On the necessity of the postulate of the collapse of the wave function: an explicit calculation of the "double-slit" problem

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Recibido el 7 de diciembre de 2004; aceptado el 28 de febrero de 2005

Based on numerical calculations of the two-slit problem, we object van Kampen's proposal that the collapse of the wave function is an unnecessary postulate of Quantum Mechanics if the measuring apparatus is included in a unitary evolution of the system-apparatus. We argue that van Kampen's interpretation does not substitute the collapse postulate, it is not precise and it appears impracticable.

Keywords: Wavepacket collapse; quantum measurement.

Basados en cálculos numéricos del problema de las dos rejillas, objetamos la propuesta de van Kampen de que el colapso de la función de onda sea un postulado innecesario de la Mecánica Cuántica si el aparato de medición se incluye en una evolución unitaria del conjunto sistemaaparato. Argüimos que la interpretación de van Kampen no sustituye el postulado del colapso, no es precisa y parece ser impracticable.

Descriptores: Colapso de la función de onda; medición cuántica.

PACS: 03.65.Bz

The meaning of the collapse of the wave function is already an old and unabated discussion. In the formalism set up by von Neumann [1], and in fact already pointed out by Heisenberg [2], it is discussed that there is a need for an additional postulate that establishes that once a measurement of a quantum system has been performed, the unitary evolution of the system wave vector ceases to be valid and the state suddenly collapses and takes one of the allowed eigenstates of the operator corresponding to the measured quantity. What constitutes a *measurement* and how the system takes on the given value of the wave function are also age-old questions and there is a plethora of discussions in the literature [3]. It is certainly not our purpose to discuss them here. There is, however, an alternative point of view very clearly summarized by van Kampen [4] and certainly advocated by other authors, in which it is argued that there is actually no need to stipulate the additional postulate of the collapse of the wave function. Instead, the alternative idea is based on the fact that the measuring apparatus is also a quantum system and that the act of measuring is just an interaction with the system under investigation. Therefore, the full description of the combined system-apparatus should yield and explain the "collapse" of the wave function. The requirement for the measuring apparatus to be thus considered is that it should have many degrees of freedom and that initially be prepared in a metastable state, such that the measurement leaves it in a stable state and the transition becomes irreversible in the macroscopic sense. One first ambiguity on van Kampen's scheme is that the macroscopic states of the apparatus are not precisely defined, or identified, with single quantum eigenstates; rather, they are loosely defined in terms of collections or densities of states. Because of the many degrees of freedom involved, the macroscopic states tend to lose coherence among themselves such that after a certain characteristic time (i.e. the coherence time) the interference among those states is almost negligible. Hence, for times after the system-apparatus interaction took place, longer than the coherence time, the probabilities for occurrence of the macroscopic states can be considered as "classical". That is, even though there may be many possible macroscopic states, only one will occur without any possible interference from the other macroscopic states. Since each macroscopic state is entangled with a given state of the microscopic system under investigation, following van Kampen, one can identify that when the interaction took place the wave function of the microscopic system collapsed, it collapsed from the wave function before the interaction with the apparatus to that entangled with the given macroscopic state of the apparatus.

Although the previous explanation is very appealing and appears to demote the postulate of the wave function collapse to a *derived* concept, we do not find it satisfactory. The purpose of this note is to raise a several-fold objection to van Kampen's interpretation of the collapse of the wave function. In the same vein as van Kampen's, it is not our intention to philosophize about Quantum Mechanics, but rather to limit ourselves as to how Quantum Mechanics is and should be used in understanding physical phenomena. It is mainly in this sense that we find that van Kampen's interpretation does not substitute for the collapse postulate, it is not precise, and it appears impossible to put in to practice.

The collapse of the wave function and the problem of measurement are intimately linked but they are not exactly the same problem. The first is the necessary concept of making contact between, on the one hand, theoretical descriptions and predictions of the evolution of a *closed* system under a given system Hamiltonian and, on the other hand, a given experimental situation that is supposed to be able to extract the eigenvalues and probabilities of the different states of the system *without* the apparatus. The concept is necessary because of the nature of the quantum mechanical description of nat-

ural phenomena, the uncertainty principle, the core of Quantum Mechanics, can only be consistent with a multitude of realizations rather than with a single one. Moreover, Quantum Mechanics is very precise as to all the possible outcomes of a given measurable variable and of their probabilities of occurrence, but it is silent as to which of them will actually be realized. For the purposes of discussing our objections to van Kampen's arguments, we would also like to pose the concept of the collapse in an equivalent manner. That is, one can argue that one does not need to follow the time evolution of the wave function and that what really matters is the measurable quantities such as the eigenvalues of the observables and the transition probabilities between given initial and final states (corresponding to a complete set of commuting observables, or, in the standard interpretation, to a complete set of specified measurements) notwithstanding what happens in the interim. This is equivalent to the collapse since the transition probabilities indicate the multitude of the final states and in a given experiment only one of them occurs.

The measurement problem deals more with the loss of coherence among macroscopic states. That is, what van Kampen's has clearly shown is that interaction of microscopic systems with macroscopic objects leads, to a large extent, to a conversion of quantum probabilities into classical probabilities and that this result can be used to understand how a measuring apparatus works. Indeed, the Schrödinger cat "paradox" can easily be resolved in this way: the cat acts as a measuring apparatus for the decaying atom that triggers the "killing" mechanism. Thus, van Kampen's measurement description is very helpful in understanding how macroscopic, "classical" objects (such as ourselves) appear to follow the rules of classical probabilities (based on our ignorance) rather than those of quantum mechanics with their "nonintuitive" interference effects.

In order to show our objections explicitely we shall analyze the Young two-slit problem using a (streamlined) version of van Kampen's measuring apparatus. We mention that van Kampen himself [5] has given an approximated calculation of his model, and for that matter Feynman in his Lectures [6] also describes those results. Here we show an exact (numerical) solution of the diffraction of a wave packet through a two-slit screen with and without the presence of a measuring apparatus. Let us briefly review the model for the two-slit experiment and the typical explanation given when trying to find out which slit the particle went through [7]. A two-dimensional "electron" initially prepared in a wave packet state (or in a plane-wave state) incides normally on a screen with two "holes" and its position is registered on a "photographic plate" placed on the other side of the screen, see Fig. 1. Under these conditions the probability of finding the particle on the plate, at any given time, shows the interference caused by the presence of the two slits. It is argued that if one tries to "see" which hole the particle went through, the interference is destroyed. (For definiteness, let us consider the case where one looks for the electron after it passes through the holes.) The simplest explanation is that the measuring apparatus that detects the position of the electron makes the wave function *collapse* to that position and, from there, a new wave function fans out. Since the detector is "on the other side" of the screen, the new wave function does not pass through the holes and, therefore, shows no interference. Van Kampen argues that it is unnecessary to appeal to the collapse of the wave function. Instead, he proposes that the measuring apparatus be considered as part of an overall unitary system-apparatus description. The "collapse" is then taken into account by the observation that the states of the detector, consistent when there is no detection, are orthogonal to the states when there is detection. Even though the apparatus is in a superposition of states, it is assumed that there can be no interference between those states by appealing to the behavior of macroscopic systems. When there is detection, van Kampen shows generally [4] that the interaction with the apparatus acts as a "source" and a wave function indeed fans out from the position of the apparatus. In the process of "measuring" the detector makes an irreversible transition that "permanently" registers the position of the electron. However, and because of the unitarity of the evolution, we emphasize that for the cases when the apparatus does not make the irreversible transition, and purposely does not detect, it still does detect. That is, even when it is not "detected", the resulting electron wave function is nevertheless modified from the case when there was no detector at all. In our opinion this poses a serious objection to van Kampen's arguments [4]. van Kampen himself [4] points out to this aspect but somehow plays it down and his argument is that, in any case, when the "detection" is successful the collapse occurs. We shall discuss below whether one can say



FIGURE 1. A sketch of the system, showing the box where the evolution is calculated indicating the position of the screen with the two slits. The two positions of the atom refer to the cases of Figs. 3 and 4. The dotted line is the position x_0 where the registering plate is placed. To the left of the screen the initial wave packet is also sketched.

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that in both cases, *i.e.* detection and no detection, the wave function collapses. We shall argue that this ambiguity at the macroscopic level indicates that the necessity of the collapse has not been avoided. This fact, namely, that the detector influences the evolution of the system even if the detection does not occur, has already been discussed by other authors, see Refs. [8–10], in the search for a satisfactory understanding of the measurement problem.

We now present the model and the exact (albeit numerical) results. As stated above the *system* is one electron and a screen with two slits. The measuring apparatus is an atom *together* with the electromagnetic radiation field. The atom is initially prepared in an excited state with a forbidden transition to the ground state. If the electron passes near the atom, the latter becomes polarized and the atomic transition is then possible with the emission of a photon in any direction. The photon can be further registered, but van Kampen indicates that it is not necessary to include it since the decayed atom acts as a permanent record of the passage of the electron; we return to this point below. Considering the energy of the atom in the ground state to be zero and Ω in the excited state, the Hamiltonian of the system-apparatus is ($\hbar = c = m = 1$)

$$H = -\frac{1}{2}\nabla^{2} + V_{s}(\vec{r}) + \frac{1}{2}\Omega(1 + \sigma_{z}) + \sum_{\vec{k}} k a_{\vec{k}}^{\dagger} a_{\vec{k}} - iu(r)\sigma_{x} \sum_{\vec{k}} v_{\vec{k}} \left(a_{\vec{k}} - a_{\vec{k}}^{\dagger}\right), \quad (1)$$

where $V_s(\vec{r})$ is the potential of the two-slit screen; the third and the fourth terms represent the atom and the radiation free Hamiltonians and the last term is the interaction between the electron and the apparatus; σ_i are the Pauli matrices. The interaction term is the product of the electric field times the dipole moment operator and whose strength depends on the function u(r); this, in turn, is supposed to be different from zero only in a small region of space. The coefficients v_k involve the normalization of the field modes and the form factor for the allowed values of \vec{k} in the dipole approximation.

The states of the overall system-apparatus may be written as

$$|\Psi(t)\rangle = \phi(\vec{r},t) |+;0\rangle + \sum_{\vec{k}} \psi_{\vec{k}}(\vec{r},t) |-;\vec{k}\rangle$$
 . (2)

The state $|+;0\rangle$ represents the atom in the excited state and no photons, while the state $|-;\vec{k}\rangle$ is the atom in the ground state and one photon with momentum \vec{k} . The amplitudes $\phi(\vec{r},t)$ and $\psi_{\vec{k}}(\vec{r},t)$ are the electron wave functions when there is no atomic transition and when there is, respectively. The orthogonality of the atom-radiation states guarantees no interference between these electronic amplitudes. On this point, we must assume that, *in practice*, it would be very difficult to produce a further measurement on the apparatus that would yield interference between those states. Since both the measuring atom and the wave packet have finite spatial extensions, there is a finite time during which the transition can occur. This may be considered to be the coherence time in this model.

The Schrödinger equation for this problem yields the following set of equations:

$$i\frac{\partial}{\partial t}\phi = \left(-\frac{1}{2}\nabla^2 + V_s(\vec{r}) + \Omega\right)\phi - iu(\vec{r})\sum_{\vec{k}}v_{\vec{k}}\psi_{\vec{k}} \quad (3)$$

and

$$i\frac{\partial}{\partial t}\psi_k = \left(-\frac{1}{2}\nabla^2 + V_s(\vec{r}) + k\right)\psi_{\vec{k}} + iu(\vec{r})v_{\vec{k}}\phi, \quad (4)$$

for all \vec{k} . In order to be able to perform explicit numerical calculations we now make further assumptions on the apparatus. These assumptions, although making it lose some of its generality, preserve the main requirements to qualify it as an "apparatus". These are: (a) the atomic transition is very sharp in energy; that is, the width of the transition line is considered much smaller than any other relevant energy parameter. This implies that as the atom makes the transition with energy Ω , the frequency of the emitted photon is $k \approx \Omega$. (b) The emission is isotropic; that is, there is equal probability for emission in any direction. Assumptions (a) and (b) are implemented through the value of the coupling coefficients $v_{\vec{k}}$. Namely, we approximate $v_{\vec{k}} = v_k \approx v$ if $k \approx \Omega$, and $v_{\vec{k}} \approx 0$ otherwise. Note that, still, there are an *infinite* number of possible directions for the emitted photons, and thus, one still faces an infinite number of equations (3) and (4). However, using a very particular choice of initial conditions the problem can drastically be reduced to only two equations: far in the past, the overall state is such that the electron is in a state $\phi(\vec{r}, t \to -\infty)$, the atom in the excited state $|+\rangle$, and the radiation field with no photons present,

$$|\Psi(t \to -\infty)\rangle = \phi(\vec{r}, t \to -\infty) |+;0\rangle$$
 . (5)

This implies that $\psi_{\vec{k}}(\vec{r}, t \to -\infty) = 0$ for all \vec{k} . We now choose the initial state of the electron $\phi(\vec{r}, t \to \infty)$ as a wave packet localized far away from the two-slit screen. Using this initial condition in Eqs.(3) and (4) together with assumptions (a) and (b), it is very easy to check that for all $k \neq \Omega$, $\psi_{\vec{k}}(\vec{r}, t) = 0$ for all time. For $|\vec{k}| = \Omega$ all $\psi_{\vec{k}}(\vec{r}, t)$ are indeed different from zero, but they are all *identical*. In words this means that *if* the atom emits, the probability of further finding the electron at a given position \vec{r} is *independent* of the direction in which the photon was emitted. Thus, we need to solve *only* two equations of the infinite set (3) and (4). This can clearly be seen if we set

$$\psi_{\vec{k}}(\vec{r},t) = \psi(\vec{r},t) \left(\sum_{k=\Omega}\right)^{-1/2} \tag{6}$$

and

$$\tilde{v} = v \left(\sum_{k=\Omega}\right)^{1/2},\tag{7}$$

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where $\sum_{k=\Omega}$ is the sum of all the electromagnetic modes with wave vector magnitude equal to Ω . The coupled equations to be solved now are

$$i\frac{\partial}{\partial t}\phi = \left(-\frac{1}{2}\nabla^2 + V_s(\vec{r}) + \Omega\right)\phi - iu(\vec{r})\,\tilde{v}\,\psi \qquad (8)$$

and

$$i\frac{\partial}{\partial t}\psi = \left(-\frac{1}{2}\nabla^2 + V_s(\vec{r}) + \Omega\right)\psi + iu(\vec{r})\,\tilde{v}\,\phi,\quad(9)$$

subject to the aforementioned initial conditions. It can also be verified that while the overall state is given by Eq.(2), the normalization condition reduces to

$$\int d^3r \left|\phi(\vec{r},t)\right|^2 + \int d^3r \left|\psi(\vec{r},t)\right|^2 = 1$$
 (10)

with ϕ and ψ being not orthogonal, in general. We stress that while the apparatus is still macroscopic in the sense of having a very large number of degrees of freedom, the analytic simplification to only two amplitudes is not only a consequence of assumptions (a) and (b) above, but also a consequence of the peculiar initial condition. The irreversibility of the measurement remains guaranteed, since once the atom emits it remains in the ground state $|-\rangle$.

We now proceed to discuss the numerical results of Eqs.(8) and (9). The equations were solved using the splitoperator method [11]. In Fig. 1 we show the position of the screen with the slits and the position of the atom. In order to facilitate the numerical calculation, the "walls" of the screen are not infinite but rather a very high and sharp Gaussian function. We have verified, not shown here, that the tunneling through those barriers is completely negligible for the times that we choose to analyze the wave functions. The strength function u(r) of the atom in Eqs.(8) and (9) is also taken to be a Gaussian function. The component $\phi(\vec{r}, 0)$ of the initial state of the electron is a minimum uncertainty wave packet in the propagating direction x and constant in the y direction, parallel to the screen. The center of the wave packet moves initially with a given velocity pointing towards the screen. The component $\psi(\vec{r}, 0)$ of the initial state is chosen to be zero. We chose arbitrary values for all variables such that the numerical procedure worked well and the results were easily interpretable.

In the following set of figures, we show the probabilities $|\phi(\vec{r}, t_0)|^2$ and $|\psi(\vec{r}, t_0)|^2$ of finding the electron at the position $\vec{r} = (x_0, y)$ with x_0 a fixed position to the right of the screen and as a function of the vertical variable y. The time t_0 is arbitrary but the same for all figures.

In Fig. 2 we show the probability $|\phi_f(\vec{r}, t_0)|^2$ for the case when *there is no atom* and, therefore, the component $\psi(\vec{r}, t)$ equals zero all the time. We can clearly see the expected symmetric interference pattern, with some diffraction due to the finite width of the slits. This is the result with which we shall compare the following cases.



FIGURE 2. Probability $|\phi_f(x_0, y, t_0)|^2$ of finding the electron at the registering plate placed in x_0 , see Fig. 1, for the case when *there is no atom*.

Figures 3 shows the probabilities $|\phi(\vec{r},t_0)|^2$ and $|\psi(\vec{r},t_0)|^2$ when the detecting atom is placed *near* one of the slits. The cases (A) and (B) correspond to different strengths of the interaction of the electron with the atom. We note that as the strength is increased, the wave function $\psi(\vec{r},t_0)$ of the "detected" electron is more noticeable and, indeed, it shows no interference (though again, some diffraction due to the finite size of the holes, see below). At the same time, the wave function $\phi(\vec{r},t_0)$ of the electron that was not "detected" becomes more and more deformed precisely in the region where the detected part appears, clearly showing the presence of the atom. This can easily be seen by comparing Figs. 2 and 3.

The set of Figs. 4 show the probabilities $|\phi(\vec{r}, t_0)|^2$ and $|\psi(\vec{r}, t_0)|^2$ when the detecting atom is placed exactly in the middle between the slits. Again, the different cases correspond to increasing electron-atom interaction strength and, once more, the presence of the atom in *both* probability amplitudes is evident. For a clearer conclusion of this point we ask the reader to compare Fig. 2 with the probability $|\phi(\vec{r}, t_0)|^2$ of Figs. 3 and 4.

We would like to point out to the small oscillations of the amplitude of $\psi(\vec{r}, t_0)$ in Figs. 3 and 4. These oscillations are due to the diffraction caused by the finite sizes of the slits and the detecting atom. That is, those oscillations are not a sign of interference of the wave function passing through both slits. In Figs. 3a and 3b the diffraction is more clearly seen because the atom is placed "asymmetrically" with respect to the slits (notice that the oscillations are also asymmetric). In Fig. 4a the effect is not seen at the scale of the plot but in Fig. 4b the effect is again visible because of the larger intensity of the electron-atom interaction, and it is symmetric in this case.

It is also instructive to verify that if one chooses "not to see" whether the atom emitted or not, the sum of the probabilities $|\phi(\vec{r}, t_0)|^2 + |\psi(\vec{r}, t_0)|^2$ does not equal the probability $|\phi_f(\vec{r}, t_0)|^2$ when there is no atom. That is, contrary to many pedagogical and qualitative discussions, the result is different if the detector is present from the case when there is no de-



FIGURE 3. Probabilities of finding the electron at the registering plate placed in x_0 , when the atom is placed *above* one of the slits, see Fig. 1. $|\phi(x_0, y, t_0)|^2$ is the pattern when the atom *does not* emit a photon, and $|\psi(x_0, y, t_0)|^2$ is when the atom *emits* a photon. (A) and (B) refer to different strengths \tilde{v} of the electron-atom interaction.

tector, whether one chooses to see or not, simply because the evolution of the system occurs under different Hamiltonians. See Fig. 5.

Our main objection is that van Kampen's connotation of collapse applies equally well to the case when the detection is successful and to the case when it is not, as opposed to his stance that the collapse occurs only when the detection actually happened. In the figures above one sees that the wave function $\phi_f(\vec{r},t)$ collapses to *either* to $\phi(\vec{r},t)$ or to $\psi(\vec{r},t)$ when the apparatus is present. This is more clearly seen in cases (B) of Figs. 3 and 4. To call only to the case when there is detection one of "collapse", because the wave function seems to spatially collapse to the position of the atom, is arbitrary: The usual connotation of the concept of collapse refers to the reduction of an arbitrary wave function to an eigenstate of the measured observables. The apparatus of van Kampen is designed to measure the position of the electron at the atom's location and, it could be argued, the collapse should only apply to the case when the atom makes the transi-



FIGURE 4. Probabilities of finding the electron at the registering plate placed in x_0 , when the atom is placed in the *center* between the slits, see Fig. 1. $|\phi(x_0, y, t_0)|^2$ is the pattern when the atom *does not* emit a photon, and $|\psi(x_0, y, t_0)|^2$ is when the atom *emits* a photon. (A) and (B) refer to different strengths \tilde{v} of the electronatom interaction.

tion. However, it is evident that the wave function is modified in either case and there is no right to call one case a "measurement" and not the other one. This is a very important objection to van Kampen's arguments; that is, for the case when there is no transition, even though the apparatus remains in a metastable state, one must consider it as "stable" as when the transition occurred. This is the only way that one could a posteriori distinguish one case from the other and attach a "classical" probability $\int d\vec{r} |\psi|^2$ to the "successful" case and $\int d\vec{r} |\phi|^2$ to the "unsuccessful" one. However, even with this observation, van Kampen's idea still seems very attractive because it appears to show that the interaction of a macroscopic system with a microscopic one always "collapses" the wave function of the latter, in the sense that after the interaction has taken place (and for times longer than the coherence time) the different macroscopic states occur with "classical" probabilities. However, the origin of the probabilities is not classical since there is no underlying dynamics that if we knew, we could in principle predict the outcome with probability equal



FIGURE 5. Comparison of the probabilities $|\phi_f(x_0, y, t_0)|^2$ when *there is no atom*, with the sum $|\phi(x_0, y, t_0)|^2 + |\psi(x_0, y, t_0)|^2$ of Fig. 3A. The latter is the total probability of finding the electron at the plate, independently of whether the atom registers or not. Note that these quantities are *different*.

to one. It is impossible because the origin is quantum mechanical. In other words, the macroscopic states also collapse. We have seen that even when the purported measurement did not take place, the apparatus itself also "collapsed" to other of its macroscopic states, and we do not know which one. And again, in a given realization of the experiment, only one state will occur. Thus, we are still facing the "problem" that we can only predict the transition probabilities between initial and final states, which we have argued is equivalent to the concept of the collapse. It is the lack of coherence between macroscopic states, and therefore of interference, that seems to free us from the particle-wave dilemma and, thus, of the apparent need for the collapse. But the collapse concept is concealed there and we have appealed to it in order to give meaning to the realization of the different macroscopic states.

The second objection, not unrelated to the previous one, is that even if we accept that the collapse happened only when the transition occurred, the result is manifestly apparatusand measurement-dependent. In van Kampen's original treatment, Ref. [4], he plays down the role of the photon that the atom emits. It is said that the photon can be registered by a photographic plate that one needs not to include in the description since the decayed atom acts as a permanent record. Indeed. However, if one includes the photographic plate into the unitary description, the interference patterns will be different from the case when it is not included. Thus, we end up with a theory whose results depend on what we use to measure them. Although this critique may seem more of a practical use rather than of a fundamental difficulty, we find that this interpretation, in a sense, defeats the purpose of the theory of Quantum Mechanics. That is, as a theory, the measurable predictions of Quantum Mechanics are not only the eigenvalues of the operators but also the probabilities of such measurements. The values of those probabilities are completely dependent on the Hamiltonian used to evolve the system. The actual probabilities in a given set of actual experiments may serve to deduce, if not the whole state, at least a partial knowledge of it. And in most practical cases one is interested in the probabilities (or states) corresponding to the evolution under a given system Hamiltonian in the absence of the apparatus. In other words, Quantum Mechanics is used in everyday problems (such as atomic spectra, specific heats and superconductivity) to find out states of systems that have attached a particular system Hamiltonian. How one measures the corresponding physical quantities in order to extract the probabilities pertaining to a given state is not a problem of Quantum Mechanics, but rather, it is part of our tasks to be able to subtract the effect of the measuring apparatus and find out the state in the absence of the detector. As a simple example, we all believe it is meaningful and useful to know the natural linewidth of, say, an atomic transition; such knowledge can only be *inferred* from experiments that necessarily include the effects of the surroundings (other than the electromagnetic vacuum) and the measuring apparatus [12].

We find that van Kampen's analysis is an excellent example of an actual measuring apparatus that shows that Quantum Mechanics can indeed describe a measurement; specially the fact that in order to go from "quantum" to "classical" probabilities the apparatus must have a large number of degrees of freedom and that the partial knowledge of the whole wave function yields an increase in entropy of the macroscopic state [4]. And indeed, van Kampen's analysis reflects the fact that a measurement "collapses" the wave function. However, many questions remain blurred and imprecise. For instance, when exactly do the macroscopic states lose their coherence? how precise and definite is this incoherence? how large a system must be to be considered macroscopic? how rigorous and precise can we make these statements as a matter of principle? We should not disregard the experimentalist ingenuity in preparing macrostates with longer and longer coherence times. As a matter of fact, the possible success of the quantum computers will rest on the ability of maintaining many qubits coherent for a long time [13].

To summarize, the point of van Kampen is whether the collapse *follows* from the rules of Quantum Mechanics or not. Our point is that the "problem" of the collapse, and its need, is a consequence of the uncertainty principle, the core of Quantum Mechanics, and we cannot dispose of it *within* the theory. The collapse postulate is what tells us that only one of the realizations occurs. For microscopic systems the collapse concept is more poignant because the interference effects cannot be neglected, but it remains true for macroscopic systems.

Acknowledgment

We acknowledge CONACYT (Mexico) for its partial support from through grant No. 32634-E.

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