A nonlinear R_ξ–gauge for the electroweak theory

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A Gauge–fixing procedure for the electroweak theory, based in the BRST symmetry and covariance under the electromagnetic group, is proposed. It is found that in order to have a renormalizable theory, four–ghost interactions must be included in the BRST invariant action, since in this class of gauges these couplings are induced at the one–loop level. This type of gauges allows us to remove several unphysical vertices appearing in conventional linear gauges, which greatly simplifies the loop calculations, since the resultant theory satisfies QED–like Ward identities. Explicit expressions for the Lagrangian of the bosonic sector, including the corresponding ghost term, are presented.

Keywords: Electroweak theory; nonlinear gauges.

Se propone un procedimiento de fijación de la norma para la teoría electrodébil basado en la simetría BRST y covariancia bajo el grupo electromagnético. Se encuentra que, a fin de tener una teoría renormalizable, se deben incluir en la acción, invariante bajo BRST, interacciones de cuatro ghosts, ya que en esta clase de normas estos acoplamientos se inducen a orden de un rizo. Este tipo de normas nos permite remover varios vértices no físicos que aparecen en las normas lineales convencionales, lo cual simplifica grandemente los cálculos de loop, ya que la teoría resultante satisface identidades de Ward tipo QED. Se presentan expresiones explícitas para el lagrangiano del sector bosónico, incluyendo el término correspondiente para los *ghost*.

Descriptores: Teoría electrodébil; normas no lineales.

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1. Introduction

One very interesting feature of systems subject to first–class constraints [1] is the existence of many physically equivalent theories. These systems are known as gauge or degenerate systems. Yang–Mills theories, which play an important role in the quantum description of the strong and electroweak interactions, are a special case of this type of systems. In order to quantize this class of systems, it is necessary to define a unique theory through a gauge–fixing procedure. The possibility to quantize several physically equivalent Lagrangians could offer important advantages in practical loop calculations. To achieve that, the gauge–fixing functions must be defined in the appropriate way. The simplest functions which can be defined are the linear ones, which can depend linearly on gauge and scalar fields [2, 3]. In this paper we are interested in studying the most general structure of a gauge–fixing procedure for the electroweak theory (EWT), nonlinear in both the vector and scalar sectors. The motivation to introduce this type of gauges in theories with spontaneous symmetry breaking (SSB) arises from the possibility of removing an important number of unphysical interactions involving gauge bosons, pseudo–goldstone bosons (PGB), and physical Higgs bosons, always present in any linear gauge. As will be seen below, such possibility exists and it is not arbitrary at all.

The main idea concerning the definition of nonlinear R_ξ–gauges in theories with SSB can be explained as follows. In general, one has a gauge group G that has been broken down to a subgroup H. After SSB, all or part of the massive gauge bosons associated with the broken generators of G -and consequently, the corresponding PGB- may become part of some representation of H. Then, we can define the gauge–fixing functions for these massive gauge fields in terms not of the ordinary derivative, as it is done in the conventional linear R_ξ–gauges, but in terms of the covariant derivative associated with the H subgroup. In this way, the gauge–fixing functions for these fields will transform covariantely under the H group, which would induce an ordered readjustment (dictated by the H subgroup) between the Higgs kinetic energy term $(D_\mu\varphi_a)^\dagger(D^\mu\varphi_a)$ (which is responsible for the existence of unphysical vertices) and the gauge–fixing Lagrangian. Except by terms that fix the gauge for the fields of the H group, the gauge–fixing and the ghost sectors will become separately invariant under this subgroup, which in turn would lead to a more reduced number of unphysical vertices. In the case of the EWT, which is the subject of this work, the gauge groups of interest are $G = SU_L(2) \times U_Y(1)$ and $H = U_e(1)$, although we will also extend the method to remove some unphysical vertices involving the Z weak gauge boson.

A nonlinear renormalizable gauge (R_ξ–gauge) was introduced by Fujikawa [4] three decades ago to remove the unphysical $W^\pm G_W^\mp \gamma$ vertex ($G_W^\mp$ is the PGB associated with the $W^\pm$ weak gauge boson) of the EWT. This procedure was later extended to remove both $W^\pm G_W^\mp \gamma$ and $W^\pm G_W^\mp Z$ vertices [5]. A more detailed study has been presented in [6]. The procedure has been extended to includes certain models beyond the SM [7], and also in the context of effective gauge theories [8]. In all these works the gauge–fixing functions defined are nonlinear in the gauge fields, but linear in the scalar fields. The main goal of this work is to define a R_ξ gauge–fixing procedure nonlinear in both the gauge and the scalar sectors. In particular, we are interested in

the most general structure of nonlinear gauge–fixing functions that respect the electromagnetic gauge symmetry. These functions will be defined in terms of the electromagnetic covariant derivative modified to include the Z gauge boson. With a gauge so defined, it will be possible to remove all unphysical vertices $W^{\pm}G_W^{\mp}\gamma$, $W^{\pm}G_W^{\mp}Z$, $HW^{\pm}G_W^{\mp}\gamma(Z)$ and $G_ZW^{\pm}G_W^{\mp}\gamma(Z)$, with G_Z and H, the neutral PGB, and the physical Higgs boson, respectively. It will be shown that, as a consequence of covariance under the $U_e(1)$ group of the gauge–fixing functions, all charged particles satisfy QED–like Ward identities.

A gauge of this class was used in a loop calculation in Ref. 9. In this paper we take one step forward and present a comprehensive study which comprise explicit expressions for the Lagrangian of the Higgs and Yang–Mills sectors, including the ghost one. Another point which will be treated with some detail in this paper is the most general structure of the ghost sector that arises from the BRST symmetry [10]. As we will see the BRST symmetry play a fundamental role when one introduces a nonlinear gauge fixing procedure because the Faddeev–Popov method does not work in this case. It results that the Faddeev–Popov method [11] does not lead to the most general renormalizable action, since it can not yield four–ghost interactions, which are allowed by the BRST symmetry, and by the power counting criterion of renormalization theory. It results that in linear gauges of the form $\partial^\mu A^a_\mu$, the Faddeev–Popov method works well because four–ghost interactions can not arise from loop effects due to the existence of antighost translation invariance, that is, invariance under the transformation $\bar{C}^a \to \bar{C}^a + c^a$, with $\bar{C}^a$ stands for antighost fields, and c^a arbitrary constant parameters. In this case, the antighost fields appear only through their derivatives, so translation is a symmetry of the theory. However, in the case of nonlinear gauges, this invariance is lost due to the presence of a term of the form $A^a_\mu A^{b\mu}$ in the gauge–fixing functions. That term would be responsible for the presence at one–loop level of four–ghost interactions. This means that renormalizability become ruined if it is used the Faddeev–Popov method with this type of gauge–fixing procedure. It is, therefore, convenient to give up the Faddeev–Popov approach, and instead build up the most general action consistent with the BRST symmetry and renormalization theory. In the following, we will adopt this point of view.

The paper has been organized as follows. In Sec. 2 we present a brief review of the bosonic sector of the EWT. We will take advantages of this to present our notation and conventions. Sec. 3 is dedicated to discuss the structure of a nonlinear gauge for the EWT, as well as its dynamical implications on the gauge invariant Lagrangian of the theory. In Sec. 4 conclusions are presented.

2. Preliminaries

To begin with, we present a brief review of the bosonic sector of the EWT. This is necessary not only to define our notation and conventions, but also to analyze the impact on this sector of the nonlinear gauge that will be introduced in the next section. A R_ξ gauge–fixing procedure, linear or nonlinear, can only affect the structure of the Higgs and Yang–Mills sectors of the theory[The U–gauge is formally equivalent to the R_ξ gauge in the limit $\xi \to 0$. The equivalence here is formal, in the sense that Feynman amplitudes in the two formulations are equal in the limit $\xi \to 0$ is taken before the Feynman integral is performed. See in Ref. 3]. So, we only need to discuss these sectors. The Higgs sector of the EWT comprise the Higgs kinetic energy term and the potential, which can be written as

$$\mathcal{L}_H = (D_\mu\varphi)^\dagger(D^\mu\varphi) - V(\varphi^\dagger,\varphi), \tag{1}$$

where $\varphi^\dagger = (\varphi^-, \varphi^{0*})$ is the Higgs doublet with hypercharge $Y = 1$. D_μ is the covariant derivative in the fundamental representation of the $SU_L(2) \times U_Y(1)$ group, given by

$$D_\mu = \partial_\mu - \frac{ig}{2}\tau^i W^i_\mu - \frac{ig'}{2}YB_\mu, \tag{2}$$

where W^i_μ and B_μ are the gauge fields associated with the groups $SU_L(2)$ and $U_Y(1)$, respectively. τ^i are the Pauli matrices, being g and g' the corresponding coupling constants. The Higgs potential has the following renormalizable structure

$$V(\varphi^\dagger,\varphi) = \mu^2(\varphi^\dagger\varphi) + \lambda(\varphi^\dagger\varphi)^2. \tag{3}$$

After SSB, $\varphi^0 = (v + H + iG_Z)/\sqrt{2}$, with v the vacuum expectation value of the Higgs doublet. For subsequent discussions, it is convenient to write the Higgs kinetic energy term as follows:

$$(D_\mu\varphi)^\dagger(D^\mu\varphi) = \mathcal{L}_{HK1} + \mathcal{L}_{HK2}, \tag{4}$$

where

$$\begin{aligned}\mathcal{L}_{HK1} &= \frac{1}{4}[(\bar{D}_\mu + \hat{D}_\mu)G^+_W]^\dagger[(\bar{D}^\mu + \hat{D}^\mu)G^+_W] \\ &\quad + \frac{g^2}{2}W^-_\mu W^{+\mu}(\varphi^{0*}\varphi^0 + G^-_W G^+_W),\end{aligned} \tag{5}$$

$$\begin{aligned}\mathcal{L}_{HK2} &= \frac{1}{4}[(\bar{D}_\mu + \hat{D}^\dagger_\mu)\varphi^0]^\dagger[(\bar{D}^\mu + \hat{D}^{\mu\dagger})\varphi^0] \\ &\quad + \frac{ig}{2\sqrt{2}}[W^-_\mu\varphi^{0*}(\bar{D}^\mu + \hat{D}^\mu)G^+_W \\ &\quad - W^+_\mu\varphi^0(\bar{D}^{\mu\dagger} + \hat{D}^{\mu\dagger})G^-_W \\ &\quad + W^+_\mu G^-_W(\bar{D}^\mu + \hat{D}^{\mu\dagger})\varphi^0 \\ &\quad - W^-_\mu G^+_W(\bar{D}^{\mu\dagger} + \hat{D}^\mu)\varphi^{0*}].\end{aligned} \tag{6}$$

On the other hand, the Higgs potential can be conveniently written as

$$\begin{aligned}-V(\varphi^\dagger,\varphi) &= \lambda[v^2 - (\varphi^{0*}\varphi^0 + G^-_W G^+_W)] \\ &\quad \times (\varphi^{0*}\varphi^0 + G^-_W G^+_W).\end{aligned} \tag{7}$$

In the above expressions we have introduced the following definitions

$$\bar{D}_\mu = \partial_\mu - ig' B_\mu, \tag{8}$$

$$\hat{D}_\mu = \partial_\mu - igW^3_\mu. \tag{9}$$

Notice that these operators contain the electromagnetic covariant derivative $D^e_\mu = \partial_\mu - ieA_\mu$. The combinations of these operators that appear in the Higgs kinetic energy term are given by: $\bar{D}^\mu + \hat{D}^\mu = 2D^\mu_e - (igc_{2W}/c_W)Z^\mu$ and $\bar{D}^{\dagger\mu} + \hat{D}^\mu = 2\partial^\mu - (ig/c_W)Z^\mu$, where $c_W = \cos\theta_W$ and $c_{2W} = \cos 2\theta_W$. This means that any term containing the first of this combinations will transform covariantely under the $U_e(1)$ group. Notice that the $\mathcal{L}_{HK2}$ Lagrangian contains a bilinear term in the weak gauge bosons and the PGB. It is a well known fact that one of the main purposes of the linear gauge–fixing procedure [3] is to eliminate these terms via a surface term. The main goal of this work is to introduce a nonlinear gauge–fixing procedure that allows to eliminate not only these bilinear terms, but also the maximum number of unphysical vertices appearing in the $\mathcal{L}_{HK2}$ Lagrangian.

We now turn our attention to the Yang–Mills sector. In terms of the operator $\hat{D}_\mu$, the corresponding Lagrangian can be written as follows:

$$\begin{aligned}\mathcal{L}_{YM} &= -\frac{1}{4}W^i_{\mu\nu}W^{i\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\ &= -\frac{1}{2}(\hat{D}_\mu W^+_\nu - \hat{D}_\nu W^+_\mu)^\dagger(\hat{D}^\mu W^{+\nu} - \hat{D}^\nu W^{+\mu}) \\ &\quad - \frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - ig\Big[s_W F_{\mu\nu} + c_W Z_{\mu\nu} \\ &\quad + i\frac{g}{2}(W^-_\mu W^+_\nu - W^-_\nu W^+_\mu)\Big]W^{-\mu}W^{+\nu}. \end{aligned} \tag{10}$$

This Lagrangian is manifestly invariant under the electromagnetic $U_e(1)$ group. It should be remember that a linear gauge–fixing procedure spoil such symmetry in the charged sector, since it introduces the term $\partial_\mu W^{\pm\mu}$, which does not transform covariantely. The main goal of this work is to define the propagator of the $W^\pm_\mu$ field without spoiling the $U_e(1)$–gauge symmetry of the charged sector.

3. A $U_e(1)$–covariant nonlinear R_ξ–gauge

3.1. Structure of the BRST invariant action

The main goal in the classical study of a gauge system is to find its gauge algebra, which consists of certain relations that must be satisfied by the gauge–structure tensors of the theory. In simple gauge systems, as Yang–Mills theories, it is easy to define the gauge algebra, but it can be a quite complicated problem for most general gauge systems. In the last year, a powerful technique based on the antifields Batalin–Fradkin–Vilkovisky formalism [12] has been developed to deal with this problem and also with the issue of covariant quantization of gauge systems, which constitutes the ultimate goal. That method is known as the antifield–antibracket formalism, in which the extended BRST symmetry play a fundamental role [13]. The starting point of this method is the introduction of an antifield for each field in the theory. It is assumed that the dynamical degree of freedom of the gauge system are characterized by the matter, gauge, ghost (C^a), antighost ($\bar{C}^a$), and auxiliary (B^a) fields. The original action, which will denoted by S_0, is a functional of matter and gauge fields only, but this configuration is extended to include the ghost fields because they are necessary to quantize the theory. A ghost for each gauge parameter is introduced. The ghost fields have opposite statistic to that of the gauge parameters. To gauge fix the theory and also to quantize it, it is necessary to introduce the so–called trivial pairs, namely the antighost and auxiliary fields. We let Φ^A run over all these fields. For each Φ^A, an antifield Φ^*_A is introduced, with opposite statistics to Φ^A and a ghost number equal to $-gh(\Phi^A) - 1$, where $gh(\Phi^A)$ is the ghost number of Φ^A, which is 0 for matter, gauge, and auxiliary fields, $+1$ for ghosts and -1 for antighosts. In this extended configuration space a symplectic structure is introduced through left and right differentiation, defined for two functionals F and G as:

$$(F, G) = \frac{\partial_r F}{\partial\Phi^A}\frac{\partial_l G}{\partial\Phi^*_A} - \frac{\partial_r F}{\partial\Phi^*_A}\frac{\partial_l G}{\partial\Phi^A} \tag{11}$$

In particular, the fundamental antibrackets are given by

$$(\Phi^A, \Phi^*_B) = \delta^A_B, \quad (\Phi^A, \Phi^B) = 0 = (\Phi^*_A, \Phi^*_B). \tag{12}$$

The extended action is a bosonic functional on fields and antifields, $S[\Phi, \Phi^*]$, with ghost number zero, which satisfy the master equation defined by

$$(S, S) = 2\frac{\partial_r S}{\partial\Phi^A}\frac{\partial_l S}{\partial\Phi^*_A} = 0, \tag{13}$$

The antibracket serves to define the extended BRST transformations as follows:

$$\delta_B\Phi^A = (S, \Phi^A) = -\frac{\partial_r S}{\partial\Phi^*_A}, \tag{14}$$

$$\delta_B\Phi^*_A = (S, \Phi^*_A) = \frac{\partial_r S}{\partial\Phi^A}, \tag{15}$$

We can see that the extended action is invariant under BRST symmetry as a consequence of the master equation, since its variation is given by $\delta S = (S, S)$. Not all the solutions of the master equation are of interest, but only those called proper solutions [13]. A proper solution must make contact with the initial theory, which means to impose the following boundary condition on S:

$$S[\Phi, \Phi^*]|_{\Phi^*=0} = S_0[\phi], \tag{16}$$

where ϕ runs only over the original fields, *i.e.* matter and gauge fields. The proper solution S can be expanded in a power series in antifields:

$$S[\Phi, \Phi^*] = S_0[\phi] + (\delta_B\Phi^A)\Phi^*_A + \ldots, \tag{17}$$

where in the series appear all gauge–structure tensors characterizing the gauge system. In this sense the proper solution S is the generating functional of the gauge–structure tensors. S also generate the gauge algebra through the master equation. So, classically a gauge system is completely determined when it is established the proper solution S, and calculated the master equation, which gives the relations that must be satisfied by the gauge–structure tensors. In simplest gauge systems, as Yang–Mills theories, a solution of the master equation is given by

$$S[\Phi,\Phi^*] = S[\phi] + (\delta_B\Phi^A)\Phi^*_A. \tag{18}$$

This action is bosonic and has ghost number zero, as it is required. It is easy to show that this action is a solution of the master equation, and that it reproduces correctly the well–known gauge algebra of Yang–Mills theories.

We now turn our attention to the quantum analysis of the gauge system. To quantize the theory, one first needs to fix the gauge. Since the extended action is degenerate, it can not be quantized in a direct way. Besides, the antifields do not represent true degrees of freedom, so they must be removed before the quantization of the theory. They can not be simply equaled to zero, since S_0 is degenerate. However, one can remove the antifields through a nontrivial procedure, and at the same time lift the degeneration of the theory. Following Batalin–Vilkovisky [14], the antifields can be eliminated by introducing a fermionic functional of the fields only, $\Psi[\Phi]$, with ghost number -1, such that

$$\Phi^*_A = \frac{\partial\Psi[\Phi]}{\partial\Phi^A}. \tag{19}$$

Notice that in this case it is not necessary to distinguish between left– and right–differentiation. In defining a gauge–fixing procedure, the presence of the trivial pairs, $\bar{C}^a$ and B^a, is necessary since the only fields with ghost number -1 are precisely the antighost ones. Noting that $(\delta_B\Phi^A)\Phi^*_A = (\delta_B\Phi^i)(\partial\Psi[\Phi]/\partial\Phi^A) = \delta_{B_\Psi}\Psi[\Phi]$, the proper solution takes the form

$$S[\Phi,\delta\Psi/\delta\Phi] = S[\phi] + \delta_{B_\Psi}\Psi[\Phi]. \tag{20}$$

This is the gauge–fixed BRST action, which is invariant under the usual BRST transformation [10] defined by

$$\delta_{B_\Psi}A^a_\mu = \mathcal{D}^{ab}_\mu C^b, \tag{21}$$

$$\delta_{B_\Psi}\psi_m = -it^a_{mn}C^a\psi_n, \tag{22}$$

$$\delta_{B_\Psi}C^a = -\frac{1}{2}f^{abc}C^bC^c, \tag{23}$$

$$\delta_{B_\Psi}\bar{C}^a = B^a, \tag{24}$$

$$\delta_{B_\Psi}B^a = 0, \tag{25}$$

where ψ_m stands for matter fields, and $\mathcal{D}^{ab}_\mu = \delta^{ab}\partial_\mu - gf^{abc}A^c_\mu$ is the covariant derivative in the adjoint representation of the group, being f^{abc} the corresponding structure constants. In general, the nilpotency of δ_{B_Ψ} only is guaranteed on–shell, *i.e.* only after using the equations of motion, but in the case of Yang-Mills theories, $\delta^2_{B_\Psi} = 0$ even off–shell.

We now proceed to define the most general fermionic functional Ψ for the EWT, consistent with the renormalization theory. In this case, the gauge group is $SU_L(2)\times U_Y(1)$. The most general renormalizable functional Ψ with ghost number -1 can be written as

$$\Psi = \int d^4x[\bar{C}^i(f^i + \frac{\xi}{2}B^i + \epsilon^{ijk}\bar{C}^jC^k) + \bar{C}(f + \frac{\xi}{2}B)], \tag{26}$$

where f^i and f are the gauge–fixing functions associated with the groups $SU_L(2)$ and $U_Y(1)$, respectively. They are restricted by renormalizability to be, at most, quadratical functions of gauge and scalar fields. The bosonic constant ξ is the so–called gauge parameter, in general one for each group, but we have used the same for simplicity. Notice that the term $\epsilon^{ijk}\bar{C}^i\bar{C}^jC^k$ does not exist in the Faddeev–Popov method, though its presence is necessary to get renormalizability. Using the above BRST transformations, we obtain for the action $\delta_{B_\Psi}\Psi$:

$$\delta_{B_\Psi}\Psi = \int d^4x\Big\{\frac{\xi}{2}B^iB^i + (f^i + 2\epsilon^{ijk}\bar{C}^jC^k)B^i + \frac{\xi}{2}BB + fB - \bar{C}^i(sf^i) - \bar{C}(sf) - \bar{C}^i\bar{C}^jC^iC^j\Big\}. \tag{27}$$

Since the auxiliary fields B^i and B appear quadratically, they can be integrated out in the generating functional. Since the coefficients of the quadratical terms do not depend of the fields, their integration is equivalent to use the corresponding equations of motion in the gauge–fixed BRST action. After doing this, we obtain an effective action defined by the following effective Lagrangian

$$\mathcal{L}_{eff} = \mathcal{L}_{EWT} + \mathcal{L}_B + \mathcal{L}_F, \tag{28}$$

where $\mathcal{L}_{EWT}$ is the gauge invariant electroweak Lagrangian, and $\mathcal{L}_B$ is the well–known gauge–fixing Lagrangian given by

$$\mathcal{L}_B = -\frac{1}{2\xi}f^if^i - \frac{1}{2\xi}f^2. \tag{29}$$

On the other hand, $\mathcal{L}_F$ depends on the ghost and anitghost fields in the way

$$\mathcal{L}_F = -\bar{C}^i(\delta_{B_\Psi}f^i) - \bar{C}(\delta_{B_\Psi}f) - \frac{2}{\xi}\epsilon^{ijk}f^i\bar{C}*C^k + \left(\frac{2}{\xi}-1\right)\bar{C}^i\bar{C}^jC^iC^j. \tag{30}$$

It should be noticed that the last two terms in this expression are not present when it is used the Faddeev–Popov method. The third term arises as a consequence of integrating out the auxiliary B^i fields. We are ready now to discuss the structure of the gauge–fixing functions f^i and f.

3.2. The gauge–fixing functions

We now turn to discuss the most general structure of the gauge–fixing functions f^i and f. The general structure of the nondegenerate Lagrangian has been discussed in previous section. Then, we only need to present explicitly the gauge–fixing functions and use the general expressions given by Eqs.(29) and (30) to construct the $\mathcal{L}_B$ and $\mathcal{L}_F$ Lagrangians. Our main purpose is to define nonlinear gauge–fixing functions that allow to remove the maximum number of the unpysical vertices appearing in the Lagrangian $\mathcal{L}_{HK2}$. It is clear that the sole type of gauge which allows to eliminate all these vertices is the unitarity gauge, but it is a not manifestly renormalizable gauge. This gauge is defined for the massive gauge fields using only the PGB as supplementary conditions. In this procedure the gauge–fixing functions posses the Lorentz and $U_e(1)$ symmetries. This is a relevant property which we will adopt in order to define a R_ξ–gauge that allows us to remove the maximum number of unphysical vertices, *i.e.* we adopt as fundamental criterion Lorentz and electromagnetic covariance to construct the gauge–fixing functions that define the propagators of the charged massive gauge bosons, $W^\pm_\mu$. Besides, these gauge–fixing functions must satisfy the power counting criterion of renormalizability, which means that they would depend, at most, quadratically on the gauge and scalar fields. Taking into account these considerations, we introduce the following gauge–fixing conditions:

$$f^i = (\delta^{ij}\partial_\mu - g'\epsilon^{3ij}B_\mu)W^{j\mu} + \xi\frac{ig}{2}[\varphi^\dagger(\tau^i - i\epsilon^{3ij}\tau^j)\phi_0 - \phi_0^\dagger(\tau^i + i\epsilon^{3ij}\tau^j)\varphi + i\epsilon^{3ij}\varphi^\dagger\tau^j\varphi], \tag{31}$$

$$f = \partial_\mu B^\mu + \xi\frac{ig'}{2}(\varphi^\dagger\phi_0 - \phi_0^\dagger\varphi), \tag{32}$$

where $\phi_0^\dagger = (0, v/\sqrt{2})$. The conventional linear gauge is obtained from these expressions by putting equal to zero all terms proportional to the factor ϵ^{3ij}. Notice that f is linear in the fields, while f^i are nonlinear in both the vector and scalar fields. Due to this, not only the vertices of the Yang–Mills and Higgs kinetic energy term would be affected, but also the Higgs potential. To clarify this point, it is convenient to write the gauge–fixing functions in terms of mass eigenstate components. For this purpose we define the mass eigenstate fields as

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp iW^2_\mu), \tag{33}$$

$$Z_\mu = c_W W^3_\mu - s_W B_\mu, \tag{34}$$

$$A_\mu = s_W W^3_\mu + c_W B_\mu. \tag{35}$$

Then, we define

$$f^\pm = \frac{1}{\sqrt{2}}(f^1 \mp if^2), \tag{36}$$

$$f^Z = c_W f^3 - s_W f, \tag{37}$$

$$f^A = s_W f^3 + c_W f. \tag{38}$$

Using these expressions, we can write the gauge–fixing functions as follows:

$$f^+ = \bar{D}_\mu W^{+\mu} - \xi\frac{ig}{\sqrt{2}}\varphi^{0*}G^+_W, \tag{39}$$

$$f^Z = \partial_\mu Z^\mu - \xi M_Z G_Z, \tag{40}$$

$$f^A = \partial_\mu A^\mu, \tag{41}$$

where $f^- = (f^+)^\dagger$. We can see that in virtue that the operator $\bar{D}_\mu$ contains the electromagnetic covariant derivative, this gauge–fixing procedure is covariant under the $U_e(1)$ group. The Lagrangian $\mathcal{L}_B$ takes then the form

$$\mathcal{L}_B = \mathcal{L}_{GFV} + \mathcal{L}_{GFS} + \mathcal{L}_{GFSV}, \tag{42}$$

where

$$\mathcal{L}_{GFV} = -\frac{1}{\xi}(\bar{D}_\nu W^{+\nu})^\dagger(\bar{D}_\mu W^{+\mu}) - \frac{1}{2\xi}(\partial_\mu Z^\mu)^2 - \frac{1}{2\xi}(\partial_\mu A^\mu)^2, \tag{43}$$

$$\mathcal{L}_{GFS} = -\frac{\xi}{2}g^2\varphi^{0*}\varphi^0 G^-_W G^+_W - \frac{\xi}{2}m_Z^2 G_Z^2, \tag{44}$$

$$\mathcal{L}_{GFSV} = \frac{ig}{\sqrt{2}}[\varphi^{0*}G^+_W(\bar{D}_\mu W^{+\mu})^\dagger - \varphi^0 G^-_W(\bar{D}_\mu W^{+\mu})] + m_Z G_Z \partial_\mu Z^\mu. \tag{45}$$

Some remarks concerning the implications of this gauge–fixing Lagrangian on the Yang–Mills and Higgs sectors are in order. First of all, notice that the term $\mathcal{L}_{GFV}$ not only define the propagators of the gauge fields, but also introduce nontrivial modifications in the Lorentz structure of the trilinear and quartic vertices appearing in the Yang–Mills Lagrangian. Explicitly, the vector sector of the nondegenerate theory takes the final form

$$\mathcal{L}_{YM} + \mathcal{L}_{GFV} = -\frac{1}{2}(\hat{D}_\mu W^+_\nu - \hat{D}_\nu W^+_\mu)^\dagger \times(\hat{D}^\mu W^{+\nu} - \hat{D}^\nu W^{+\mu}) - \frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - ig\Big[s_W F_{\mu\nu} + c_W Z_{\mu\nu} + i\frac{g}{2}(W^-_\mu W^+_\nu - W^-_\nu W^+_\mu)\Big] \times W^{-\mu}W^{+\nu} - \frac{1}{\xi}(\bar{D}_\nu W^{+\nu})^\dagger(\bar{D}_\mu W^{+\mu}) - \frac{1}{2\xi}(\partial_\mu Z^\mu)^2 - \frac{1}{2\xi}(\partial_\mu A^\mu)^2. \tag{46}$$

Notice that the term that introduces modifications in these vertices is covariant under the electromagnetic group. Due to this, the trilinear electromagnetic vertices satisfy QED–like Ward identities. The Feynman rules for this sector are given

by

$$A_\eta(k_1)W^+_\lambda(k_2)W^-_\rho(k_3) \to -ie\Gamma^{WWA}_{\lambda\rho\eta}(k_1,k_2,k_3), \quad (47)$$

$$Z_\eta(k_1)W^+_\lambda(k_2)W^-_\rho(k_3) \to -ie\Gamma^{WWZ}_{\lambda\rho\eta}(k_1,k_2,k_3), \quad (48)$$

$$A_\alpha A_\beta W^+_\lambda W^-_\rho \to -ie^2\Gamma^{WWAA}_{\alpha\beta\lambda\rho}, \quad (49)$$

$$A_\alpha Z_\beta W^+_\lambda W^-_\rho \to -iegc_W\Gamma^{WWAZ}_{\alpha\beta\lambda\rho}, \quad (50)$$

$$Z_\alpha Z_\beta W^+_\lambda W^-_\rho \to -ig^2c^2_W\Gamma^{WWZZ}_{\alpha\beta\lambda\rho}, \quad (51)$$

$$W^+_\alpha W^-_\beta W^+_\lambda W^-_\rho \to +ig^2\Gamma^{WWWW}_{\alpha\beta\lambda\rho}, \quad (52)$$

where

$$\Gamma^{WWA}_{\lambda\rho\eta}(k_1,k_2,k_3)=(k_3-k_2)_\eta g_{\rho\lambda}+(k_1-k_3-\frac{1}{\xi}k_2)_\lambda g_{\rho\eta} + (k_2-k_1+\frac{1}{\xi}k_3)_\rho g_{\lambda\eta}, \quad (53)$$

$$\Gamma^{WWZ}_{\lambda\rho\eta}(k_1,k_2,k_3)=(k_3-k_2)_\eta g_{\rho\lambda}+(k_1-k_3+\frac{t^2_W}{\xi}k_2)_\lambda g_{\rho\eta} + (k_2-k_1-\frac{t^2_W}{\xi}k_3)_\rho g_{\lambda\eta}, \quad (54)$$

$$\Gamma^{WWAA}_{\alpha\beta\lambda\rho} = \Gamma^{WWZZ}_{\alpha\beta\lambda\rho} = 2g_{\alpha\beta}g_{\lambda\rho} - \left(1-\frac{1}{\xi}\right) \times (g_{\alpha\lambda}g_{\beta\rho}+g_{\alpha\rho}g_{\beta\lambda}), \quad (55)$$

$$\Gamma^{WWAZ}_{\alpha\beta\lambda\rho} = 2g_{\alpha\beta}g_{\lambda\rho} - \left(1+\frac{t^2_W}{\xi}\right) \times (g_{\alpha\lambda}g_{\beta\rho}+g_{\alpha\rho}g_{\beta\lambda}), \quad (56)$$

$$\Gamma^{WWWW}_{\alpha\beta\lambda\rho} = 2g_{\alpha\lambda}g_{\beta\rho} - g_{\alpha\rho}g_{\beta\lambda} - g_{\alpha\beta}g_{\lambda\rho}. \quad (57)$$

In the above expressions, all momenta are token as incoming. Using these Feynman rules, we can see that the vertex functions of three and two points satisfy the Ward identity

$$k^\eta_1\Gamma^{WWA}_{\lambda\rho\eta}(k_1,k_2,k_3) = \Gamma^{WW}_{\lambda\rho}(k_2) - \Gamma^{WW}_{\lambda\rho}(k_3). \quad (58)$$

Also notice that the only vertex which is not affected by this gauge–fixing procedure is the quartic one, $WWWW$. It is interesting to notice that the Eqs.(47-58) are quite similar to those presented in Ref. 15 in Eqs.(A.29-A.34,39) within the context of the Background Field Method technique [16].

As for the $\mathcal{L}_{GFS}$ term, it defines the unphysical masses of the PGB and introduce modifications in the couplings arising from the Higgs potential. These terms can be grouped as follows:

$$-V(\varphi^\dagger,\varphi)+\mathcal{L}_{GFS}=\lambda(v^2-\varphi^{0*}\varphi^0)\varphi^{0*}\varphi^0-\frac{1}{2}\xi m^2_Z G^2_Z - \lambda(G^-_W G^+_W)^2+[\lambda v^2-(2\lambda+\frac{\xi g^2}{2})\varphi^{0*}\varphi^0]G^-_W G^+_W. \quad (59)$$

From this expression, it is clear that the gauge–fixing procedure introduces modifications in the unphysical vertices $HG^-_W G^+_W$, $H^2G^-_W G^+_W$, and $G^2_Z G^-_W G^+_W$.

Let us now discuss the dynamical implications of the $\mathcal{L}_{GFSV}$ term. This term has an important impact on the unphysical Higgs sector of the theory. In fact, after an integration by parts to remove the bilinear terms we obtain

$$\mathcal{L}_{HK2}+\mathcal{L}_{GFSV}=(\partial_\mu\varphi^{0*})(\partial^\mu\varphi^0)+\frac{g^2}{4c^2_W}Z_\mu Z^\mu\varphi^{0*}\varphi^0 + \frac{g}{2c_W}Z^\mu(H\partial_\mu G_Z - G_Z\partial_\mu H) + i\sqrt{2}g(W^+_\mu G^-_W\partial^\mu\varphi^0 - W^-_\mu G^+_W\partial^\mu\varphi^{0*}). \quad (60)$$

We can see that the unphysical vertices $WG_W\gamma$, WG_WZ, $HWG_W\gamma$, HWG_WZ, $G_ZWG_W\gamma$, and G_ZWG_WZ appearing in $\mathcal{L}_{HK2}$ have been removed of the theory. The advantages of using this nonlinear gauge can be appreciated now. The absence of these unphysical vertices has important consequences in practical loop calculations where one or more external photons are involved [6–9]. In particular, there is a considerable reduction in the number of Feynman diagrams. Besides, $U_e(1)$ gauge invariance is transparent since the theory satisfies QED–like Ward identities.

3.3. The ghost sector

Let us now discuss the implications of this nonlinear gauge on the ghost sector characterized by the Lagrangian $\mathcal{L}_F$. Using the following definitions for the ghost fields

$$C^\pm = \frac{1}{\sqrt{2}}(C^1 \mp C^2), \quad (61)$$

$$C^Z = c_W C^3 - s_W C, \quad (62)$$

$$C^A = s_W C^3 + c_W C \quad (63)$$

and similar expressions for the antighost fields, we can write this Lagrangian as a sum of two terms:

$$\mathcal{L}_F = \mathcal{L}_{F1} + \mathcal{L}_{F2}, \quad (64)$$

where

$$\mathcal{L}_{F1} = -\bar{C}^-(\delta_{B_\Psi}f^+) - \bar{C}^+(\delta_{B_\Psi}f^-) - \bar{C}^Z(\delta_{B_\Psi}f^Z) - \bar{C}^A(\delta_{B_\Psi}f^A) \quad (65)$$

and

$$\mathcal{L}_{F2} = -\frac{2i}{\xi}[(f^-\bar{C}^+ - f^+\bar{C}^-)(c_WC^Z+s_WC^A) + (c_W\bar{C}^Z+s_W\bar{C}^A)(f^+C^- - f^-C^+) + (c_Wf^Z+s_Wf^A)(\bar{C}^-C^+ - \bar{C}^+C^-)] + 2(1-\frac{2}{\xi})[\bar{C}^+\bar{C}^-C^+C^- + (c_W\bar{C}^Z+s_W\bar{C}^A) \times (\bar{C}^+C^- + \bar{C}^-C^+)(c_WC^Z+s_WC^A)]. \quad (66)$$

The term $\mathcal{L}_{F2}$ does not exist if it is used the Faddeev–Popov method. The variations of the charged gauge–fixing functions are given by

$$\delta_{B_\Psi} f^+ = \bar{D}_\mu \hat{D}^\mu c^+ + ig(C_W c^Z + s_W C^A)(\bar{D}_\mu W^{+\mu}) + \frac{ig}{c_W} W^{+\mu}(\partial_\mu C^Z) + \frac{\xi g^2}{2}[(\varphi^{0*}\varphi^0 - G_W^- G_W^+)C^+ + \sqrt{2}(c_W C^Z + s_W C^A)\varphi^{0*} G_W^+], \quad (67)$$

with $\delta_{B_\Psi} f^- = (\delta_{B_\Psi} f^+)^\dagger$. Notices that these functions transform covariantly under the $U_e(1)$ group. On the other hand, the variations of the neutral functions are given by

$$\delta_{B_\Psi} f^Z = \Box C^Z + igc_W \partial_\mu (W^{-\mu} C^+ - W^{+\mu} C^-) - \frac{\xi g}{2} m_Z (G_W^+ C^- + G_W^- C^+) + \frac{\xi g}{2\sqrt{2} c_W} m_Z (\varphi^{0*} + \varphi^0) C^Z, \quad (68)$$

$$\delta_{B_\Psi} f^A = \Box C^A + ie\partial_\mu (W^{-\mu} C^+ - W^{+\mu} C^-). \quad (69)$$

We can see that these Lagrangians are invariant under the $U_e(1)$ group. Due to this, they contain new vertices not present in the linear gauges. For example, the presence of the vertices $\bar{C}^\pm C^\mp \gamma\gamma$ is a direct consequence of the $U_e(1)$–gauge invariance. It is clear that the charged anticommuting fields satisfy QED–like Ward identities. Indeed, all charged particles of the theory satisfy this type of identities.

4. Summary

In this paper we have presented a nonlinear R_ξ–gauge for the electroweak theory. This gauge–fixing procedure was defined on the basis of the BRST symmetry. It was found that this gauge modifies in a nontrivial way both the Yang–Mills and the Higgs sectors. In contrast with the conventional linear gauges, in this nonlinear gauge the ghost sector is manifestly invariant under the electromagnetic gauge group and includes four–ghost interactions. At one–loop, the four–ghost interactions are only necessary for off–shell renormalization, but in two–loop binary processes they would play an important role in the determination of the corresponding S–matrix element. The method allows us to eliminate the unphysical vertices $W^\pm G_W^\mp \gamma$, $W^\pm G_W^\mp Z$, $HW^\pm G_W^\mp \gamma$, $HW^\pm G_W^\mp Z$, $G_Z W^\pm G_W^\mp \gamma$, and $G_Z W^\pm G_W^\mp Z$, which are always present in conventional linear gauges. An important feature of this procedure is that all charged particles of the theory satisfy QED–like Ward identities, which greatly simplifies the loop calculations.

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