The fundamental optimal relations and the bounds of the allocation of heat exchangers and efficiency for a non-endoreversible brayton cycle

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In this paper we analyze a Brayton cycle with external and internal irreversibilities. The external ones come from heat transfer with counterflow heat exchangers in the cold and hot sides; the internal irreversibilities are given by the isentropic efficiencies of the compressor and turbine. Optimization is carried out with respect to the pressure ratio and the total inventory of the heat transfer units, using the ε-NTU method. We show the analytical expressions for the efficiency that mazimizes work and the optimal allocation (size) of heat exchangers. We also analyze the asymptotic behavior of these expressions. The results obtained extend a Bejan's model and are more general and useful.

Keywords: Thermodynamics Optimization, heat engines. internal and external irreversibilities.

En este trabajo analizamos un ciclo Brayton con irreversibilidades externas e internas. Las externas provienen de la transferencia de calor mediante intercambiadores de calor de flujo cruzado de los lados frío y caliente; las irreversibilidades internas son producidas por las eficiencias isoentrópicas del compresor y la turbina. La optimización se realiza con respecto a la razón de presiones y al inventario total del número de unidades de trasferencia de calor, empleando el método ε-NTU. Mostramos las expresiones analíticas para la eficiencia que maximiza el trabajo y la dimensión óptima de los intercambiadores de calor. También analizamos el comportamiento asintótico de estas expresiones. Los resultados obtenidos extienden un modelo de Bejan y son más útiles y generales.

Descriptores: Optimización termodinámica, máquinas térmicas, irreversibilidades internas y externas.

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1. Introduction

The classical air standard Brayton cycle has been used as a model of the gas turbine heat engine. This cycle results in unrealistically high performance predictions. Recently, there has been renewed the analysis of Brayton-like cycles by various researchers, considering the more practical aspects of entropy generation, power, power-density and the ecological and efficiency optimization.

Bejan [1] considered a closed ideal Brayton cycle (endoreversible Brayton cycle) operating between an infinite heat source and an infinite heat sink. He showed that, when the entropy generation is minimum, the efficiency corresponds to the efficiency of Curzon-Ahlborn [2] and the optimal allocation (size) of hot-side and cold-side heat exchangers is balanced.

Formerly, Leff [3] focused on the idealized Brayton cycle and obtained a Curzon-Ahlborn-like efficiency. Wu [4] looked at a closed non-isentropic Brayton cycle, without external irreversibilities, and found that the efficiency that maximizes the work corresponds to a Curzon-Ahlborn-like efficiency (see also Ref. 5). Later, Sahin *et. al* [6] determined the efficiency that maximizes the density of the work. In Ref. 5 we optimize the efficiency of a closed non-isentropic Brayton cycle.

The optimization of Brayton-like cycles with external irreversibilities started with Bejan. Later, Swanson [7] optimized the Bejan model using a log-mean temperature difference representation for both the high and low temperatures heat exchangers and assumed that it is internally operated by a reversible Carnot cycle. Cheng and Chen [8] made the numerical power optimization for a non-endoreversible Brayton cycle and, later, an ecological optimization in Ref. 9. Blank [10] optimizes the power for an open Brayton cycle with a finite interactive heat reservoir. Chen *et al.* [11] carried out the numerical optimization for density power and distribution of heat exchanger operation for the endoreversible Brayton cycle. In the optimization made in these latter works, except Blank, the proposal made by Andresen and Gordon [12] was used: for the heat exchanger operation in the hot-side and the cold-side, a single-pass counterflow heat exchanger can represent the optimal solution.

Recently, in Ref. 13 we analyzed the maximum efficiency of a non-endoreversible Brayton cycle: the internal irreversibilities are given by the isentropic efficiencies of the compressor and turbine, and the external irreversibilities corresponding to the heat transfer in the isobaric processes were modelled by the coupled, Andresen-Gordon proposal, single-pass counterflow heat exchangers, using the $\varepsilon-NTU$ method (effectiveness- number of transfer unit, see Ref. 14).

In this work we analyze the same non-endoreversible Brayton cycle. We do the optimization by a parametrization of the total inventory of the heat transfer units and the pressure ratio; the compressor and turbin efficiencies are fixed. We find optimal expressions for the efficiency that maximizes work for the allocation (size) of the heat exchanger inventory.

We determine bounds for both expressions and compare the numerical results respecting the endoreversible Brayton cycle.

This paper is organized as follows. In the Sec. 2, we use the $\varepsilon - NTU$ method, for the external irreversibilities, and the isentropic efficiencies of turbine and compressor, for the internal irreversibilities, to find the relation for the dimensionless work. In the section 3, we show the optimal expressions for efficiency (maximum work) and allocation (size) of the heat exchangers, we present some limit cases of our model and we analize the asymptotic behavior of the model with respect to the endoreversible model. In Sec. 4, numerical results are presented, on the behavior of the allocation with respect to the total number of transfer units and of the efficiency (maximum work) with respect to mininimum and maximum temperature ratio. In the conclusions, we propose that the allocation for the heat exchangers should be approximately 2–4% less than the Bejan's value; thus, the size of the heat exchanger in the hot side decreases.

2. The relation for the dimensionless work.

We consider a non-endoreversible Brayton cycle shown in Fig. 1.

The reversible Brayton cycle $(1-2s-3-4s-1)$ efficiency is given by:

$$\eta = 1 - x \tag{1}$$

where $x = \varepsilon^{1-(1/\gamma)}$, with $\varepsilon = (p_{2s}/p_1)$ the pressure ratio (maximum pressure divided by minimum pressure) and $\gamma = (c_p/c_v)$, where c_p is the constant-pressure specific heat and c_v is the constant-volume specific heat. Furthermore, for the reversible cycle the following temperature relations are satisfied:

$$T_{2s} = \frac{T_1}{x} \tag{2}$$

$$T_{4s} = T_3 x \tag{3}$$

where x is given by the Eq. (1). Henceforth, x denotes the working substance temperature ratio.

Considering a non-isentropic Brayton cycle, without external irreversibilities (see $1-2-3-4-1$, cycle in Fig. 1) with the isentropic efficiencies of the turbine and compressor η_1 and η_2 given by ([15]):

$$\eta_1 = \frac{T_3 - T_4}{T_3 - T_{4s}} \tag{4}$$

$$\eta_2 = \frac{T_{2s} - T_1}{T_2 - T_1} \tag{5}$$

we obtained the following temperature relations ([5]):

$$T_2 = T_1\left(1 + \frac{1-x}{\eta_2 x}\right) \tag{6}$$

$$T_4 = T_3\left(1 - \eta_1(1-x)\right) \tag{7}$$

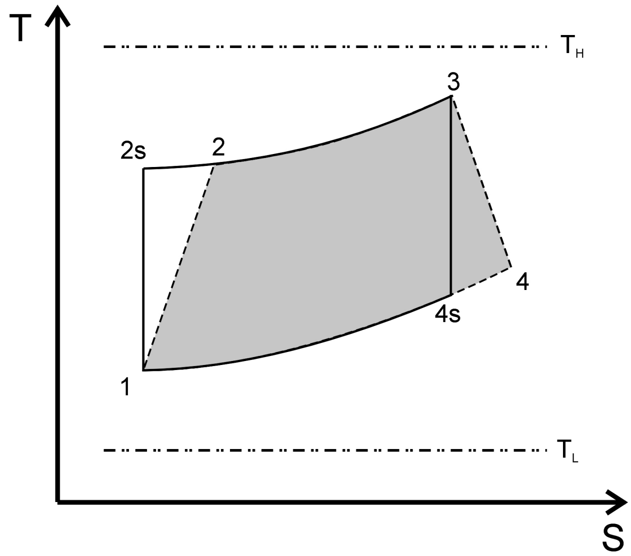

FIGURE 1. A non-endoreversible Brayton cycle.

where T_3, T_1 are the maximum and minimum temperatures achieved in the reversible cycle.

We now, consider an endoreversible Brayton cycle with external irreversibilities, temperature reservoirs given by the constant temperatures T_H and T_L (since the substance can be in phase change or infinite temperature reservoirs) and internally reversible: $\eta_1 = \eta_2 = 100\%$ (see Fig. 1). In this cycle, two single-pass counterflow heat exchangers are coupled to T_H and T_L. We calculate the heat transfer between the reservoir and the working substance using the log mean temperature difference ($LMTD$). The heat transfer balance for the hot-side is:

$$Q_H = U_H A_H LMTD_H = mc_p\left(T_3 - T_{2s}\right)$$

where H denotes the hot-side, U is the global heat transfer coefficient per area unit, A is the superficial area of the exchanger and m is the substance working mass. The $LMTD_H$ is given by [14]:

$$LMTD_H = \frac{T_H - T_{2s} - (T_H - T_3)}{\log\frac{T_H - T_{2s}}{T_H - T_3}}$$

The number of transfer units (NTU) of the hot-side, N_H, is [14]:

$$N_H = \frac{U_H A_H}{mc_p} = \frac{T_3 - T_{2s}}{LMTD_H}$$

and so,

$$e^{N_H} = \frac{T_H - T_{2s}}{T_3 - T_{2s}}$$

the effectiveness is:

$$\varepsilon_H = 1 - e^{-N_H} = \frac{T_H - T_3}{T_H - T_{2s}} \tag{8}$$

Assuming that the heat exchangers are counterflow, the heat conductance of the hot-side (cold side) is $U_H A_H$ ($U_L A_L$) and the thermal capacity rate (mass and specific heat product) of the working substance is C_W.

The heat transfer balance results in:

$$Q_H = C_W \varepsilon_H (T_H - T_{2s}) = C_W (T_3 - T_{2s}) \tag{9}$$

Similarly, the balance for the cold-side is given by:

$$Q_L = C_W \varepsilon_L (T_{4s} - T_L) = C_W (T_{4s} - T_1) \tag{10}$$

with ε_L given by:

$$\varepsilon_L = 1 - e^{-N_L} = \frac{T_1 - T_L}{T_{4s} - T_L} \tag{11}$$

As the effectiveness has expressions analogs to the isentropic efficiencies (Eqs. (4), (5) and (8), (11)), we can make a similar analysis to the non-isentropic cycle, without external irreversibilities.

The temperature reservoirs T_H and T_L are fixed. By Eqs. (9) and (10) we obtain the temperatures T_{2s} and T_{4s}:

$$T_{2s} = \frac{T_3 - \varepsilon_H T_H}{1 - \varepsilon_H}$$

$$T_{4s} = \frac{T_1 - \varepsilon_L T_L}{1 - \varepsilon_L}$$

Combining these Eqs. with Eqs. (2) and (3), we obtain:

$$T_{2s} = \frac{T_{4s} x^{-1} - \varepsilon_H T_H}{1 - \varepsilon_H}$$

$$T_{4s} = \frac{x T_{2s} - \varepsilon_L T_L}{1 - \varepsilon_L}$$

In these Eqs. there are two temperatures, one of the cycle and another of the exchanger. Resolving for T_{2s} and T_{4s}, we obtain (cf. [11]):

$$T_{2s} = \frac{\varepsilon_L \mu x^{-1} + \varepsilon_H (1 - \varepsilon_L)}{\varepsilon_L + \varepsilon_H (1 - \varepsilon_L)} T_H \tag{12}$$

$$T_{4s} = \frac{\varepsilon_H x + \varepsilon_L \mu (1 - \varepsilon_H)}{\varepsilon_L + \varepsilon_H (1 - \varepsilon_L)} T_H \tag{13}$$

where $\mu = T_L / T_H$.

We are interested in optimizing the dimensionless work, w (of the work W) of the non-endoreversible Brayton cycle (see Fig. 1) with respect to the maximum energy by mass unit attained in the cycle:

$$w = \frac{W}{C_W T_H}$$

where C_W is as above (Eqs. (9) and (10)).

The dimensionless expressions, $q = Q/(C_W T_H)$, for the hot-side and cold-side are:

$$q_H = \varepsilon_H \left(1 - \frac{T_2}{T_H}\right) \tag{14}$$

$$q_L = \varepsilon_L \left(\frac{T_4}{T_H} - \mu\right) \tag{15}$$

with $\mu = (T_L/T_H)$ and ε_H, ε_L given by the Eqs. (8) and (11); but instead of the temperatures T_{2s}, T_{4s}, now the temperatures involved are T_2, T_4. Thus, ε_H and ε_L are given by:

$$\varepsilon_H = \frac{T_H - T_3}{T_H - T_2} \quad \text{and} \quad \varepsilon_L = \frac{T_1 - T_L}{T_4 - T_L}$$

We find expressions for temperatures T_2 and T_4, including the isentropic efficiencies η_1 and η_2, effectiveness ε_H and ε_L, and the parameter μ (that is, the ratio between the temperatures of hot-side and cold-side). Combining Eqs. (2), (3), (6), (7), (12) and (13), we have:

$$T_2 = \frac{\left[\varepsilon_L \mu x^{-1} + \varepsilon_H (1 - \varepsilon_L)\right] \left(\frac{1-x}{\eta_2} + x\right)}{\left[\varepsilon_L + \varepsilon_H (1 - \varepsilon_L)\right]} T_H \tag{16}$$

$$T_4 = \frac{\left[\varepsilon_H x + \varepsilon_L \mu (1 - \varepsilon_H)\right] \left(\frac{1}{x} - \frac{(1-x)\eta_1}{x}\right)}{\left[\varepsilon_L + \varepsilon_H (1 - \varepsilon_L)\right]} T_H \tag{17}$$

The work w of the non-endoreversible cycle is (Eqs. (14) and (15)):

$$w = \varepsilon_H \left[1 - \frac{T_2}{T_H}\right] - \varepsilon_L \left[\frac{T_4}{T_H} - \mu\right].$$

Substituting the Eqs. (16) and (17), we obtain the analytical relation:

$$w = \varepsilon_H \left[1 - \frac{\varepsilon_L \mu x^{-1} + \varepsilon_H (1-\varepsilon_L)}{\varepsilon_L + \varepsilon_H (1-\varepsilon_L)} \left(\frac{1-x}{\eta_2} + x\right)\right]$$
$$\times \varepsilon_L \left[\frac{\varepsilon_H x + \varepsilon_L \mu (1-\varepsilon_H)}{\varepsilon_L + \varepsilon_H (1-\varepsilon_L)} \left(\frac{1}{x} - \frac{(1-x)\eta_1}{x}\right) - \mu\right] \tag{18}$$

3. Analytical optimal expressions for the efficiency (maximum work) and the allocation (size) of the heat exchangers.

First, let us look at some special cases. If $\varepsilon_H = \varepsilon_L = \eta_1 = \eta_2 = 1$ in the Eq. (18), then we have the reversible Brayton cycle ($1 - 2s - 3 - 4s - 1$ in Fig. 1), with $T_H = T_3$ and $T_L = T_1$. But $mC_H T_H = mc_P T_3$; thus, the dimensionless work, $w = (W/mc_P T_3)$, is:

$$w = (1 - x)\left(1 - \frac{\mu^*}{x}\right)$$

with $\mu^* = T_1/T_3$ and x is the working substance temperature ratio [Eq. (1)].

This last expression is a function of only the temperature ratio x. To maximize it, we obtain the Curzon-Ahlborn efficiency ([2]; see also Leff [3]):

$$\eta_{RT} = 1 - \sqrt{\mu^*} \tag{19}$$

Now, we suppose that $\varepsilon_H = \varepsilon_L = 1$, η_1 and η_2 positives and less than one, in Eq. (18). Then, we have the non-isentropic Brayton cycle $1-2-3-4-1$ (see Fig. 1), with $T_H = T_3$ and $T_L = T_1$. Again, $mC_H T_H = mc_P T_3$, and the dimensionless work, w, is:

$$w = (1-x)\left[\eta_1 - \frac{1}{\eta_2 x}\mu^*\right]$$

with $\mu^* = (T_1/T_3)$; which reaches its maximum value in:

$$x_{NI} = \sqrt{I\mu^*} \tag{20}$$

with $I = (1/\eta_1\eta_2)$. By Eq. (1) we find the efficiency that maximizes work:

$$\eta_{NI} = 1 - \sqrt{I\mu^*} \tag{21}$$

Now $\eta_1 = \eta_2 = 1$, ε_H and ε_L positive and less than one, in Eq. (18). We have the endoreversible Brayton cycle (see Fig. 1), with $T_H > T_3$ and $T_L < T_1$. The dimensionless work, w, is:

$$w = \frac{\varepsilon_H\varepsilon_L(1-x)\,x - (1-x)\,\mu}{x\left[\varepsilon_L + \varepsilon_H(1-\varepsilon_L)\right]} \tag{22}$$

Now we include the following parametrization of the total inventory of heat transfer in Eq. (22). The total number of transfer units, N, of both heat exchangers is:

$$N_H + N_L = N \tag{23}$$

$$N_H = yN \tag{24}$$

$$N_L = (1-y)\,N \tag{25}$$

Optimizing Eq. (22) with respect to the ratio of temperatures, x, and to the allocation (size) of both heat exchangers inventory, y, we obtain the following:

$$y = \frac{1}{2} \qquad \eta = 1 - \sqrt{\mu} \tag{26}$$

The physical interpretation of these values is the following: in $y = 1/2$ the hot-side and cold-side heat exchangers will have the same size -allocation balanced- and it corresponds to the efficiency of Curzon-Albhorn:

$$\eta_{CA} = 1 - \sqrt{\mu} \tag{27}$$

(see Ref. 1 who obtained the same result but minimizing the entropy generation).

The variations of the total number of transfer units N are related to the heat transfer between the reservoirs and the working substance. When N increases, the temperature of the working substance (T_3 or T_1) tends toward the temperature of the reservoirs (T_H or T_L). If there is a decrease in N, the temperatures difference increases.

Obviously, if the temperatures corresponding to the heat exchangers are very close to the working substance temperatures, T_3 tends to T_H and T_1 tends to T_L, the total number of heat transfer units N increases very fast, excluding any practical application. Thus, the effectiveness tends to one and the efficiency will correspond to (Eq. (19)):

$$\eta_{RT} = 1 - \sqrt{\mu^*}$$

Now we consider the optimization of the non-endoreversible Brayton cycle. The parameters and variables η_2 and η_1, ε_H and ε_L, $\mu = (T_L/T_H)$, $x = 1-\eta$ are all positive and less than one. We include in Eq. (22) the same parametrization that in above cycle, for the total inventory N of transfer units of heat (Eqs. (24) and (25)). Then, w is a function of only x and y.

Applying the extreme conditions:

$$\frac{\partial w}{\partial x} = 0 \quad \text{and} \quad \frac{\partial w}{\partial y} = 0$$

we obtain the following analytical expressions for η [see Eq. (1)] and y:

$$\eta_{NE} = 1 - \sqrt{I_y\mu} \tag{28}$$

$$y_{NE} = \frac{1}{2} + \frac{1}{2N}\ln(I_x) \tag{29}$$

where I_y and I_x are given by:

$$I_y = \frac{\left(e^N - e^{yN}\right)\left[e^N\left(e^{yN}-1\right) + \eta_2(1-\eta_1)\left(e^N - e^{yN}\right)\right]}{e^N\left(e^{yN}-1\right)\left[\eta_1\eta_2\left(e^N - e^{yN}\right) - (1-\eta_2)\left(e^{yN}-1\right)\right]}\mu \tag{30}$$

$$I_x = \frac{\eta_1\eta_2 e^N(x-\mu) - (1-\eta_2)\left(e^N\mu - x\right)}{e^N(x-\mu) - \eta_2\left(e^N x - \mu\right)(1-\eta_1)} \tag{31}$$

The Eqs. for η_{NE} and y_{NE} [(28) and (29)] are coupled and are difficult to uncouple. But we can establish the following bounds for η_{NE} and y_{NE}:

$$o < \eta_{NE} \le \eta_{NI} \tag{32}$$

$$0 < y_{NE} \le \frac{1}{2} \tag{33}$$

with η_{NI} given by the Eq. (21). The inequality (32) is satisfied because $I_y \ge I \ge 1$, where $I = (1/\eta_1\eta_2)$ is the irreversible factor of the non-isentropic cycle (Eq. (20)).

To see the inequality (33), first we write I_x (Eq. (31)) as:

$$I_x = \frac{Ax - B\mu}{Bx - C\mu} = \frac{A}{B} - \mu\frac{B^2 - CA}{(Bx - C\mu)\,B} \tag{34}$$

where $A = \eta_1\eta_2 e^N + 1 - \eta_2$; $B = e^N(\eta_1\eta_2 + 1 - \eta_2)$ and $C = e^N - \eta_2 + \eta_1\eta_2$. Clearly $A, B, C > 0$; $A \le B$ and $B^2 - CA > 0$.

Furthermore, from the last inequality we find that $x > \frac{C}{B}\mu$ if $x > \frac{B}{A}\mu$. Therefore,

$$0 < I_x = \frac{Ax - B\mu}{Bx - C\mu} \le \frac{A}{B} \le 1$$

and the inequality (33) is satisfied.

When $I = 1$ ($\eta_1 = \eta_2 = 100\%$), we obtain the values:

$$\eta_{NE} = 1 - \sqrt{\mu}$$

$$y_{NE} = \frac{1}{2}$$

corresponding to the endoreversible Brayton cycle. Therefore, Eqs. (28) and (29) generalize Eqs. (26).

The optimal allocation (size) of the heat exchangers has the following asymptotic behavior:

$$\lim_{N\to\infty} y_{NE} = \frac{1}{2}$$

That is, y_{NE} is asymptotic to the value of $\frac{1}{2}$ found by Bejan [1], as the inventory of the total number of heat transfer units is increased. This differs from the result obtained by Swanson ([7]).

Also, the efficiency η_{NE} has the following asymptotic behavior:

$$\lim_{N\to\infty} \eta_{NE} = \eta_{NI}$$

where η_{NI} is the efficiency that maximizes work in the Brayton cycle with only internal irreversibilities (see Eq. (21))

4. Numerical results

In Ref. 4 the influence of the isentropic efficiencies η_1 and η_2 on the maximum power output is established for a non-isentropic Brayton cycle. Several realistic values were taken for the compressor and the turbine efficiencies, all of them above 0.90. In Ref. 9 the power optimization of an irreversible Brayton heat engine is discussed, taking the compressor efficiency as $\eta_2 = 0.85$ and the turbine efficiency as $\eta_1 = 0.9$. We can then, take the following realistic values for the isentropic efficiencies of turbine and compressor: $\eta_1 = \eta_2 = 0.8$, 0.9. For the total number of heat transfer units N, we take the value of 3, so there is a finite difference of temperatures, since $N = 6$ tends to the non-isentropic. Brayton cycle without external irreversibilities.

Also, if we approximate y_{NE} with

$$I_x \backsimeq \frac{A}{B} < 1$$

since A/B is an upper bound of I_x and $A = B$ if $\eta_1 = \eta_2 = 100\%$ [see Eq. (34)], we obtain:

$$y_{NE} \backsimeq \frac{1}{2} + \frac{1}{2N}\ln\left(\frac{A}{B}\right) \tag{35}$$

And with $\eta_1 = \eta_2 = 0.8, 0.9$, we get Fig. 2.

Now, with the same approximation (Eq. (35)) in Eq. (30) and (28), we obtain the Figs. 3 and 4.

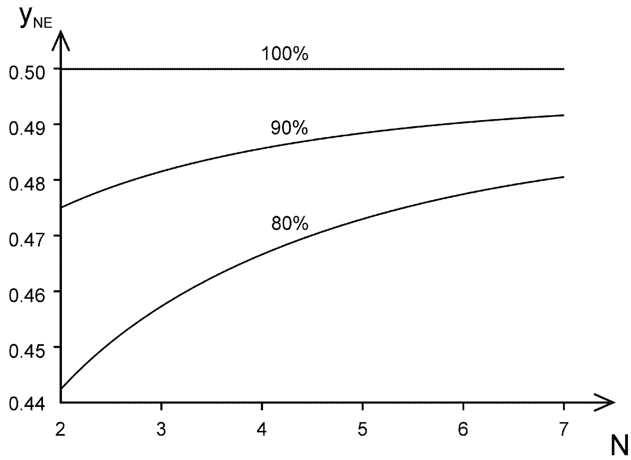

FIGURE 2. Behavior of y_{NE} versus N, using the approximation $I_x \backsimeq (A/B)$ in Eq. (29) $\eta_1 = \eta_2 = 0.8$, 0.9.

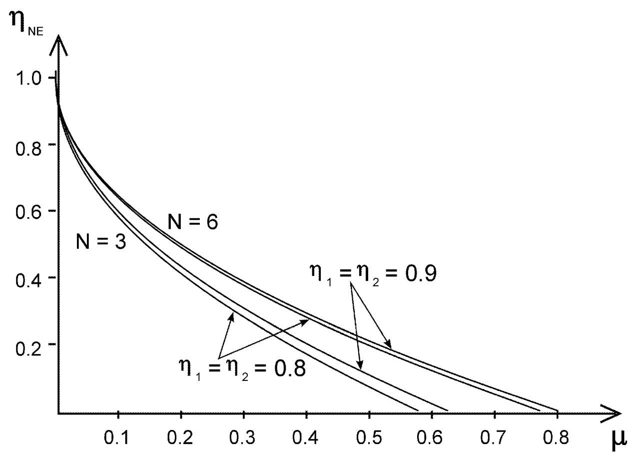

FIGURE 3. Behavior of η_{NE} versus μ, using the approximation $I_x \backsimeq (A/B)$ in Eq. (29). $\eta_1 = \eta_2 = 0.8, 0.9$ and $N = 3, 6$.

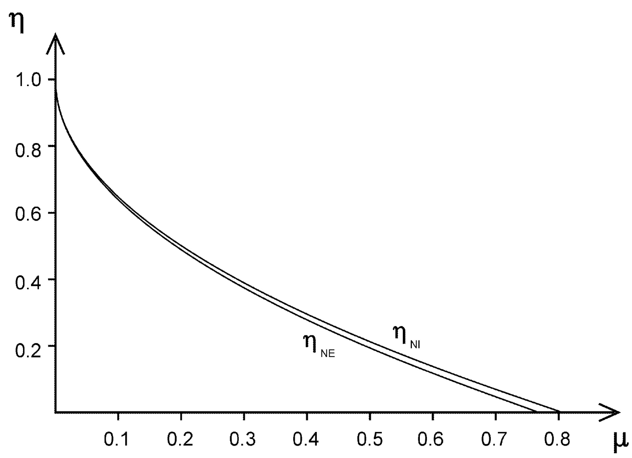

FIGURE 4. Behavior of η_{NE} and η_{NI} versus μ, using the approximation $I_x \backsimeq (A/B)$ in the Eq. (29). $\eta_1 = \eta_2 = 0.9$ and $N = 3$.

In Fig. 2 we can see the behavior of the allocation (size) of the heat exchangers in the hot and cold sides. In the Figs. 3 and 4 we can see that the efficiency η_{NE} can be well approached by the efficiency η_{NI}.

5. Conclusions

The results found provide us with important information about the performance of Brayton-like cycles, including the ideal cycle and those with internal and external irreversibilities (endoreversible and non-endoreversible).

This study combines the first and second law in order to develop new analytical expressions, both for efficiency and allocation, when we optimize work output by the $\varepsilon - NTU$ method. Optimization is carried out by the use of temperature ratio (x) and optimal allocation (size) for the heat exchangers (y).

The analysis done in this paper for the non-endoreversible Brayton cycle generalizes the Bejan model for irreversible power plants, and it is asymptotic to Bejan's value, for the allocation. This result is more realistic than the one obtained by Swanson.

The maximum work output occurs when the NTU increases ($N = 6$). This happens because of the temperatures in the cycle (T_1, T_3) tend toward the temperatures of the heat exchangers (T_L, T_H). This situation can be reached just if the heat exchangers operate with phase change or greater flow rates.

The analytical expressions found for the efficiency that develops maximum work and the optimal allocation (size) of heat exchangers are:

$$\eta_{NE} = 1 - \sqrt{I_y \mu}$$

$$y_{NE} = \frac{1}{2} + \frac{1}{2N} \ln (I_x) \, .$$

The procedure for bounding these coupled expressions and their asymptotic behavior are outstanding, since with these expressions a more realistic approximation than Bejan's model is obtained, moreover this model could be adjusted. We find some expressions for efficiency and allocation (size) that can be used for real gas turbine plants. In particular, our results can be used to maximize the work output relating it to the temperature ratio, optimal heat exchangers allocation (size) with the endoreversible cycle ($y = 0.5$), or optimal heat exchangers allocation (size) with the non-endoreversible cycle ($y_{NE} = 1/2 + (1/2N) \ln I_x$).

The approximation made for y_{NE}, with realistic values for the isentropic efficiencies of turbine and compressor (see Fig. 2), gives an allocation for the heat exchangers y That is approximately $2 - 4\%$ less than Bejan's value. This result shows that the size of the heat exchanger in the hot side decreases. This optimal allocation will lead to better power plant designs, lower pumping and maintenance costs.

Finally, we can apply the method developed in Ref. 13 with the expressions found (Eqs. (28) and (29)) to maximize the efficiency, with respect to the pressure ratio and the total inventory of the heat transfer units, of this cycle. Also, the results of this work can be compared, to the results obtained for regenerative Brayton cycles in Ref. 16 and extended. This work is under way.

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