# An all-optical 4-bit register based on a four-order scattering of light by coherent acoustic phonons in single crystals

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A specific case of a four-order non-collinear light scattering in anisotropic media is presented. Compared to our previous studies, an innovation lies in the fact that now we consider passing just the quartet of incident light waves through a single crystal that is perturbed by a coherent stream of acoustic phonons. The exact and closed analytical model for describing this strongly nonlinear phenomenon is developed. In fact, a specially designed regime of a four-order light scattering, when transitions of four input light beams into four output light modes are allowed and electronically controlled, is examined. The feasibility of applying such an effect to perform an all-optical switching is analyzed. An opportunity for arranging the digital 4-bit register is revealed and algorithmically analyzed.

Keywords: All-optical switching; logic-based data processing; acousto-optical device.

Se presenta el caso específico de la dispersión de la luz no-colineal de cuarto orden en un medio anisotrópico. Comparado con nuestros previos estudios, la innovación yace en la consideración de hacer pasar cuatro haces incidentes de ondas de luz a través de un cristal el cuál es perturbado por una columna de fonones acústicos coherentes. La solución analítica y exacta al modelo que describen este fenómeno fuertemente no lineal es desarrollada. De hecho, se examina el régimen especialmente diseñado de la dispersión de la luz de cuarto orden, cuando las transiciones de cuatro ondas de luz de entrada hacia cuatro haces de luz de salida son permitidas y electrónicamente controladas. Se describe también la posibilidad de aplicar tal efecto para desarrollar el proceso de un interruptor todo-óptico. Se presenta la posibilidad de implementar un registro digital de 4 bits la cuál es analizada algorítmicamente.

Descriptores: Interruptores todo-ópticos, lógica basada en el procesamiento de datos; dispositivos acusto-ópticos.

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## 1. Introduction

We present the current progress in both the theoretical investigations and the computer simulations in the field of creating novel opto-electronic components for an all-optical logic-based data processing, founded on special regimes of acousto-optical interaction in bulk crystals [1,2]. Previously, the results obtained due to studying all-optical logic gates and switching via a two-phonon light scattering in single crystals, had been reported [3-7]. The case at hand is the opto-electronic component fulfilling an all-optical switching through the mechanism of a four-order light scattering in optically anisotropic media. A special approach to the Bragg regime of scattering the light in four modes by coherent acoustic phonons in single crystals is under examination. Because of the limitations by the phase-synchronism conditions in uniaxial crystals, a four-order scattering manifests itself due to transitions between the only neighboring orders. Moreover, the peculiarity of a four-order scattering consists in the fact that both normal and anomalous regimes of acousto-optical interaction are involved in this process, thus, relative efficiencies play a remarkable role. The proposed regime is potentially suitable for realizing 100% efficiency during the light scattering into each mode, so it gives us an opportunity to fulfill switching all-optically with improved efficiency. As this takes place, a certain amount of the multi-beam regimes can be realized depending on the relations between the intensities and the phases of incident light waves. In doing so, particular attention is placed on searching the points of extrema for the intensities of scattered light beams that hold the greatest interest from the viewpoint of controlling light by using light. A lot of combinations can be created due to varying the intensity and the carrier frequency of coherent acoustic phonons, which take part in the process of light scattering on equal terms and shape some dynamic acoustic grating. One possibility to apply this effect is connected with the design of all-optical logic-based switches. The properties of a four-order scattering give us an opportunity to investigate both binary and even non-binary logic operations. This opportunity has its origin in exploiting an all-optical nonlinearity inherent in this phenomenon. Such a nonlinearity can be varied electronically within wide limits according to the power density of the incoming phonon stream. The analysis permits realizing a few logic-based switching operations with or without the optical pump, so both possibilities: breaking and selecting switches can be performed. Here, a four-order light scattering in an uniaxial single crystal is applied to design the all-optical digital 4-bit register, being interferometric in behavior. In Sec. 2, we present the quantum approach to a four-order scattering of light by acoustic phonons in optically uniaxial anisotropic media, and introduce the diagrams of wave vectors that give

clear view on the processes of non-collinear acousto-optical interaction in crystals. Then, the exact and closed form of amplitude equations for describing a four-order light scattering is derived, analytically solved, and analyzed from the viewpoint of possible applications in Sec. 3. Because the final version of this analytical solution is too bulky, the corresponding graphic representation for the results of numerical simulation is given as well. Finally, in Sec. 4, we consider one of the possible applications of the above-described phenomenon to the design of an all-optical digital 4-bit register.

## 2. Quantum approach to a four-order scattering of light by acoustic phonons in optically uniaxial crystals

There is a good reason to take advantage of the quantum approach to the phenomenon of scattering of light by elastic waves in crystals, which can be interpreted as scattering the light quanta - photons by the quanta of the acoustic field phonons. When the phonons' length of propagation is large enough, it is reasonable to believe that phonons are passing through an infinite medium and, consequently, they have well-determined magnitudes of the momentum. Under such condition, each partial act of acousto-optical interaction represents a coherent three-particle process, so one may use the conservation laws for both the momentum  $\vec{p} = \hbar k$ , and the energy  $E = \hbar \omega$  and these laws determine, in fact, the wave vectors k and the angular frequencies  $\omega$  of interacting particles. Because similar relations are true for photons as well as for phonons, henceforward we will use small letters for denoting the photons' parameters and capital letters for the phonons' parameters. Thus, the conservation laws for partial three-particle process can be written as

$$\omega_1 = \omega_0 \pm \Omega, \ \vec{k_1} = \vec{k_0} \pm \vec{K}, \tag{1}$$

where  $\omega_0$ ,  $\omega_1$  and  $\vec{k_0}$ ,  $\vec{k_1}$  are the angular frequencies and wave vectors for the incident and scattered photons, respectively; while  $\Omega$  and  $\vec{K}$  are the angular frequency and wave vector of the injected phonons. In Eq.(1), the plus sign corresponds to creating an anti-Stokes photon, whereas the minus sign meets a Stokes photon. By this it is meant that there are two processes, manifesting the annihilation of a phonon (anti-Stokes process) or creation of a Stokes phonon. It is well known from the quantum mechanics that the probabilities of annihilating and creation the phonons are proportional to  $N^{(1/2)}$  and  $(N+1)^{(1/2)}$ , respectively. Due to the contribution of spontaneous process in the last case (here N is the number of acoustic phonons per unit volume in oscillation mode), the number N of heat phonons with the temperature T in oscillation mode is determined by the statistical mechanics as

$$N = \left[ \exp\left[\frac{\hbar\Omega}{\kappa T} - 1\right] \right]^{-1}, \tag{2}$$

where  $\kappa$  is the Boltzmann constant. Substituting the parameters for ultra-high frequency acoustic phonons in Eq.(2), we arrive at the inequality

$$N \approx \frac{\kappa T}{\hbar \Omega} \gg 1.$$
 (3)

This result is true as well for coherent acoustic phonons, injected into a crystal, because an effective temperature inherent in the oscillation mode under excitation of coherent acoustic phonons is much higher than the temperature of the crystal lattice [8]. Thus, at room temperature the contribution of spontaneous processes may be neglected and, consequently, the probabilities of annihilating and creating the acoustic phonons or, what is the same, the probabilities of creating Stokes and anti-Stokes photons are almost equal to each other. An upper angular frequency of acoustic phonons in the first Brillouin zone of solid states may be estimated in approximation of line lattice as  $\Omega_{\rm max} \approx 2V/\zeta \approx 10^{13} rad/s$  (here V is the velocity of passing acoustic phonons in a low-frequency limit, and  $\zeta$  is the lattice constant), while in usual practice  $\Omega \leq 10^{10} rad/s$ . Comparing these estimations with the lowest photon's angular frequencies in the visible range, for instance, at the wavelength of  $\lambda = 633nm$ , we obtain  $\omega \approx 3.10^{15} rad/s$ and, consequently,  $\omega \gg \Omega_{\max} \gg \Omega$ . By this it is meant that leaving aside the cases when coherence of light is of the first importance, one may assume that the angular frequency of photons does not change with scattering and use, for instance, an approximate relation  $\omega_1 \approx \omega_0$  when finding the coefficients for amplitude equations. Moreover, under conventional experimental conditions, when the intensities of the light and the acoustic beams are approximately equal to each other, the number of phonons is  $10^5$  times more than the number of photons, and up to 100% of photons can be scattered due to three-particle processes without appreciable effect on a stream of acoustic phonons. Consequently, the process of light scattering by coherent acoustic phonons can be considered in the approximation of a prescribed phonon field.

In the long run, the linkage between wave vectors of interacting particles can be expressed in the form of wave vector diagrams on cross-sections of the wave vector surfaces inherent in a crystal. Similar diagrams represent a graphic version of the conservation laws, see Eq.(1), and they may be exploited for the analysis of scattering. For example, Fig. 1a) illustrates an opportunity for one-fold scattering of the incident photon by one acoustic phonon in a single-axis crystal, when the initial and final states of polarization for these photons are different. Then, under certain conditions, i.e. at set angles of light incidence on the phonon beam and at fixed angular frequencies of phonons, one can observe the phenomenon of three-fold scattering of light caused by three participating acoustic phonons. The main peculiarity of this phenomenon lies in conserving both the energy and the momentum for the three transitions simultaneously. In turn, these laws determine the angular frequencies and wave vectors of



FIGURE 1. The diagrams of wave vectors for the light scattering in optically uniaxial crystal;  $\vec{k}_p$  is the wave vector of the light wave scattered in the p-th order: (a) the one fold scattering (p = 0, 1); (b) the three-fold scattering (p = 0, 1, 2, 3).

all four interacting waves

$$\omega_{1} = \omega_{0} + \Omega, \quad \omega_{2} = \omega_{0} + 2\Omega, \quad \omega_{3} = \omega_{0} + 3\Omega,$$
  
$$\vec{k_{1}} = \vec{k_{0}} + \vec{K}, \quad \vec{k_{2}} = \vec{k_{0}} + 2\vec{K}, \quad \vec{k_{3}} = \vec{k_{0}} + 3\vec{K}, \quad (4)$$

where  $\omega_p$  and  $k_p$  (p = 0, 1, 2, 3) are the angular frequencies and wave vectors of the interacting photons. This fact leads the creating of three orders scattering, besides the zero-th one, each by satisfying itself the conservation laws. Again, Fig. 1 presents the diagrams of wave vectors, dealing with a one- and four-order light scattering quanta in an uniaxial crystal.

Such a diagram offers rather small angles of deflection and occurs at the specific angular frequency of acoustic phonons and the angle  $\theta_i$  of incidence

$$\Omega = 2\pi\lambda^{-1}V\sqrt{2|n_1^2 - n_2^2|},$$
  

$$\sin(\theta_i) = 3\sqrt{(8n_1^2)^{-1}|n_1^2 - n_2^2|}.$$
(5)

It follows from this diagram that the polarization of light in the first and the second maxima of scattering is orthogonal to the polarization of light in the zero-th and the third maxima. Furthermore, the carrier frequencies of light in this first, second, and third maxima are shifted by  $\Omega$ ,  $2\Omega$ ,  $3\Omega$ , respectively, with respect to the zero-th maximum.

# **3.** Amplitude equations for a four-order light scattering

Let us consider an approach to the particular case of a fourorder light scattering by coherent acoustic phonons in an optically anisotropic uniaxial medium. To deduce the amplitude equations, describing a four-order light scattering, we use a classical approach to the phenomenon under consideration, based on the dispersion relation for the light waves in a crystalline medium, perturbed by elastic waves, with following parabolic approximation. Let the perfectly polarized plane light wave propagates through a crystal, wherein a plane elastic wave with the angular frequency  $\Omega$  as well as with the wave number K, are passing along z-axis. Initially, the dispersion relation can be written as  $\vec{k} \cdot \vec{k} = k^2 = \omega^2 n^2 / c^2$ , where c is the light velocity,  $\vec{k} = (k_x, k_y)$ , and  $\omega$  are the wave vector and angular frequency of the light wave passing through a medium, whose perturbed refractive index n is

$$n = n_1 + \Delta n_2 \sin\left(Kz - \Omega t\right). \tag{6}$$

Here,  $n_1$  and  $n_2$  are perturbed and non-perturbed refractive indices, respectively;  $\Delta n_2$  is the amplitude of perturbation. We assume  $k_z \ll k_x$ , so  $k \approx k_x + k_z^2/2K$ , with  $k_x \approx k$ in the denominator of the second summand. Thus, the dispersion relation can be rewritten in parabolic approximation as

$$k_x + \frac{k_z^2}{2k} = \frac{\omega}{c} \left[ n_1 + \Delta n_2 \sin(Kz - \Omega t) \right].$$
 (7)

Equation (7) represents the linear dispersion relation, because it does not contain any amplitude parameters of the optical field. Consequently, the normalized optical field strength E(x, z, t) can be expressed via the Fourier integral. In this case the well-known correspondences  $ik_x \rightarrow \partial/\partial x$ ,  $ik_z \rightarrow \partial/\partial z$ , and  $-i\omega \rightarrow \partial/\partial t$ , which are applicable to linear system, can be exploited, so we arrive at the following partial differential equation for the optical field strength

$$\frac{\partial E}{\partial x} - \frac{i}{2k} \frac{\partial^2 E}{\partial z^2} + \frac{n_1}{c} \frac{\partial E}{\partial t} + \frac{\Delta n_2}{c} \frac{\partial}{\partial t} [E \sin (Kz - \Omega t)] = 0.$$
(8)

The proposed solution to Eq.(8) is taken in the form of Fourier series with partial amplitudes  $C_p(x)$  as

$$E(x, z, t) = \sum_{p} C_{p}(x) \exp\left[i \left(k_{p,x} x + k_{p,z} z - \omega_{p} t\right)\right].$$
 (9)

We substitute Eq.(9) into Eq.(8). In the resulting equation, one can separate the approximate dispersion relation in terms of partial light waves:

$$\sum_{p} \left( k_{p,x} + \frac{k_{p,z}^2}{2k} - \frac{\omega_p}{c} \right) = 0$$

While the remained terms give a set of the amplitude equations

$$\frac{dC_p}{dx} = q_p \left\{ C_{p-1} \exp\left(i\eta_{p-1}x\right) - C_{p+1} \exp\left(i\eta_p x\right) \right\}.$$
 (10)

Here  $q_p = \Delta n_2 k_p (2n_1)^{-1}$ , and  $\eta_p = k_{p,x} - k_{p+1,x}$ . It follows from Eq.(10) that the redistribution of energy in each pth order of scattering is governed by the only neighboring order with number  $p\pm 1$ ,  $p\pm 2$ ,  $p\pm 3$ . In the case of a four-order light scattering presented in Fig.1b), Eq.(10) can be considerably simplified. First, one can disregard all the amplitudes  $C_p(x)$  in Eq.(10) with the exception of the amplitudes  $C_0$ ,  $C_1$ ,  $C_2$ , and  $C_3$ . And putting that the Bragg conditions are fulfilled perfectly,  $\theta \ll 1$ , we obtain a set of equations for the three-fold Bragg scattering in the anisotropic medium

$$\frac{dC_0}{dx} = -q_a C_1 \exp\left[-i\eta_0 x\right], 
\frac{dC_1}{dx} = q_a C_0 \exp\left[i\eta_0 x\right] - q_n C_2 \exp\left[-i\eta_1 x\right], 
\frac{dC_2}{dx} = q_n C_1 \exp\left[i\eta_1 x\right] - q_a C_3 \exp\left[-i\eta_2 x\right], 
\frac{dC_3}{dx} = q_a C_2 \exp\left[-i\eta_2 x\right].$$
(11)

Second, when a four-order light scattering is realized, it is seen from the wave vector diagram in Fig.1b) that  $\eta_p = 0$ . Consequently, in the case of stationary scattering we obtain the following set of simplified ordinary differential equations for the amplitudes of the light modes:

$$\frac{dC_0}{dx} = -q_a C_1, \qquad \qquad \frac{dC_1}{dx} = q_a C_0 - q_n C_2, \\ \frac{dC_2}{dx} = q_n C_1 - q_a C_3, \qquad \qquad \frac{dC_3}{dx} = q_a C_2.$$
(12)

In the deduction of Eq.(12), the shifts in carrier frequencies of light waves in different maxima were not taken into account. Then, the evident notations were introduced for the parameters  $q_p$ , namely  $q_a$ , and  $q_n$ , indicating the scattering with and without the change of polarization in the light beams. In the general case  $q_a \neq q_n$ , even if  $\theta_i \ll 1$ , because these two types of the scattering processes are determined by different photo-elastic constants.

The exact and closed analytical solutions to Eq.(12), with the stationary boundary conditions

$$\begin{split} C_{0} & (x=0) = A_{0} \exp \left(i\varphi_{0}\right), \\ C_{1} & (x=0) = A_{1} \exp \left(i\varphi_{1}\right), \\ C_{2} & (x=0) = A_{2} \exp \left(i\varphi_{2}\right), \\ C_{3} & (x=0) = A_{3} \exp \left(i\varphi_{3}\right), \end{split} \tag{13}$$

where  $A_0$ ,  $A_1$ ,  $A_2$ ,  $A_3$ , and  $\varphi_0$ ,  $\varphi_1$ ,  $\varphi_2$ ,  $\varphi_3$ , are the amplitudes and phases of the incident light waves on the plane x = 0. Additionally, we use  $\beta = \sqrt{1 + 4q^2}$ ,  $\alpha = (\sqrt{2})^{-1}$ , and two simplified expressions for the roots as

$$P = \sqrt{1 + 2q^2 + \sqrt{1 + 4q^2}},$$
  

$$S = \sqrt{1 + 2q^2 - \sqrt{1 + 4q^2}},$$
(14)

then we obtain

$$\begin{split} C_{0}\left(q_{n}x\right) &= \frac{A_{o}\exp\left(i\varphi_{o}\right)}{2\beta} \left[\left(-2q^{2}+P^{2}\right)\cos\left(\alpha Sq_{n}x\right) + \left(2q^{2}-S^{2}\right)\cos\left(\alpha Pq_{n}x\right)\right] \\ &\quad - \frac{\alpha A_{1}\exp\left(i\varphi_{1}\right)}{2q\beta} \left[S\left(-2q^{2}+P^{2}\right)\sin\left(\alpha Sq_{n}x\right) + P\left(2q^{2}-S^{2}\right)\sin\left(\alpha Pq_{n}x\right)\right] \\ &\quad + \frac{A_{2}\exp\left(i\varphi_{2}\right)}{\beta} \left[\cos\left(\alpha Sq_{n}x\right) - \cos\left(\alpha Pq_{n}x\right)\right] - \frac{\alpha A_{3}\exp\left(i\varphi_{3}\right)}{2q^{2}\beta} \left[SP^{2}\sin\left(\alpha Sq_{n}x\right) - PS^{2}\sin\left(\alpha Pq_{n}x\right)\right], \quad (15) \\ C_{1}\left(q_{n}x\right) &= \frac{q\alpha A_{o}\exp\left(i\varphi_{0}\right)}{SP\beta} \left[P\left(2q^{2}-S^{2}\right)\sin\left(\alpha Sq_{n}x\right) + S\left(-2q^{2}+P^{2}\right)\sin\left(\alpha Pq_{n}x\right)\right] \\ &\quad + \frac{A_{1}\exp\left(i\varphi_{1}\right)}{2\beta} \left[\left(2q^{2}-S^{2}\right)\cos\left(\alpha Sq_{n}x\right) + \left(-2q^{2}+P^{2}\right)\cos\left(\alpha Pq_{n}x\right)\right] \\ &\quad - \frac{\alpha A_{2}\exp\left(i\varphi_{2}\right)}{SP\beta} \left[SP^{2}\sin\left(\alpha Pq_{n}x\right) - PS^{2}\sin\left(\alpha Sq_{n}x\right)\right] + \frac{A_{3}\exp\left(i\varphi_{3}\right)}{\beta} \left[\cos\left(\alpha Sq_{n}x\right) - \cos\left(\alpha Pq_{n}x\right)\right] \\ &\quad - \frac{\alpha A_{2}\exp\left(i\varphi_{0}\right)}{\beta} \left[\cos\left(\alpha Sq_{n}x\right) - \cos\left(\alpha Pq_{n}x\right)\right] - \frac{\alpha A_{1}\exp\left(i\varphi_{1}\right)}{\beta} \left[S\sin\left(\alpha Sq_{n}x\right) - P\sin\left(\alpha Pq_{n}x\right)\right] \\ &\quad + \frac{A_{2}\exp\left(i\varphi_{3}\right)}{2\beta} \left[\left(2q^{2}-S^{2}\right)\cos\left(\alpha Sq_{n}x\right) + \left(-2q^{2}+P^{2}\right)\cos\left(\alpha Pq_{n}x\right)\right] \\ &\quad - \frac{\alpha A_{3}\exp\left(i\varphi_{3}\right)}{2\beta} \left[S\left(-2q^{2}+P^{2}\right)\sin\left(\alpha Sq_{n}x\right) + P\left(2q^{2}-S^{2}\right)\sin\left(\alpha Pq_{n}x\right)\right] \\ &\quad - \frac{\alpha A_{3}\exp\left(i\varphi_{3}\right)}{SP\beta} \left[P\left(2q^{2}-S^{2}\right)\sin\left(\alpha Sq_{n}x\right) + P\left(2q^{2}-S^{2}\right)\sin\left(\alpha Pq_{n}x\right)\right] \\ &\quad + \frac{q\alpha A_{2}\exp\left(i\varphi_{3}\right)}{SP\beta} \left[P\left(2q^{2}-S^{2}\right)\sin\left(\alpha Sq_{n}x\right) + S\left(-2q^{2}+P^{2}\right)\sin\left(\alpha Pq_{n}x\right)\right] \\ &\quad + \frac{q\alpha A_{2}\exp\left(i\varphi_{3}\right)}{SP\beta} \left[P\left(2q^{2}-S^{2}\right)\sin\left(\alpha Sq_{n}x\right) + S\left(-2q^{2}+P^{2}\right)\sin\left(\alpha Pq_{n}x\right)\right] \\ &\quad + \frac{q\alpha A_{2}\exp\left(i\varphi_{3}\right)}{SP\beta} \left[P\left(2q^{2}-S^{2}\right)\sin\left(\alpha Sq_{n}x\right) + S\left(-2q^{2}+P^{2}\right)\sin\left(\alpha Pq_{n}x\right)\right] \\ &\quad + \frac{q\alpha A_{2}\exp\left(i\varphi_{3}\right)}{SP\beta} \left[P\left(2q^{2}-S^{2}\right)\sin\left(\alpha Sq_{n}x\right) + S\left(-2q^{2}+P^{2}\right)\sin\left(\alpha Pq_{n}x\right)\right] \\ &\quad + \frac{q\alpha A_{2}\exp\left(i\varphi_{3}\right)}{SP\beta} \left[P\left(2q^{2}-S^{2}\right)\sin\left(\alpha Sq_{n}x\right) + S\left(-2q^{2}+P^{2}\right)\sin\left(\alpha Pq_{n}x\right)\right] \\ \\ &\quad + \frac{q\alpha A_{2}\exp\left(i\varphi_{3}\right)}{2\beta} \left[\left(-2q^{2}+P^{2}\right)\cos\left(\alpha Sq_{n}x\right) + S\left(-2q^{2}+P^{2}\right)\sin\left(\alpha Pq_{n}x\right)\right] \\ \\ &\quad + \frac{q\alpha A_{2}\exp\left(i\varphi_{3}\right)}{2\beta} \left[\left(-2q^{2}+P^{2}\right)\cos\left(\alpha Sq_{n}x\right) + S\left(-2q^{2}+P^{2}\right)\sin\left(\alpha Pq_{n}x\right)\right] \\ \\ &\quad + \frac{q\alpha A_{2}\exp\left(i\varphi_{3}\right)}{SP} \left[\left(-2q^{2}+P^{2}\right)\cos\left(\alpha Sq_{n}x\right) + S\left(-2q^{2}+P^{2}\right)\sin\left(\alpha Pq_{n}x\right)\right] \\ \\ &\quad + \frac{q\alpha A$$

Eqs.(15) - (18) make it possible to analyze a four-order light scattering. The transition probabilities are electronically controllable, and they may be varied within wide limits according to the level of incoming power density in elastic waves. For further analysis Eqs.(15) - (18) can be performed in terms of their intensities, *i.e.* we shall consider the intensities  $|C_p(q_n x)|^2$  as functions of the coordinate  $q_n x$ , and exploit the value q as a parameter with the incoming light intensities  $A_p^2$ , and the initial phases  $\varphi_p$  chosen in a specific way. For simplicity sake, the cases are chosen when the only one incoming light intensity has a non-zero with magnitude. If  $A_i^2 = 1$ ,  $q = \{1/2, 1.1, 2, 3\}$ , and  $\varphi_p = 0$ , while  $A_j^2 = A_k^2 = A_l^2$  for  $k \neq i \neq j$ ,  $j \neq l \neq i$ ,  $j \neq k \neq l$ , i, j, k, l = 0, 1, 2, 3, we arrive at a set of diagrams shown in Figs.2-3.

## 4. Application of a four-order light scattering

An opportunity to apply a four-order light scattering can be connected with the design of an all-optical switch. The above Eqs.(15) - (18) show us that for this purpose a set of specific parameters should be taken into consideration. These parameters include four input amplitudes  $(A_0, A_1, A_2, A_3)$  with the corresponding phases  $(\varphi_0 = \varphi_1 = \varphi_2 = \varphi_3 = 0)$ . This means that together with the input amplitudes, we have the following independent degrees of freedom: three phase differences, the ratio  $q = q_n \swarrow q_a$ , and the normalized length  $q_n x$ of interaction. Initially, we can consider the acousto-optical cell as a black box with four inputs and four outputs.

Analysis of the plots presented in Fig.2 shows that one can realize the switching processes with  $1 \le q \le 3$ . In particular, one can choose q=2,  $\varphi_0 = \varphi_1 = \varphi_2 = \varphi_3 = 0$ , and assume that the input amplitudes are normalized to unity, while the normalized length x of interaction can be exploited as a variable parameter. It follows from Fig.2 and Fig.3 that there is a discrete sequence of values for the length  $q_n x$ , which are optimal from the viewpoint of switching. One of the most appropriate lengths of interaction is  $q_n x = 443/143$ . This value makes it possible to reduce Eqs.(15) - (18) to

$$|C_{0} (443/143, q = 2)|^{2} = |A_{3}|^{2},$$
  

$$|C_{1} (443/143, q = 2)|^{2} = |A_{2}|^{2},$$
  

$$|C_{2} (443/143, q = 2)|^{2} = |A_{1}|^{2},$$
  

$$|C_{3} (443/143, q = 2)|^{2} = |A_{0}|^{2}.$$
 (19)

Because there are four inputs inherent in the abovementioned black box, Eq.(19) allow us to consider various regimes of operation, *i.e.* four different possible combinations relative to the input amplitudes.

The first regime will be realized, when the only one input amplitude, for instance,  $A_i$  is equal to unity  $(A_i = 1)$ , while the three remaining input amplitudes are equal to zero  $(A_{j,k,l} = 0)$ . The second one is in operation, when  $(A_{i,j} = 1)$ , and  $(A_{k,l} = 0)$ . The third regime will be with and  $A_{i,j,k} = 1$ , and  $A_l = 0$ , finally, the fourth possibility exists with  $A_{i,j,k,l} = 1$ . Here,  $k \neq i \neq j, j \neq l \neq i, j \neq k \neq l$  and i = j = k = l = 0, 1, 2, 3.

Now we propose to apply the obtained relations to the field of digital circuits. Digital logic circuits are usually made up of combinational elements such as NAND and NOR logic gates and memory elements, which might be single bit memory elements such as discrete flip-flops [9,10], and here we meet the medium-scale integration (MSI) circuit that contains storage cells within it, which is, by definition, a sequential circuit. These MSI circuits are classified in one of three categories: registers, counters, or random-access memory. We suggest applying a four-order light scattering in single crystal to design a 4-bit digital register [11-13], see Fig. 4. Of course, to manifest optical bistability any digital device should include a feedback, but here for the sake of simplicity we will leave aside the problem of arranging an all-optical feedback.

A circuit with flip-flops is considered a sequential circuit, even in the absence of combinational gates. Circuits that include flip-flops are usually classified by the function they perform, rather than the name of the sequential circuit. Two examples of such sequential circuits are: registers and counters. Binary information is stored in digital systems, in devices such as flip-flops. Each cell can store 1 bit of data. The content or state of the cell can be changed from 1 to 0 or from 0 to 1 by the signals on its inputs, while the content of a cell is determined by sensing its outputs. A collection of such storage cells is called a register. The number of bits in the most often manipulated data unit in the system determines the word size of the system. Common word sizes are powers of two; such as 4, 8, 16 or 32 bits. An n-bit register is a group of n flip-flops and is capable of storing n-bits. In addition to flip-flops, a register may have combinational gates that control when and how new information is transferred into the register [9,10]. An all-optical version of the device under proposal is shown in Fig.5, then the data stream from the channel  $A_0$  (dotted line) is switched into the output  $C_3$ , the data stream from the channel  $A_1$  (dashed line) is switched into the output  $C_2$ , and so on. The device, presented in Fig. 5, was simulated using the Workbench 4.0, which is a simulation program for testing both analog and digital circuits. Table I indicates the results of such a simulation, *i.e.* all the possible 4-input combinations for this device. Judging from the presented truth table, the proposed device represents a 4-bit digital register, which is capable of loading 4-bit binary words. Moreover, this device realizes the functional scheme displayed in Fig. 4.

To illustrate the principles of operation for such an alloptical 4-bit register we can select a pair of combinations of the input signals presented in Table I. For example, if we take the rows 11 and 12 from Table I they give us the schemes of the algorithmic realizations shown in Fig. 6.



FIGURE 2. Plots of the intensities of light waves versus the normalized length  $q_n x$  of interaction with a four order light scattering: (a)  $A_0 = 1, A_1 = A_2 = A_3 = 0$ , (b)  $A_1 = 1, A_0 = A_2 = A_3 = 0$ ; the dotted line are for  $|C_0(x)|^2$  order, the dashed line is for  $|C_1(x)|^2$  order; the dot-dashed line is for  $|C_2(x)|^2$  order, the solid line is for  $|C_3(x)|^2$  order.



FIGURE 3. Plots of the intensities of light waves versus the normalized length  $q_n x$  of interaction with a four order light scattering: (a)  $A_2 = 1$ ,  $A_0 = A_1 = A_2 = 0$ , (b)  $A_3 = 1$ ,  $A_0 = A_1 = A_2 = 0$ ; the dotted line are for  $|C_0(x)|^2$  order, the dashed line is for  $|C_1(x)|^2$  order; the dot-dashed line is for  $|C_2(x)|^2$  order, the solid line is for  $|C_3(x)|^2$  order.

Inputs				Outputs			
$ A_{0} ^{2}$	$ A_{1} ^{2}$	$ A_2 ^2$	$ A_{3} ^{2}$	$\left C_{3}\left(x ight) ight ^{2}$	$\left C_{2}\left(x ight)\right ^{2}$	$\left C_{1}\left(x ight) ight ^{2}$	$\left  C_{0}\left( x ight)  ight  ^{2}$
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	0
0	0	1	1	0	0	1	1
0	1	0	0	0	1	0	0
0	1	0	1	0	1	0	1
0	1	1	0	0	1	1	0
0	1	1	1	0	1	1	1
1	0	0	0	1	0	0	0
1	0	0	1	1	0	0	1
1	0	1	0	1	0	1	0
1	0	1	1	1	0	1	1
1	1	0	0	1	1	0	0
1	1	0	1	1	1	0	1
1	1	1	0	1	1	1	0
1	1	1	1	1	1	1	1

TABLE I. An all-optical realization of the truth table for a 4-bit register via a four-order light scattering, see Fig. 5.



FIGURE 4. Functional scheme of a 4-bit digital register.



FIGURE 5. An all-optical version of a 4-bit digital register.



FIGURE 6. Optical schemes of algorithmic realizations for alloptical registering via a four-order light scattering in a single crystal. The dotted lines represent the data stream for the transmission  $A_0^2 \rightarrow |C_3|^2$ , the dot-dashed lines are for the transmission  $A_2^2 \rightarrow |C_1|^2$ , and the solid line is for the transmission  $A_3^2 \rightarrow |C_0|^2$ : a) the row 11, b) the row 12 in Table I.

### 5. Conclusions

The possibility of applying the scheme of a four-order light scattering to an all-optical switching has been examined. For this purpose, an exact analytical solution describing this phenomenon in uniaxial single crystals has been developed, analyzed, and numerically simulated. The results involve an alloptical 4-bit digital register, which has been algorithmically estimated. The above-mentioned 4-bit digital register makes

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it possible to provide an all-optical switching at the efficiency of close to 100% together with a high speed of operation.

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