ttR} \\ + \frac{2}{3}\sin^2\theta_W \left(\frac{\sin\alpha}{r_{\theta_W}} - \cos\theta_W \right) \Bigg] \equiv C'_{ttR} \end{aligned} \tag{11}$$

For the Z boson with the b quark one has:

$$\frac{g}{\cos\theta_W}\left[-\frac{1}{2}\left(\cos\alpha-\frac{\sin^2\theta_W\sin\alpha}{r_{\theta_W}}\right)(A^{+}A)_{bbL}\right.$$
$$\left.-\frac{1}{3}\sin^2\theta_W\left(\frac{\sin\alpha}{r_{\theta_W}}-\cos\alpha\right)\right]\equiv A''_{bbL} \quad (12)$$

$$\frac{g}{\cos\theta_W}\left[-\frac{1}{2}\left(\frac{\cos^2\theta_W\sin\alpha}{r_{\theta_W}}\right)(C^{+}C)_{bbR}\right.$$
$$\left.-\frac{1}{3}\sin^2\theta_W\left(\frac{\sin\alpha}{r_{\theta_W}}-\cos\theta_W\right)\right]\equiv C''_{bbR} \quad (13)$$

Finally, the W coupling with these quarks is:

$$-\frac{1}{\sqrt{2}}g(A^*_{bbL}A^*_{ttL}). \quad (14)$$

The associated amplitudes to Figs. 1(a) and 1(b) are then:

$$M_{a)}+M_{b)}\equiv M_{ab}=-\frac{1}{\sqrt{2}}g(A^*_{bbL}A^*_{ttL}A''_{ttL}\bar{b}_L\gamma^\mu\Delta_b\gamma^\nu t_L$$
$$+A^*_{bbL}A^*_{ttL}A'_{ttL}\bar{b}_L\gamma^\nu\Delta_t\gamma^\mu t_L)\epsilon_{W\nu}\epsilon_{Z\mu}, \quad (15)$$

with

$$\Delta_t=i\frac{(k_1\cdot\gamma+m_t)}{k_1^2-m_t^2}, \qquad \Delta_b=i\frac{(k_2\cdot\gamma+m_b)}{k_2^2-m_b^2} \quad (16)$$

M_{ab} can be rewritten as:

$$M_{ab}=A_{\nu\mu}\sum_{i=1}^{2}a_iM_i^{\nu\mu}, \quad (17)$$

where

$$A_{\nu\mu}=-\frac{1}{\sqrt{2}}g\epsilon_{W_\nu}\epsilon_{Z\mu} \quad (18)$$
$$a_1=\frac{A^*_{bbL}A^*_{ttL}A''_{ttL}}{k_2^2-m_b^2}$$
$$a_2=\frac{A^*_{bbL}A^*_{ttL}A'_{ttL}}{k_1^2-m_1^2}$$

and

$$M_1^{\nu\mu}=\bar{u}(p_b)(1+\gamma_5)\gamma^\mu(k_2\cdot\gamma+m_b)\gamma^\nu(1-\gamma_5)u(p_t)$$
$$M_2^{\nu\mu}=\bar{u}(p_b)(1+\gamma_5)\gamma^\nu(k_1\cdot\gamma+m_t)\gamma^\mu(1-\gamma_5)u(p_t)$$

The amplitude for Fig. 1(c) is

$$M_{c)}=-\frac{1}{\sqrt{2}}g(-ig\cos\theta_W)\left(\frac{-i}{k_3^2-M_W^2}\right)$$
$$\times\left(g^{\lambda\rho}-\frac{k_3^\lambda k_3^\rho}{M_W^2}\right)\epsilon_W^\mu\epsilon_\nu^n\left\{(A^*_{bbL}A^*_{ttL})\overline{u}_L(p_b)\gamma_\lambda\right.$$
$$\times\left[(p_Z+k_3)_\mu g_{\rho\nu}+(p_W-p_Z)_\rho g_{\mu\nu}\right]$$
$$\left.-(k_3+p_W)_\nu g_{\rho\mu}]u_L(p_t)\right\} \quad (19)$$

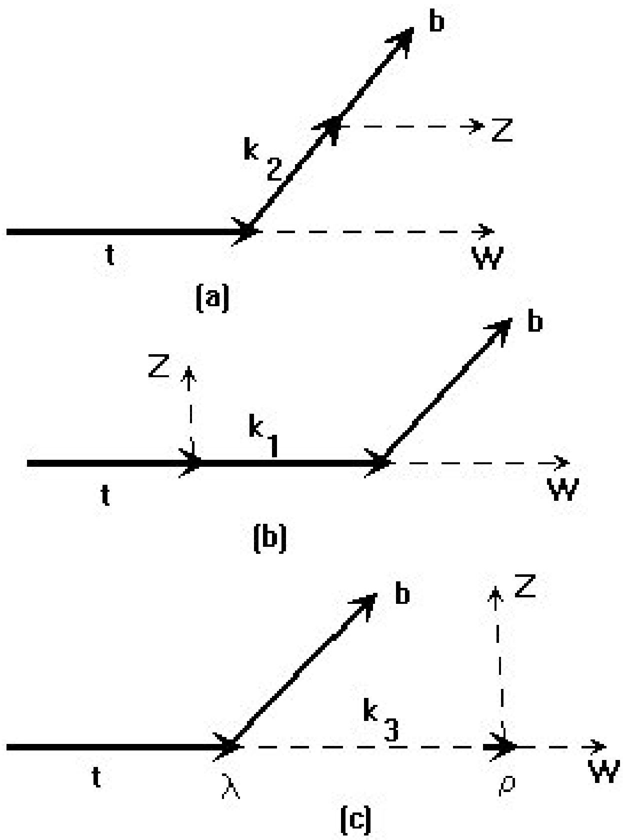

FIGURE 1. $t\longrightarrow bW^+Z$. Feynman diagrams for the amplitudes $M_{a)}$, $M_{b)}$, and $M_{c)}$.

After combination of these equations one finds for the complete amplitude of this process:

$$M{=}-\frac{1}{\sqrt{2}}g\epsilon W^\mu\epsilon_Z^\nu\{a_L\bar{b}_L\gamma^\mu\Delta_b\gamma^\nu t_L-b_L\bar{b}_L\gamma^\nu\Delta_t\gamma^\mu t_L$$
$$-\left(\frac{g\cos\theta_W}{k_3^2-M_W^2}\right)\left(g^{\lambda\rho}-\frac{k_3^\lambda k_3^\rho}{M_W^2}\right)c_L\overline{u_L}(p_b)\gamma_\lambda$$
$$\times[(p_Z+k_3)_\mu g_{\rho\nu}+(p_W-p_Z)_\rho g_{\mu\nu}$$
$$-(k_3+p_W)_\nu g_{\rho\mu}]u_L(p_t)\} \quad (20)$$

with

$$a_L=A^*_{bbL}A^*_{ttL}A''_{ttL},$$
$$b_L=A^*_{bbL}A^*_{ttL}A'_{ttL},$$
$$c_L=A^*_{bbL}A^*_{ttL}. \quad (21)$$

We take as a first approximation α small (otherwise the number of parameters is increased), *i.e.*, a nearly diagonal mixing matrix if we consider the extra Z' neutral boson to be sufficiently more massive than the Z boson, obtaining:

$$A''_{ttL}\approx A'_{ttL}\approx\frac{g}{\cos\theta_W}\left(g_{bL}+\frac{\alpha\ \sin^2\theta_W}{6r_{\theta W}}\right) \quad (22)$$

$$g_{bL} = -\frac{1}{2} + \frac{1}{3}\ \mathrm{sen}^2\theta_W$$

so that

$$M \approx c_L(M_{a)} + M_{b)})_{SM}(1+\delta) + c_L M_{c)\ SM} \tag{23}$$

where c_L is given by Eq. (21) and

$$\delta = \alpha \ \ \sin^2\theta_W/6r\theta_W. \tag{24}$$

Now we assume that the new contribution of the model to $M_{c)}$ is of the same order of δ, then

$$M \approx c_L(1+\delta)M_{SM} \tag{25}$$

and the width is

$$\Gamma \approx |c_L|^2(1+2\delta)\Gamma_{SM}. \tag{26}$$

Since the WZW coupling has not been discussed yet in the model, we assume it is roughly of the same form that for the vertices $\bar{b}Zb$ and $\bar{t}Zt$, to first order of g. However, to our knowledge there is no experimental bounds for $\Gamma(t \to bWZ)$. For the main decay mode $t \to bW$ one has

$$M = \left(\frac{1}{2}\sqrt{2}\right) V_{tb} g c_L^* \bar{u}(p_b)\gamma^\mu(1-\gamma_5)u(p_t) \tag{27}$$

or

$$M = c_L^* M_{SM} \tag{28}$$

This implies [13]:

$$\Gamma = |c_L|^2\Gamma_{SM} \approx 1.4|c_L|^2 GeV \tag{29}$$

With the result [14]:

$$R \equiv B(t \to bW)/\sum_{q=d,s,b} B(t \to qW)$$
$$= \Gamma(t \to bW)/\sum_{q=d,s,b}\Gamma(t \to qW) = 0.94, \tag{30}$$

which tell us that the contributions of the d and s quarks are almost negligible, we obtain

$$\Gamma(t \to bW) = 0.94 \sum_{q=d,s,b}\Gamma(t \to qW) \tag{31}$$

4. Conclusions

We have found the transition amplitude for the decay $t \to bWZ$ in the $LRMM$, according to Eqs. (20) and (21). By means of these relations and confronting with the expression given by the Standard Model [15], one can compare with experiment to predict parameters of the model such as A_{tt}. However, to our knowledge, there is no measured value reported for the width of the decay $t \to bWZ$. This is because the sensibility of the current experiments are below (10^{-4}) of the required theoretical width [9]. As it is known, the decay $t \to bW\gamma$ has a larger BR as compared with the decay we are dealing with, ($\approx 10^{-3}$, [16]), and even increases one order of magnitude according to a recent report [17]. On the other hand, the above formulae can be corrected and simplified if one works in the t quark center of mass system and the mass of the b quark is neglected. These approximations will not be so large due to the expected smallness of the involved parameters; that is why we have used the reduced Eqs. (25-26). Other investigation has been made recently in this model including Higgses [18].

Concerning one loop radiative corrections to our process, we must include in Fig. 1 diagrams containing self-energies in the fermion lines mediated by γ photon, Z boson, and the neutral Higgs particles h and H. There will also appear diagrams with vertex corrections mediated by the same particles, as well as those diagrams like vacuum polarization in the Z and W boson lines with the ordinary fermions in the internal lines. We asumme that all of these (and higher order) corrections do not change the order of magnitude of Eq. (26). However, at present time there is no available experimental bound for the decay $t \to bWZ$ which would allow one to check our prediction at tree level.

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