Pinch technique prescription to compute the electroweak corrections to the muon anomalous magnetic moment

L.G. Cabral-Rosetti

Departamento de Física de Altas Energías, Instituto de Ciencias Nucleares, UNAM,
Apartado Postal 70-543, 04510 México, D.F., México,
e-mail: luis@nuclecu.unam.mx

G. López Castro

Departamento de Física, Centro de Investigación y de Estudios Avanzados del IPN,
Apartado Postal 14-740, 07000 México D.F., México,
e-mail: glopez@fis.cinvestav.mx

J. Pestieau

Institut de Physique Théorique, Université Catholique de Louvain,
B-1348 Louvain-La-Neuve, Belgium,
e-mail: pestieau@fyma.ucl.ac.be

We apply a simple prescription derived from the framework of the Pinch Technique formalism to check the calculation of the gauge-invariant one-loop bosonic electroweak corrections to the muon anomalous magnetic moment.

Keywords: Muon anomalous magnetic moment; pinch technique; gauge-invariance.

Aplicamos la simple prescripción derivada en el marco de la Pinch Technique para corroborar la invariancia de norma en los cálculos a un lazo de las correcciones electrodébiles al momento magnético anómalo del muón.

Descriptores: Momento magnético anómalo del muón; pinch technique; invariancia de norma.

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A definition of the neutrino charge radius that satisfies good physical requirements, *i.e.* it is a physical observable, has been provided recently [1] in the framework of the Pinch Technique (PT) formalism [2]. Usual gauge dependencies encountered in the calculation of neutrino electromagnetic form factors can be removed by adopting the PT philosophy of defining the form factors from an observable (gauge-invariant and gauge-independent) scattering amplitude instead of using the (non-observable) one-loop vertex functions alone [1, 3]. We can summarize the results of Ref. 1 by saying that the *effective* charge form factor defined from the 'pinched' one-loop corrected νe scattering amplitude is the same as the charge form factor obtained from the one-loop corrections to the $\nu\nu\gamma$ vertex provided the Feynman rules given below are used in the second case.

In the PT formalism, the construction of a gauge-independent and gauge-invariant one-loop vertex and, in particular, of an *effective* electromagnetic form factor for the neutrino amounts to compute [1] the one-loop vertex corrections using a simple prescription in the linear R_ξ^L gauge, where gauge-boson propagators

$$P_{\mu\nu}^v(q) = \frac{-i}{q^2 - M_V^2}\left[g_{\mu\nu} + (1-\xi)\frac{q_\mu q_\nu}{\xi q^2 - M_V^2}\right] \quad (1)$$

are taken in the 't Hooft-Feynman gauge $\xi = 1$, and the usual three-boson vertex

$$\Gamma_{\alpha\mu\nu}(q, k, -q-k) = (q-k)_\nu g_{\alpha\mu}$$
$$+ (2k+q)_\alpha g_{\mu\nu} - (2q+k)_\mu g_{\alpha\nu} \quad (2)$$

is replaced by the truncated vertex [4]:

$$\Gamma_{\alpha\mu\nu}^F = (2k+q)_\alpha\, g_{\mu\nu} + 2q_\nu g_{\alpha\mu} - 2q_\mu g_{\alpha\nu}, \quad (3)$$

which satisfies [1] a simple Ward identity:

$$q^\alpha\Gamma_{\alpha\mu\nu}^F = (k+q)^2\, g_{\mu\nu} - k^2\, g_{\mu\nu}.$$

In this paper we argue that this prescription can be used also to compute the electromagnetic form factors of other fermions and, in particular, their static electromagnetic properties [5]. Since this prescription has been derived using the PT rearrangement of one-loop corrections to the νe scattering amplitude [1] *a priori* it is not a trivial issue that it will give the correct results for the vertex corrections of other fermions. In this note we apply the PT prescription to give an alternative derivation of the well known one-loop W-boson contribution to the anomalous magnetic moment of the muon, $a_\mu \equiv (g-2)/2$.

The complete one-loop electroweak corrections to a_μ were computed long time ago in Refs. 6 (the very small Higgs boson contribution and subleading muon mass terms are ne-

glected):

$$a_\mu^{weak} = \frac{G_F m_\mu^2}{8\pi^2\sqrt{2}}\left\{\frac{10}{3} + \frac{1}{3}[(1 - 4\sin_{\theta_W}^2)^2 - 5]\right\}. \quad (4)$$

The first term in Eq. (4), which is the focus of our interest, accounts for the W-boson (plus unphysical scalars) contributions, and the second term for the Z^0-boson correction to the vertex. Each one of these contributions is independent of the ξ-gauge parameters (in the linear R_ξ^L gauges) [6]. It is worth mentioning that, in contradistinction with the Pinch Technique, the evaluation of the muon anomalous magnetic form factor (for a non-vanishing q^2 value) is gauge-dependent with the methods used in Refs. 6.

Instead of performing an explicit evaluation of the W-boson corrections to the vertex, we can take advantage of a result derived, in another context, by Brodsky and Sullivan, and independently by Burnett and Levine in the late sixties [7]. Using the W-boson propagator of Eq. (1) and the electromagnetic vertex of the W-boson as proposed by Lee and Yang [8] (all particles are incoming, namely $k_1+k_2+k_3=0$):

$$\begin{aligned}V_{\mu\alpha\beta} = ie\{&g_{\alpha\beta}(k_1 - k_2)_\mu\\ &- g_{\alpha\mu}(k_1 + \kappa_W k_1 + \xi k_2 + \kappa_W k_2)_\beta\\ &+ g_{\beta\mu}(k_2 + \kappa_W k_2 + \xi k_1 + \kappa_W k_1)_\alpha\},\end{aligned} \quad (5)$$

it can be shown that the prescription of the PT formalism for the W-boson propagator and electromagnetic vertex [see Eqs. (1) and (3)] is obtained by choosing [The usual electro-

(a)   (b)

(c)   (d)

FIGURE 1. W-boson (and would-be Goldstone) contributions to a_μ.

magnetic vertex for the W-boson in gauge theories is recovered for the special choice $\xi = 0$ and $\kappa_W = 1$ in Eq. (5)]:

$$\xi = 1 \text{ and } \kappa_W = 1. \quad (6)$$

The W-boson contribution (Fig. 1a) to a_μ^{weak} obtained in Refs. 7 using the Feynman rules of Eqs. (1) and (5) is:

$$a_\mu^{WW} = \frac{G_F m_\mu^2}{8\pi^2\sqrt{2}}\left\{2(1 - \kappa_W)\ln\xi + \frac{10}{3}\right\}. \quad (7)$$

As it can be easily checked by inserting the values given in Eq. (6), the PT prescription for this correction gives the correct result for the W-boson contributions to a_μ (first term in Eq. (4)). The contribution from the Z^0-boson corresponding to the PT prescription ($\xi = 1$) computed in [6] must be added to Eq. (7) in order to complete the evaluation of the electroweak contributions. Therefore, we recover, in the leading muon mass approximation, the usual result for the electroweak corrections to a_μ at the one-loop level. In addition, we can address the following interesting remark: our derivation of a_μ^{WW} shows that the old-fashioned quantization ξ-procedure of Lee and Yang [8] makes sense only in the limit defined by Eq. (6).

In summary, the application of the prescription given in Eqs. (1) (with $\xi = 1$) and (3), shows the robustness and simplicity of the PT formalism. In particular, the PT could be useful to verify the independence of the result with respect to the gauge-parameter in a given gauge structure, and to clarify the evaluation of the complete contributions to the two-loop electroweak corrections to a_μ, since it has been proved that gauge invariance is satisfied to all orders [9, 10] using this method. Note that the two-loop electroweak contributions to a_μ were computed in Ref. 11. These corrections were computed using the linear R_ξ gauge in the 't Hooft-Feynman gauge and also a nonlinear gauge structure, and neglecting the contributions that involve two or more scalar couplings [11] since they are supressed by additional powers of m_μ^2/m_W^2. The two-loop electroweeak corrections amount to a reduction of –22.6% with respect to the one-loop electroweak result and it is at the level of the sentitivies expected in current experiments. The PT formalism can therefore provide an additional check of these results in a consistent, gauge-invariant and gauge-parameter independent way.

Finally, we would like to emphasize that although our work only reproduces well known results for the muon anomalous magnetic moment, it is interesting because it confirms the validity of the simple prescription derived in the context of the Pinch technique formalism in the calculation of an independent observable.

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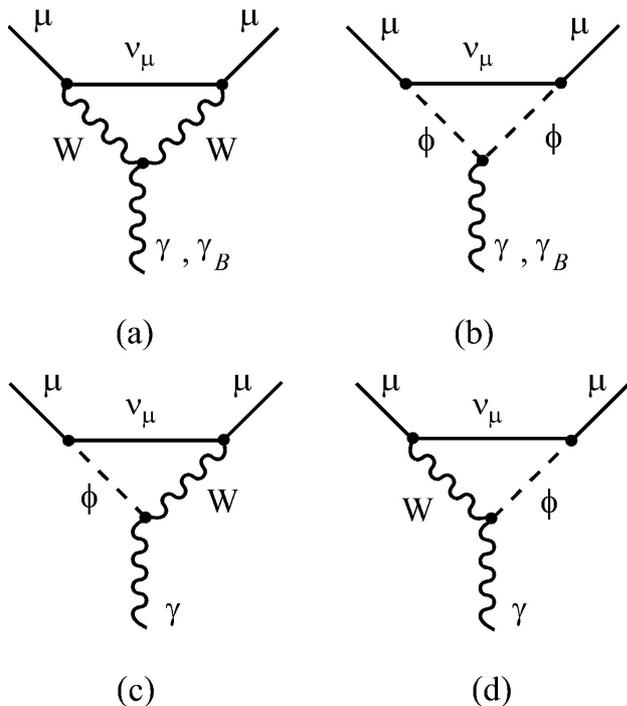
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