# Baryon magnetic moments in the SU(3) and the SU(2)×U(1) flavor groups

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Working within the non relativistic quark model, a two parameter fit to the magnetic moments of baryons is presented. The fit has an excellent $\chi^2$. The model is based on taking different flavor groups to describe the different magnetic moments. The selection of which group to assign to each baryon is guided by the structure of its wavefunction. The model corresponds to assigning different effective masses to a quark depending on which baryon is being considered. Using the values extracted from the fit, the magnetic moments of the $\Omega^-$ and the $\Delta^{++}$ have been predicted, and the comparison to the existing experimental values is quite satisfactory.

*Keywords:* Magnetic moments; flavor groups; baryons.

Trabajando en el contexto del modelo no relativista de quarks, se presenta un ajuste de dos parámetros a los momentos magnéticos de los bariones. El ajuste tiene un $\chi^2$ excelente. El modelo está basado en tomar diferentes grupos de sabor para describir diferentes momentos magnéticos. La asignación de grupos a bariones se basa en la estructura de su función de onda. El modelo corresponde a asignar diferentes masas efectívas a los quarks dependiendo de que barión se trate. Usando los valores proporcionados por el ajuste, los momentos magnéticos de la $\Omega^-$ y la $\Delta^{++}$ has sido calculdos y la comparasión con valores experimentales existentes es muy satisfactoria.

*Descriptores:* Momentos magnéticos; grupos de sabor; bariones.

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## 1. Introduction

There has recently been a renewed interest in the magnetic moments and spin structure of baryons within a variety of models. For example, the chiral quark model [1,2], quenched lattice gauge theory [3], the 1/N$_c$ expansion [4], and extensions to the non-relativistic quark model (NRQM) [5] to name a few. These models are more ambitious than the NRQM. Nonetheless it has been argued that due to some subtle cancellations the NRQM is a good approximation to the magnetic moments [6], so a simple model may extract the physics of the problem more easily than a complicated one.

It is well known since a long time that the magnetic moments of the octet baryons can be described only approximately via a SU(3) flavor group [7]. Using this approach within the NRQM, it is assumed that the breaking of the flavor symmetry acts equally in all states of the octet. However this is not physically acceptable. Take, for example, the relation $\mu_p/\mu_n = -1.5$. It can be obtained either from the SU(2) as well as from the SU(3) flavor group. It is known that the SU(3) symmetry is valid at the 30% level, so the relation should be valid at this level. But the relation is experimentally valid at the 1.5% percent level as expected from the SU(2) symmetry. Therefore, in this case, it can not be accepted that the breaking of SU(3) acts as it does for the case of, say, the magnetic moment of the $\Lambda$.

Accordingly, in this letter it is proposed to change the idea to use one broken flavor group to describe all magnetic moments, to the idea of describing some magnetic moments with one exact flavor group and some with another exact flavor group. Furthermore we give criteria, based on the structure of the wavefunctions, as to which flavor group to use to describe the magnetic moment of a given baryon. A physical interpretation of the model in terms of effective quark masses is provided.

Section II introduces the wave functions of the baryons under the SU(2)×U(1) flavor group (times the SU(2) spin group). Section III presents the magnetic moments of the baryons keeping the masses of the three quarks as different parameters. Remarkably, although the wave functions are different under the SU(3) and the SU(2)×U(1) flavor groups, the magnetic moments turn out to be the same.

In section IV the fits to the measured magnetic moments are presented. First we note that each magnetic moment can be written either in the form $\mu = a\mu_d(1 + b(\mu_s/\mu_d))$, or as $\mu = c\mu_s(1 + d(1 - \mu_s/\mu_d))$ (from here on the approximation $m_u = m_d$ will be assumed). It is found that each baryon falls in only one of the two following sets: $i$) The baryon has either a small contribution to its magnetic moment coming from the $\mu_s/\mu_d$ factor, *i.e.* the coefficient $b$ is less than one, or $ii$) it has a small contribution to its magnetic moment coming from the $1 - \mu_s/\mu_d$ factor, *i.e.* the coefficient $d$ is less than one. Note that the exact SU(3) flavor limit implies $\mu \rightarrow c\mu_s$ ($d = 0$), and the SU(2)×U(1) exact flavor limit with $m_d \ll m_s$ implies $\mu \rightarrow a\mu_d$ ($b = 0$). This two set division of the magnetic moments can be readily explained using the wave functions of the baryons, and it provides criteria to know which group should be used to calculate the magnetic moment of a given baryon. Then a fit is performed using the formulas for the magnetic moments from the SU(2)×U(1) exact flavor limit, with $m_d \ll m_s$, for the baryons in the first set, and those of the exact SU(3) flavor group for the baryons in the other set.

It was found that using a two parameter fit to the experimental data a better agreement in terms of $\chi^2$ was obtained, than other two parameter fits in the literature and a comparable agreement to fits requiring four parameters (see for example [8, 9]). Using the values of the parameters obtained from the fit, the magnetic moments of the $\Omega^-$ and the $\Delta^{++}$ have been predicted. The comparison to the existing experimental values is quite satisfactory.

Section V is devoted to a discussion of our results. This letter is closed with a brief summary and an outlook of future work in section VI.

The current status on the experimental side is as follows. Seven of the magnetic moments are measured with around 1% accuracy or better [10]. The transition magnetic moment for $\Sigma^0 \rightarrow \Lambda$ is known to a 5% precision [11]. The $\Omega^-$ was measured some time ago [12], and recently a new measurement has been presented [13]. Finally the magnetic moment of the $\Delta^{++}$ has also been measured [14].

## 2. Wave functions of baryons

The procedure to obtain the wave functions for a given flavor group is standard and can be found in many textbooks. It is well known that the SU(3) flavor group (times the SU(2) spin group) produces, in the case of baryons, an **8** and a **10** multiplet. On the other hand the SU(2)×U(1) flavor group (times the SU(2) spin group) yields different wave functions (see the appendix) which form the following multiplets: **2** ($N$), **4** ($\Delta$), **3** ($\Sigma$), **1** ($\Lambda$), **2** ($\Xi$), **3** ($\Sigma^*$), **2** ($\Xi^*$), **1** ($\Omega$).

## 3. The magnetic moments of baryons

The expectation value for the magnetic moment $\mu$ of a baryon $B$ in the S wave is given by the expression

$$\mu_B = \langle \psi_B | \sum_{i=1}^{3} \hat{\mu_q}(i) \hat{\sigma_z}(i) | \psi_B \rangle$$

where $\hat{\mu_q}$ is the operator for the magnetic moment of the quarks, $\hat{\sigma_z}(i)$ is Pauli's spin operator and $i$ runs over $\{u, d, s\}$. The wave functions for the SU(2)×U(1) group are different to those found with the SU(3) flavor group (see for example [20]). In spite of this fact, the magnetic moments of the baryons are the same independently of which set of wave functions are used to calculate them.

The formulas for the magnetic moments are given in Table I. Each magnetic moment can be written using the functional form

$$\mu = a\mu_d(1 + b(\mu_s/\mu_d))$$

and as

$$\mu = c\mu_s(1 + d(1 - \mu_s/\mu_d)).$$

Each baryon falls in only one of the two following sets:

**A** The baryon has a contribution to its magnetic moment coming from the $\mu_s/\mu_d$ factor with $b < 1$. The baryons in this group are $p$, $n$, $\Sigma^+$, $\Sigma^-$, $\Sigma^0$, $\Sigma^0 \rightarrow \lambda^0$, $\Delta^{++}$, $\Delta^+$, $\Delta^0$, $\Delta^-$, $\Sigma^{*+}$, $\Sigma^{*o}$ and $\Sigma^{*-}$.

**B** The baryon has a contribution to its magnetic moment coming from the $1 - \mu_s/\mu_d$ factor with $d < 1$. The baryons in this group are $\Lambda$, $\Xi^0$, $\Xi^-$, $\Xi^{*0}$, $\Xi^{*-}$ and $\Omega^-$.

This division of the baryons in two sets has an explanation in terms of wave functions. The SU(2)×U(1) exact flavor limit with $m_d \ll m_s$ implies $\mu \rightarrow a\mu_d$ so this case can be naturally identified with set **A**. Note that none of the wave functions in this case has a dominance of the strange quark. The behavior corresponds then to a decoupling of $s$ which is described by the SU(2)×U(1) exact flavor limit with $m_d \ll m_s$. On the other hand the exact SU(3) flavor limit implies $\mu \rightarrow c\mu_s$, so it can be identified with the set **B**. Here the $s$ quark dominates the wave function ($\Xi^0$, $\Xi^-$) or the isospin structure of the wave function cancel the influence of the light quarks when calculating the magnetic moments ($\Lambda$). Note that each quark mass $m_q$ is a parameter of the group. So in principle a more precise notation would be in the lines $m_q^{SU(3)}$ and so on. This notation is rather cumbersome, so the same symbol has been used for both groups. This does not mean that the value of $m_q$ must be the same in both groups, so there is no contradiction in having $\mu_s/\mu_d$ small for one group and $1 - \mu_s/\mu_d$ small for the other.

## 4. Fit to the magnetic moments of baryons

There is an important technical point, before doing the fit. The magnetic moments of both the proton and the neutron have a very small experimental error. This precision of more than one part per million is huge when compared to the accuracy of the isospin symmetry of the ($p$,$n$) doublet. This turns meaningless a $\chi^2$ approach to the fit. To avoid this problem, it was proposed in [15] to add in quadrature a common absolute error to all the moments. Following this lead (see also [9]) an absolute error of $\sigma = 0.03\mu_N$ has been added in quadrature to the real experimental error. The measured values of the magnetic moments of baryons are shown in Table II.

First a three parameter fit was performed. For the elements of set **A** the form

$$\mu = a\mu_d[1 + \epsilon b(\mu_s/\mu_d)]$$

was used. For the elements of set **B** the form

$$\mu = c\mu_s[1 + \epsilon d(1 - \mu_s/\mu_d)].$$

$a$, $b$, $c$ and, $d$ can be easily read off Table I. The three parameters are then $\mu_d$, $\mu_s$ and $\epsilon$. The parameter $\epsilon$ turned out to be compatible with zero ($\epsilon$=0.028±0.098) while the values for $\mu_d$ and $\mu_s$ remained exactly as in the two group fit shown below. This experimental evidence strengths our assumption of separating the magnetic moments in two different nonoverlapping sets. Thus a new 2 parameter fit was performed using $\mu = a\mu_d$ (set **A**) and $\mu = c\mu_s$ (set **B**).

TABLE I. Expressions for magnetic moments $\mu$ of baryons. The standard form corresponds to SU(3) with all masses different. The following columns assume $m_u = m_d$. The formula for SU(2)×U(1) are on the limit $m_d \ll m_s$. In the case SU(3) all masses are equal. The expressions used in the fit with three parameters are shown in the last column.

| Baryon | Standard Form | SU(2)×U(1) | SU(3) | $\epsilon$ fit |
|---|---|---|---|---|
| $p$ | $\frac{1}{3}(4\mu_u - \mu_d)$ | $-3\mu_d$ | $-3\mu_s$ | $-3\mu_d$ |
| $n$ | $\frac{1}{3}(4\mu_d - \mu_u)$ | $2\mu_d$ | $2\mu_s$ | $2\mu_d$ |
| $\Lambda$ | $\mu_s$ | 0 | $\mu_s$ | $\mu_s$ |
| $\Sigma^+$ | $\frac{1}{3}(4\mu_u - \mu_s)$ | $-8/3\mu_d$ | $-3\mu_s$ | $-8/3\mu_d(1+\frac{\mu_s}{8\mu_d}\epsilon)$ |
| $\Sigma^-$ | $\frac{1}{3}(4\mu_d - \mu_s)$ | $4/3\mu_d$ | $\mu_s$ | $4/3\mu_d(1-\frac{\mu_s}{4\mu_d}\epsilon)$ |
| $\Sigma^0$ | $(2\mu_u + 2\mu_d - \mu_2)/3$ | $-2/3\mu_d$ | $\mu_s$ | |
| $\Xi^0$ | $\frac{1}{3}(4\mu_s - \mu_u)$ | $2/3\mu_d$ | $2\mu_s$ | $2\mu_s[1-\frac{1}{3}(1-\frac{\mu_d}{\mu_s})\epsilon]$ |
| $\Xi^-$ | $\frac{1}{3}(4\mu_s - \mu_d)$ | $-1/3\mu_d$ | $\mu_s$ | $\mu_s[1+\frac{1}{3}(1-\frac{\mu_d}{\mu_s})\epsilon]$ |
| $\Sigma^0 \to \Lambda$ | $\frac{1}{\sqrt{3}}(\mu_d - \mu_u)$ | $\sqrt{3}\mu_d$ | $\sqrt{3}\mu_s$ | $\sqrt{3}\mu_d$ |
| $\Delta^{++}$ | $3\mu_u$ | $-6\mu_d$ | $-6\mu_s$ | |
| $\Delta^+$ | $2\mu_u + \mu_d$ | $3\mu_d$ | $3\mu_s$ | |
| $\Delta^0$ | $\mu_u + 2\mu_d$ | 0 | 0 | |
| $\Delta^-$ | $3\mu_d$ | $3\mu_d$ | $3\mu_s$ | |
| $\Sigma^{*+}$ | $2\mu_u + \mu_s$ | $-4\mu_d$ | $-3\mu_s$ | |
| $\Sigma^{*0}$ | $\mu_u + \mu_d + \mu_s$ | $-\mu_d$ | 0 | |
| $\Sigma^{*-}$ | $2\mu_d + \mu_s$ | $2\mu_d$ | $3\mu_s$ | |
| $\Xi^{*0}$ | $2\mu_s + \mu_u$ | $-\mu_d$ | 0 | |
| $\Xi^{*-}$ | $2\mu_s + \mu_d$ | $\mu_d$ | $3\mu_s$ | |
| $\Omega$ | $3\mu_s$ | 0 | $3\mu_s$ | |

This two parameter fit can be viewed as two independent one parameter fits. For the case of the 5 magnetic moments in set **A** a $\chi^2$ per degree of freedom (dof) of 0.42 was found. The other 3 magnetic moments in set **B** yield $\chi^2$/dof=1.9. To be able to compare the quality of the fit for this model with other results in the literature which quote a single value for $\chi^2$, both fits have been performed simulaneously. In this case $\chi^2$/dof=1.4. The fitted values of the parameters are $\mu_d = -0.930 \pm 0.007$ and $\mu_s = -0.628 \pm 0.013$. The values obtained for the magnetic moments using these parameters are shown in Table II under the heading $\mu_{2G}$. The subscript is meant to stress the fact that the fit was performed using simultaneously two flavor groups; SU(2)×U(1) identified with the elements of set **A** and SU(3) corresponding to the elements of set **B**. The errors shown are the maximum spread in the values of the magnetic moments obtained by varying the parameters within their errors.

TABLE II. Measured values for baryon magnetic moments in units of $\mu_N$, along with the prediction of our two group model with two parameters.The experimental values above the middle line were used in the fits. The values below the middle line are parameter free predictions of our model

| Baryon | $\mu_{\rm exp}$ | $\mu_{\rm 2G}$ |
|---|---|---|
| $p$ | 2.79±6.3x10$^{-8}$ | 2.79±0.02 |
| $n$ | -1.91±4.5x10$^{-7}$ | -1.86±0.01 |
| $\Lambda$ | -0.613±0.004 | -0.63±0.01 |
| $\Sigma^+$ | 2.46±0.01 | 2.48±0.02 |
| $\Sigma^-$ | -1.16±0.025 | -1.24±0.01 |
| $\Xi^0$ | -1.25±0.014 | -1.26±0.02 |
| $\Xi^-$ | -0.651±0.0025 | -0.63±0.01 |
| $\Sigma^0 \to \Lambda$ | -1.61±0.08 | -1.61±0.01 |
| $\Sigma^0$ | | 0.620±0.005 |
| $\Delta^{++}$ | 4.52±0.95 | 5.58±0.04 |
| $\Delta^+$ | | 2.79±0.02 |
| $\Delta^0$ | | 0 |
| $\Delta-$ | | -2.79±0.02 |
| $\Sigma^{*+}$ | | 3.72 ±0.03 |
| $\Sigma^{*0}$ | | 0.93±0.01 |
| $\Sigma^{*-}$ | | -1.86±0.01 |
| $\Xi^{*0}$ | | 0 |
| $\Xi^{*-}$ | | -1.88±0.04 |
| $\Omega$ | -2.02±0.06 | -1.88±0.04 |

## 5. Discussion

**1.** To be able to do the fit, an extra error of $\sigma = 0.03\mu_N$ has been added in quadrature to the experimental error. This value makes sense as much as in the size of accuracy of considering the proton and the neutron as an isospin doublet, as in comparison to the errors of the other measured magnetic moments. Nonetheless to study the sensitivity of the results to this error, its value was changed to 0.02 and 0.04 $\mu_N$. As expected, the main effect was in the $\chi^2$/dof which changed from 1.4 to 2.6 and 0.9 respectively. The value of the parameters remained the same and their errors varied from $\pm$0.93 to $\pm$0.17 for $\mu_d$ and $\pm$0.005 to $\pm$0.009 for $\mu_s$. This shows that the fit is quite stable under variation of this assumption. It must be noted that other analysis have used this extra error up to $\sigma = 0.1\mu_N$ to equalize the weights, within the fit, of the different magnetic moments and to *force* a $\chi^2$/dof of the order of one [8, 16].

**2.** The wave functions obtained using the SU(2)×U(1) flavor group are different to those from the SU(3) flavor group. Nevertheless the magnetic moments in both approaches turn out to be the same when considering the three quark masses as different parameters. Note that when taking into account the physical hierarchy of quark masses the magnetic moments of both approches differed and could be classified in two different sets.

**3.** From this analysis it is clear that different baryons can be associated with different flavor groups. This new idea differs from the traditional method of fixing one flavor group for all baryons and then breaking it. This result is strengthened by the results of the the fit with the parameter $\epsilon$. One could argue that the variation of, say, the coefficients $b$ from baryon to baryon can be big (for example there is a factor of 2 between $b$ for $\Sigma^+$ and $b$ for $\Sigma^-$) and that it is too much to ask $b$ and $d$ to be zero in all cases. Nontheless the fit including the parameter $\epsilon$ implies exactly that, and from this it follows naturally the separation of the baryons in two groups governed by different exact flavor symmetries.

**4.** A criteria to decide which flavor group to use for calculating the magnetic moment of a given baryon is provided. If the $s$ quark dominates the wave function SU(3) is a good choice, if not, then SU(2)×U(1) is a better flavor group. Using these guidelines the full sets are

$$\mathbf{A} = \{N, \Sigma, \Delta, \Sigma^*\},$$

$$\mathbf{B} = \{\Lambda, \Xi, \Xi^*, \Omega\}.$$

**5.** Physically this means that the effective masses of the quarks in a baryon depend on which other quarks are bound to them to form the baryon. Pictorically, the quarks dress themselves depending on the company. This idea is not so strange as it sounds and it has already been explored [17, 18]. In the NRQM the quarks are in a potential with an energy (the total mass) which changes from baryon to baryon, so it is natural that the *effective* quark masses may depend on the baryon. There are even experimental evidence that quarks affect and are affected by their surroundings, *i.e.*, the measurement of the light quark sea asymmetry [19]. From the values for the magnetic moments the quark masses can be computed for each group. It is found that in the case of exact SU(3) flavor symmetry the three masses are 498 MeV. For SU(2)×U(1) $m_u = m_d =$336 MeV. In our model this means that when the $s$ quark dominates the wave function the effective $u$ and $d$ quarks are heavier than in the absence of the strange quark. In other words, the presence of a heavier quark induces an increase on the effective binding energy assinged to the lighter quarks.

**6.** The two flavor group model is based on the phenomenological idea that the binding energy of quarks, which is effectively assigned to their masses in NRQMs, depends on the surronding media. This approach is validated by the excellence of the fit and accuracy of its predictions. The magnetic moments of the $\Delta^{++}$ and the $\Omega$ have been compared to experimental measurements which have not been used in the fits, i.e they are independent and can be used to test the model. It is predicted that $\mu_{2G}(\Delta^{++}) = 5.58 \pm 0.04$ and $\mu_{2G}(\Omega) = -1.88 \pm 0.04$ in very good agreement with the measured values of 4.52$\pm$0.95 and -2.02$\pm$0.06 respectively.

## 6. Conclusions

The wave functions of baryons for the SU(2)×U(1) flavor group have been presented. From the wave funtions the magnetic moments of the baryons have been calculated. A two parameter fit to the magnetic moments of the baryons has been performed. The new idea behind the fit is to use two flavor groups to describe the magnetic moments. A criteria to assign a given baryon to a flavor group, based in the structure of its wave function, has been provided. In terms of $\chi^2$ the 2 parameter model presented here has an accuracy of the same order than other 4 parameter fits in the literature. The parameters have been used to predict the magnetic moments of the $\Delta^{++}$ and the $\Omega$. An excellent agreement with the measured values has been found. Given the different multiplet structure of the two flavor groups used and the different wave functions they provided, this approach could be applied to the description of other phenomena like, for example, semileptonic decays or fragmentation function of baryons.Furthermore, in view of the success of the model, the idea of two different exact flavor groups may be used as a guide to simplify calculations and define approximations in other more formal approaches based on first principles calculations within QCD.

## Appendix

### Wave functions for the SU(2)×U(1) flavor group

The baryon wave functions are given by:

$$\psi_{p\uparrow} = \frac{1}{3\sqrt{2}}[uud(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) + udu(2\uparrow\downarrow\uparrow - \uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow) + duu(2\downarrow\uparrow\uparrow - \uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow)],$$

$$\psi_{n\uparrow} = -\frac{1}{3\sqrt{2}}[ddu(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) + dud(2\uparrow\downarrow\uparrow - \uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow) + udd(2\downarrow\uparrow\uparrow - \uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow)],$$

$$\psi_{\Lambda^o\uparrow} = \frac{1}{2}(uds - dus)(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow),$$

$$\psi_{\Sigma^+\uparrow} = \frac{1}{\sqrt{6}}uus(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow),$$

$$\psi_{\Sigma^-\uparrow} = \frac{1}{\sqrt{6}}dds(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow),$$

$$\psi_{\Xi^o\uparrow} = \frac{1}{\sqrt{6}}ssu(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow),$$

$$\psi_{\Xi^-\uparrow} = \frac{1}{\sqrt{6}}ssd(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow),$$

$$\psi_{\Sigma^o\uparrow} = \frac{1}{2\sqrt{3}}(uds + dus)(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow),$$

$$\psi_{\Delta^{++}\uparrow} = uuu\uparrow\uparrow\uparrow,$$

$$\psi_{\Delta^+\uparrow} = \frac{1}{\sqrt{3}}(uud + udu + duu)\uparrow\uparrow\uparrow,$$

$$\psi_{\Delta^o\uparrow} = \frac{1}{\sqrt{3}}(udd + dud + ddu)\uparrow\uparrow\uparrow,$$

$$\psi_{\Delta^-\uparrow} = ddd\uparrow\uparrow\uparrow,$$

$$\psi_{\Sigma^{*+}\uparrow} = uus\uparrow\uparrow\uparrow,$$

$$\psi_{\Sigma^{*o}\uparrow} = \frac{1}{\sqrt{2}}(uds + dus)\uparrow\uparrow\uparrow,$$

$$\psi_{\Sigma^{*-}\uparrow} = dds\uparrow\uparrow\uparrow,$$

$$\psi_{\Xi^{*o}\uparrow} = uss\uparrow\uparrow\uparrow,$$

$$\psi_{\Xi^{*-}\uparrow} = dss\uparrow\uparrow\uparrow,$$

$$\psi_{\Omega^-\uparrow} = sss\uparrow\uparrow\uparrow\ .$$

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