Propagation of a photoinduced surface wave along an ideal metal–photorefractive crystal interface

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A mathematical model is developed in order to study self-confinement of the square beam propagating along the boundary of the *Ideal Metal-Photorefractive Crystal* media. It is shown that the square beam is self-bending and can be balanced by internal reflection at the *Photorefractive Crystal* surface to result in surface wave formation. Theoretical evidence is given.

Keywords: Spatial solitons (surface waves); interface; boundary conditions; photorefractive crystal.

Se desarrolla un modelo matemático para analizar el auto confinamiento de un haz de forma cuadrada, propagándose a lo largo entre la frontera entre un cristal fotorefractivo en contacto con un metal ideal. Se muestra que el haz es auto-deflectado y que puede ser balanceado por la reflexión interna en la superficie interna del cristal photorefractivo, resultando en la formación de la onda superficial. Se da evidencia teórica.

Descriptores: Solitones espaciales (ondas superficiales); interfase; condiciones de frontera; cristales fotorefractivos.

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1. Introduction

The propagation of spatial solitons in bulk photorefractive crystals (PRC) has generated a great deal of interest in recent years, since it provides a tool for energy localization in a non-linear medium. The spatial soliton, which is due to the quasi-local drift mechanism of photorefractive non-linearity, has been obtained both experimentally and theoretically [1–4], from the balance between linear diffraction and non-linear self-focusing and several applications have been developed.

An alternative approach to the generation of a spatial soliton-like beam may be based on non-linear surface waves. One way is when a beam falls on the interface of two different media, that is, on the wave trapped in the interface between either a crystal and a metal, or a crystal with the opposite sign of non-linear diffusion and a dielectric with a lower average refractive index than a crystal. The occurrence of self-confinement and the stability of the incident beam on the interface depends on the diffusion non-linearity (of the gradient type) of the photorefractive crystal [5].

The non-linear guiding properties in bulk photorefractive media has recently been a subject for theoretical and experimental research. The response speed of the photorefractive effect is limited by the photorefractive non-linearity. It has been predicted that, for a photorefractive crystal such as $BaTiO_3$, the diffusion mechanism can assure efficient concentration of light power in crystal layers $\sim 10\,\mu$ m thick [5].

At this moment, the spatial confinement of these waves has been theoretically predicted. The guided, spatially confined surface wave along the boundary of the crystals has been previously considered [5]; similarly, the surface wave formation is an eigen-mode problem [6] as is the self-bending of the beam [7]. Symmetrical and anti-symmetrical surface waves formation is induced by a laser beam in a $PRC' - PRC$ media, with the opposite sign of the diffusion non-linearity [5].

Experimental formation of the surface wave in $Bi_{12}TiO_{20}$ and $Bi_{12}SiO_{20}$ crystals was demostrated when the photorefractive surface waves were induced with a Gaussian beam of small diameter [8, 9].

Our attention is focused on the diffusion mechanism produced by the non-linear crystals; the non-linearity is related to the dielectric constant that depends on the spacial position. We are analyzing the one light polarization case where photoconductivity along interface media does not exist, using the square laser beam (beam distribution) propagation; the self-bending beam is spread by some diffraction effects that produce the anti-symmetrical surface wave formation that is formed by the energy beam's self-confinement.

The formation of the surface waves and their stability are produced when a beam is propagated along two media interface, *i.e.* when the photoconductivity of each medium is stabilized. This important because it produces the spatial wave (soliton-like or surface wave) propagation due to a diffusion mechanism of non-linearity (of the gradient type).

The primary goal of our paper is to develop an analytical approach to the practically important problem of laser beam propagation along the interface of two media. The question is whether it is possible to confine the light in a thin layer along the interfaces, namely is it possible to compensate for the self-bending and diffraction spreading by diffusion non-linearity and total internal reflection.

In our analysis, we use a specific beam shape that propagated along the $IM - PRC$ media (IM is the ideal metal), which allows us to obtain and predict a solution, that is the surface wave [5]. That in turn will be obtained in a form of diffraction free propagation, where the analytical model obtained from a non-linear equation is reduced to a linear equation. This is why the non-linearity has a specific form, *i.e.* it is dependent on the light intensity or it is dependent on the electric field amplitude that is real, which allows us to thoroughly determine that the surface wave obtained is a problem of photorefractive eigen-modes [6].

For some cases, it is easier to obtain surface waves experimentally than to predict them theoretically, but in both cases, the obtaining must be by the diffusion mechanism of gradient a type.

The diffusion mechanism is produced by spatially varying the dielectric constant $\varepsilon(x,z) = \varepsilon + \delta\varepsilon(x,z)$, where $\delta\varepsilon(x,z)$ represents a non-linear contribution. This contribution leads to the beam self-bending and if some reflecting surface presents the surface wave formation.

For the beam incidence, the solid media interface can be constituted by photorefractive crystal and ideal metal. For the beam propagation along the interface, the field continuity equations and its field derivative function are applied. Here is demonstrated that the square beam is stabilized by the diffusion mechanism and by the formation of spatial surfaces wave for the $IM - PRC$ media.

The present work is the result of the analytical study of the conditions of formation and propagation of surface waves of the soliton-like form as a linear approximation in $IM - PRC$ media with a diffusion non-linearity (of the gradient type) and mainly focusing on the anti-symmetrical surface wave formation generated by a specific shape laser beam or for the non-symmetrical surface waves, under the conditions they are normally produced.

It is shown that such surface waves shapes depend on the beam shape of the incident laser, also it is shown that under such conditions the existence of the surface waves formation can be predicted (for the one light polarization case).

The fundamental mathematical tool used to solve the differential equation that describes the beam propagation into the photorefractive medium is the Laplace transform for the PRC medium and the Maxwell equations for the interface media.

Below we develop a mathematical model in order to obtain the differential equation, which describes the beam amplitude behavior in each medium, the continuity conditions that satisfies the Maxwell equations in the interface media, the conditions for obtaining the surface wave formation for describing the steady solutions. Finally a dicussion and the conclusions are presented.

2. Mathematical model

Figure 1 shows the linear polarization light, this is when the beam incides in the photorefractive crystals' interface. It is assumed that the boundary between two media has no particular properties such as: surface trapping centers, blocking properties for the charge transfer, surface photogalvanic effects etc.

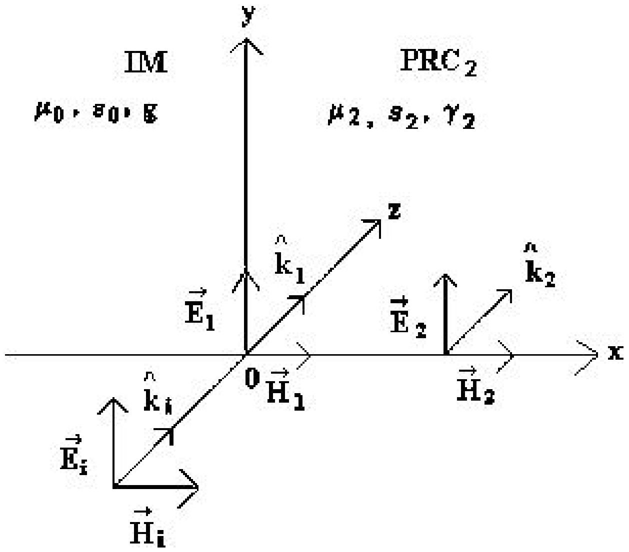

FIGURE 1. Field continuity applying the Maxwel equations in $z = 0$ and continuity conditions in $Z > 0$.

One of the media is an IM and it must be located in the $x < 0$region; the second $-PRC$ in the $x \geq 0$ region, which means both must be in mutual contact.

The wave equation shown below is used for the region $x \geq 0$ for PRC medium. It is a standard scalar three-dimensional equation for monochromatic light wave with frequency ω and complex amplitude $E_1(X,Y,Z,t)$ propagating in an unbonded optically transparent medium with small spatial variations of the dielectric constant $\varepsilon(X,Z)$.

The wave equation for the scalar electric field component is written as:

$$\nabla^2 E_1(X,Y,Z,t) - \mu\varepsilon_0\varepsilon(X,Z)\frac{\partial^2 E_1(X,Y,Z,t)}{\partial t^2} = 0 \quad \text{for} \quad x \geq 0. \qquad (1)$$

and the non-linear properties are describerd by $\varepsilon(X,Z)$.

Three different stages of internal field formation due to charges on transparent metals (electrodes) which are in contact with $PRC's$ are reported [10, 4.6 section]. But in our particular case, none of these affects the surface wave formation. Static electric fields induced by charges on the electrodes are produced for very small diffusion lengths given as

$$L_D = \sqrt{D\tau_f} = \sqrt{\mu\tau_f k_B T e^{-1}} = \sqrt{\mu\tau_f E_D K^{-1}},$$

where D is the diffusion coefficient, μ is the mobility, τ_f is the free carrier life time, k_B is the Boltzman constant, T is the absolute temperature, e the electron charge, E_D is the diffusion field, and K is the spatial frequency [11]. Usually the diffusion length is small in comparison with the surface wave width. For example, in $BaTiO_3$: $L_{D\|C} = 0.08\ \mu$m and $L_{D\perp C} = 0.32\ \mu$m that correspond both crystal orientations, where the C-axis is $\|$ parallel or $\perp$ perpendicular to the grating vector [12] and comparing these results with the surface wave width obtained by Cronin for this crystal (Fig. 4) [7], it turns out that the diffusion lengths are too small compared with the surface wave width. On the other hand, the magnitude of the internal field induced by charges on the electrodes is negligible compared with the field magnitude produced by the diffusion non-linearity. Part of these contact effects is the "bottleneck"; the charge stratification property introduced by this effect has been suggested to design an adaptive photodetector [13]. But as before, this effect does not significantly influence the surface wave formation. For all these reasons, in our next analysis these contact effects will not be taken into account.

Another simplification is that the surface current, flowing exactly on the boundary surface [14], can also be neglected in the paraxial approximation.

The space charge electric field relief to the refractive index change is ensured via a linear electro-optic effect in $\delta\varepsilon(X,Z)$, where:

$$\varepsilon(X,Z) = \varepsilon + \delta\varepsilon(X,Z). \tag{2}$$

Here ε is the spatially uniform average dielectric constant and $\delta\varepsilon(X,Z)$ is the photoinduced changes, $\varepsilon \gg |\delta\varepsilon(X,Z)|$ [5].

The distribution of the electric field amplitude $E_1(X,Y,Z,t)$ does not depend on the longitudinal coordinate Y and is assumed to be a function of the transversal coordinates X, Z. Then for a monochromatic wave we assume a solution of the form:

$$E_1(X,Z,t) = E_1(X,Z)\exp(-i\omega t). \tag{3}$$

Then (1) is transformed to:

$$\frac{\partial^2 E_1(X,Z)}{\partial X^2} + \frac{\partial^2 E_1(X,Z)}{\partial Z^2} + \omega^2\mu\varepsilon_0\varepsilon(X,Z)E_1(X,Z) = 0. \tag{4}$$

Furthermore, we will look for the solution of the form:

$$E_1(X,Z) = E(X,Z)\exp(-i\beta Z) \tag{5}$$

where β is the wave propagation constant [5, 6]; then (4) changes to:

$$\frac{\partial^2 E(X,Z)}{\partial X^2} + \frac{\partial^2 E(X,Z)}{\partial Z^2} - 2i\beta\frac{\partial E(X,Z)}{\partial Z} - (\beta^2 - \omega^2\mu\left[\varepsilon + \delta\varepsilon\left(X,Z\right)\right]\varepsilon_0)E(X,Z) = 0. \tag{6}$$

The following equation defines the photoinduced variation of the dielectric constant given by [5–7]:

$$\delta\varepsilon(X,Z) = n^4 r\frac{k_B T}{e}\frac{\partial \ln|E(X,Z)|^2}{\partial X}. \tag{7}$$

The last is valid under the steady-state conditions of illumination (when nothing is changing with time), and the space charge electric field arises as a result of drift-diffusion equilibrium

$$\delta\varepsilon(X,Z) = n^4 r E_{sc}(X,Z)$$

and

$$E_{sc}(x,z) = -k_B T e^{-1}\frac{\partial \ln I}{\partial x}.$$

For a surface mode with a planar phase [5–7]

$$(I(x,z))^{-1}\frac{\partial\, I(x,z)}{\partial x} = 2(E(x,z))^{-1}\frac{\partial E(x,z)}{\partial x},$$

here I is the light intensity. That transformation of the space charge electric field relief to the index changes is ensured via the linear electro-optic effect; the photoinduced change is $\delta\varepsilon(x,z) = n^4 r E_{sc}(x,z)$. Then, taking into account (7), one can write:

$$\delta\varepsilon(x,z) = 2n^4 r\frac{K_B T}{e}(E(x,z))^{-1}\frac{\partial E(x,z)}{\partial x}. \tag{8}$$

Here (7) is a real quantity, r is the electro-optic coefficient, and n is the average refractive index of the sample.

Parameter $\gamma = k_0 n^2 r k_B T e^{-1}$ characterizes non-linear diffusion intensity, $k_0 = \sqrt{\mu\varepsilon\varepsilon_0}$ is the light wave number in a optically linear medium, and ε is the spatially uniform average dielectric constant. Substituting the last equation into (6) and considering also the approach of slow variation of the amplitude [5, 6]:

$$\left|\frac{\partial^2 E(X,Z)}{\partial Z^2}\right| \ll \left|\beta\frac{\partial E(X,Z)}{\partial Z}\right|, \tag{9}$$

we have:

$$k_0^{-2}\frac{\partial^2 E(X,Z)}{\partial X^2} + D'\frac{\partial E(X,Z)}{\partial X} - 2bk_0^{-1}i\frac{\partial E(X,Z)}{\partial Z} - CE(X,Z) = 0 \tag{10}$$

where $C = (\beta^2 - k_0^2)k_0^{-2}$, and $D' = 2\gamma k_0^{-1}$.

Assuming that C=0, then b=$\beta k_0^{-1} = 1$, for β=k_0 and using normalized coordinates: x=$k_0 X$ and z=$2^{-1}k_0 Z$, Eq. (10) transforms to:

$$\frac{\partial^2 E(x,z)}{\partial x^2} + D\frac{\partial E(x,z)}{\partial x} - ib\frac{\partial E(x,z)}{\partial z} = 0, \tag{11}$$

The last equation describes the beam propagation only for $x \geq 0$, with $D = 2\gamma$.

Here the amplitude of optical electric field $E(x, z \longrightarrow \infty)$ is assumed to be real, corresponding to a problem of the photorefractive eigen-modes. For the ideal metal medium in the region $x < 0$ we have:

$$E_{IM}(x,z) = 0 \tag{12}$$

3. Boundary and initial conditions

Further we will consider a light beam with linear polarization along the $y-axis$ (Fig. 1).

For $z>0$ only, the optical electric field has only a tangential component. From the continuity conditions for the $IM-PRC$ case we have

$$E_1(x,z)|_{x=0}=G_1(z)=G(z)=0 \tag{13}$$

and

$$\frac{\partial E_1(x,z)}{\partial x}|_{x=0}=F(z)\ for\ z>0 \tag{14}$$

where $G_1(z), G(z)$ represents the boundary conditions for the photorefractive media and ideal metal respectively on the interface media, and $F(z)$ represents the derivative of the electric field distribution on the interface as a boundary condition too.

The input optical electric field distribution is expressed, by:

$$E_1(x,z)|_{z=0}=K_1(x). \tag{15}$$

This condition is only valid in the region z=0, which satisfies the Maxwell equations in the three media $(AIR-IM-PRC)$ (see Fig. 1).

4. Differential equation solution

To solve (11) and to obtain a non-stationary solution (with respect to the $z-$ variable), two methods already known can be applied: the Laplace and Fourier transform. However the latter is more complicated to use in the case under consideration. So we apply the Laplace Transform with respect to the variable z, $\mathcal{L}(E(x,z))=E(x,s)$. As a result, we obtain a second order non-homogeneous differential equation, for the region $x\geq 0$

$$\frac{d^2E(x,s)}{dx^2}+D\frac{dE(x,s)}{dx}-ibsE(x,s)=-ibK_1(x) \tag{16}$$

Applying the continuity conditions (13) and (14), to (16) we get:

$$E(x,s)=\exp(-\frac{Dx}{2})\left[G_1(s)\cos\left\{\frac{\beta_1(s)x}{2}\right\}\right.$$
$$\left.+H_1(s)\sin\left\{\frac{\beta_1(s)x}{2}\right\}\right]+g_1(x,s) \tag{17}$$

where

$$H_1(s)=[DG(s)+2F(s)]\,\beta_1^{-1}(s), \tag{18}$$

so that $g_1(x,s)$ is the non-homogeneous solution of (16), in the space (x,s), with the constant

$$\beta_1=-\frac{i}{2}\sqrt{D^2+4ibs}=-i\sqrt{\gamma^2+ibs} \tag{19}$$

and

$$g_1(x,s)=2ib\int_0^x K_1(t)\exp\left[-\frac{D}{2}(x-t)\right]$$
$$\times\frac{\sin\left[\beta_1(s)(x-t)\right]}{i\beta_1(s)}dt \tag{20}$$

Now we apply the inverse Laplace transform to (20), *i.e.*

$$\mathcal{L}_s^{-1}g_1(x,s)=g_1(x,z) \tag{21}$$

Eq. (16), where the solution to the non-homogeneous part is:

$$g_1(x,z)=\sqrt{\frac{ib}{4\pi z}}\exp(-\eta z)\int_0^x K_1(t)$$
$$\times\exp\left[-\frac{ib(x-t)^2}{4z}-\frac{D(x-t)}{2}\right]dt \tag{22}$$

where $\eta=D^2(4ib)^{-1}$ and

$$\mathbf{G}(x,z,t,0)=\sqrt{b(4i\pi z)^{-1}}\exp\left[-\frac{1}{2}ib(t-x)^2(2z)^{-1}\right]$$
$$\times\exp\left[\frac{1}{2}D(t-x)\right] \tag{23}$$

represents the Green function [14, 15]. Using this function, we can rewrite (22) as:

$$g_1(x,z)=\exp(-\eta z)\int_0^x K_1(t)\mathbf{G}(x,z,t,0)dt \tag{24}$$

5. Square beam and convolution integrals

Furthermore we will consider a square beam of the form:

$$K_1(x)=\begin{array}{c}0\ \text{if}\ \ 0\leq x<\zeta\\ g_0\ \text{if}\ \ \zeta\leq x\leq h\\ \text{zero in other cases}\end{array} \tag{25}$$

as an initial condition. Applying the inverse Laplace transform to the homogeneous part of (17), we get:

$$E(x,z)=\exp\left(-\frac{Dx}{2}\right)[E_I(x,z)+E_{II}(x,z)$$
$$+E_{III}(x,z)]+g_1(x,z) \tag{26}$$

where

$$E_I(x,z)=\frac{\tau x}{2\sqrt{\pi}}$$
$$\times\int_0^z\left[G(z-u)\exp\left(\frac{\tau^2x^2}{4u}-\eta u\right)\right]u^{\frac{-3}{2}}du \tag{27}$$

$$E_{II}(x,z) = \frac{D}{2\sqrt{\pi}} \times \int_0^z \left[G(z-u) \exp\left(\frac{\tau^2 x^2}{4u} - \eta u \right) \right] u^{\frac{-1}{2}} du \quad (28)$$

$$E_{III}(x,z) = \frac{1}{\sqrt{\pi}} \times \int_0^z \left[F(z-u) \exp\left(\frac{\tau^2 x^2}{4u} - \eta u \right) \right] u^{\frac{-1}{2}} du. \quad (29)$$

Here $\tau = 2i\sqrt{ib}$.

The integrals (27), (28) and (29), represent the convolution of initial conditions.

6. Analytical solution for the amplitude $E(x,z)$

Evaluating the integrals for the region $x \geq 0$, we obtain the solution of (10), that is:

$$E_{PRC}(x,z) = f_1 g_0 [f_2(f_3 + f_4) - f_5(f_6 + f_7) - f_8(f_9 + f_{10}) + f_{11}(f_{12} + f_{13})] \quad (30)$$

for $z > 0$. (see Appendix A and see Figs. from 2 to 4). (the erf function has its complex argument, so now for an approximation of it, we can use [16]).

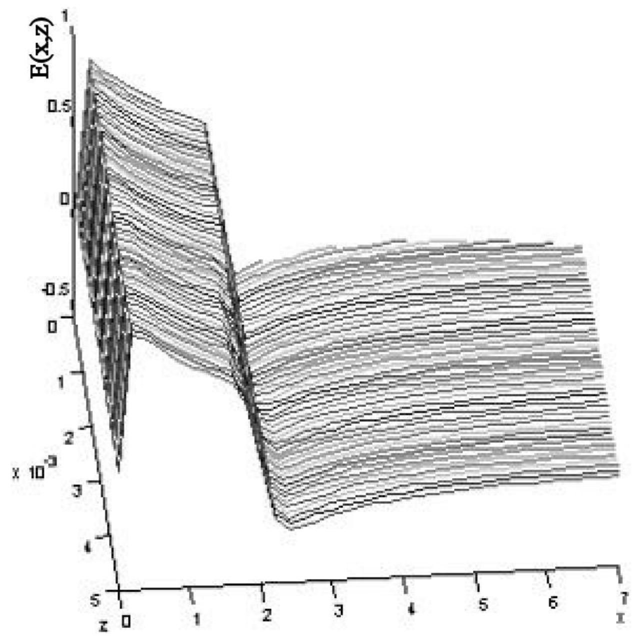

FIGURE 2. Beam propagation near to incidence plane $IM-PRC$, and the self-bending beam begins with D=0.7.

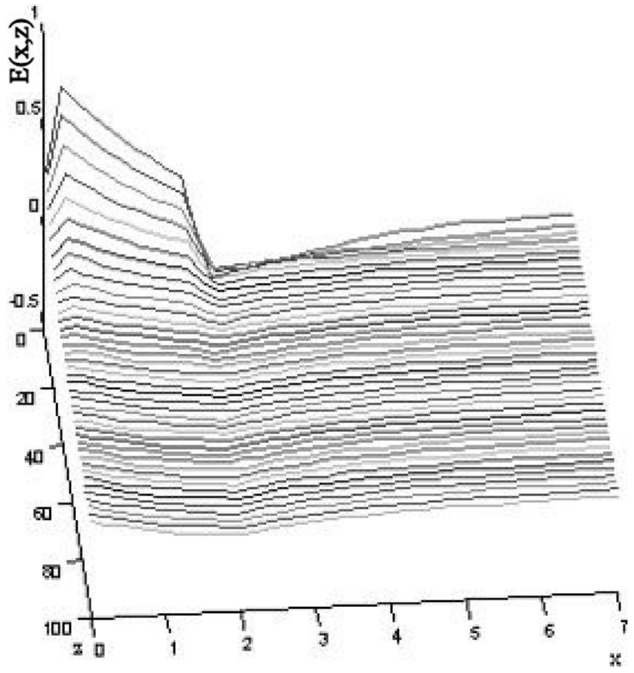

FIGURE 3. The formation of the surface wave in $IM-PRC$ starts, with D=0.7, the characteristic length of the surface wave formation decreases when the diffusion coefficient increases.

For the analysis of the asymptotic behavior of solution (30), we used the series expansion of the erf functions with a complex large values of z are considered (see Figs. 2 and 4).

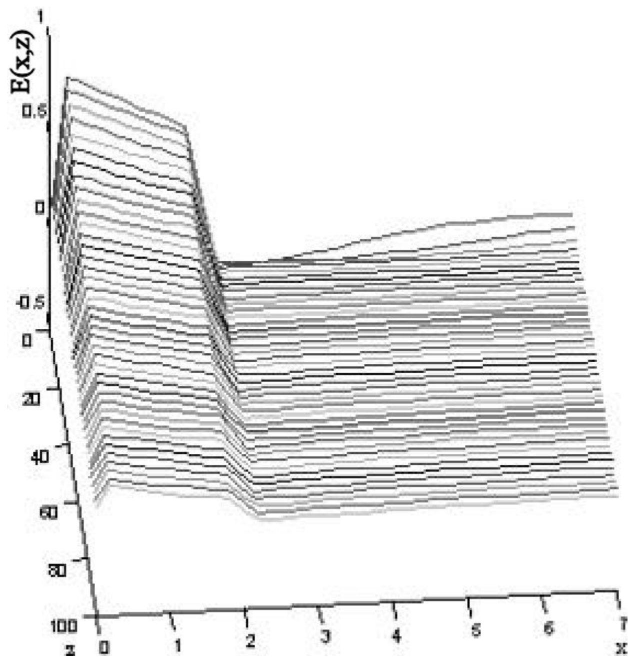

FIGURE 4. The formation of the surface wave in $IM-PRC$ starts, with D=0.3, the characteristic length of the surface wave formation increases when the diffusion coefficient decreases.

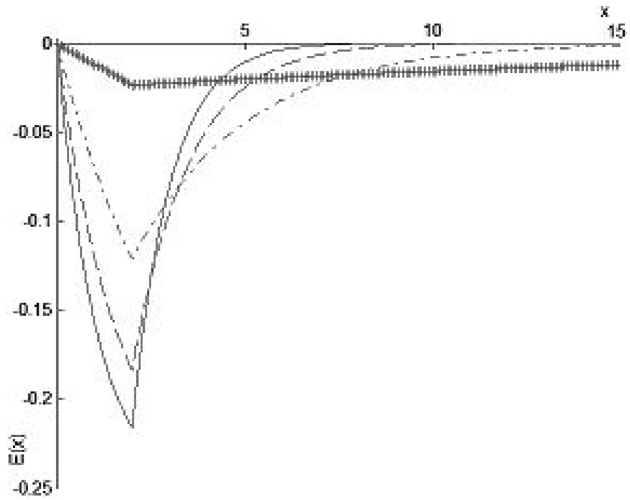

FIGURE 5. The surface waves set with D=0.05(depicted with+), D=0.333 (depicted with-), D=0.666 (depicted with-) and D=0.999 (depicted with continuous line), which represent the increase of the photorefractive response speed.

Now applying the limit when $z \to \infty$ in (30), we obtain the stationary solution, namely the surface wave:

$$\lim E(x, z \to \infty) = E(x) \tag{31}$$

where:

$$E(x) = \frac{1}{2} g_0 \exp\left(-\frac{Dx}{2}\right) [Q_1(x) + Q_2(x)] \tag{32}$$

where g_0 is a normalization constant (if necessary).

$$\begin{aligned} Q_1(x) = {} & \frac{1}{2} \exp\left(\frac{D\zeta}{2}\right) \exp\left(-\frac{D}{2}\sqrt{(x-\zeta)^2}\right) \\ & - \exp\left(-\frac{D\zeta}{2}\right) \exp\left(\frac{D}{2}\sqrt{(x+\zeta)^2}\right) \end{aligned} \tag{33}$$

$$\begin{aligned} Q_2(x) = {} & \exp\left(-\frac{Dh}{2}\right) \exp\left(\frac{D}{2}\sqrt{(x-h)^2}\right) \\ & - \frac{1}{2} \exp\left(\frac{Dh}{2}\right) \exp\left(-\frac{D}{2}\sqrt{(x+h)^2}\right) \end{aligned} \tag{34}$$

for region $x \geq 0$, (see Fig. 5).

7. Conclusions

Equation (4) was derived for a slow variation of the amplitude, and it is widely used in the analysis of the light diffraction in grating volume [5, 17], which agrees with the results given by Eqs. (27), (28), (29).

This model, in principle, could give an initial guess that can be corrected to take into account the dark conductivities.

$IM-PRC$ media generate the spatial surface wave since this clearly results from the imaging property of an ideal metal surface, for this specific crystal orientation.

As was shown, the surface wave shape essentially depends on the shape of the incident beam. Before this work, it had not been specified how the shape of the input beam affects the surface wave formation.

Therefore, there in so evidence that surface wave formation depends on $\Delta K = \beta - k_0$ as is also outlined in [5].

Here it is predicted that the anti-symmetrical surface waves (oscillatory waves) can not be obtained with this orientation crystal, as is also outlined in [5].

The light confinement of the beam is obtained from the speed of the photorefractive response, when the magnitude of the diffusion coefficient γ involved in D is increased, as shown in the arguments of the $\exp$ function in (32). If the magnitude of γ is increased, then the $\exp$ function in (32) determines that when the wave penetration is closer to the boundary of the two media $IM - PRC$, the beam spread by diffraction tends to be much less, which implies that the wave quickly reaches stability (see Fig. 5) where, if we increase the diffusion coefficient $D(\gamma)$, the incident beam energy tends to lose a bit of energy by spread.

The non-stationary solution obtained for beam propagation along an $IM - PRC$ media interface makes it possible to predict the beam behavior inside the crystal. In particular, it is possible to estimate the speed of the photorefractive response, which depends on the light intensity, *i.e.* for this analysis, to know the length from the point where the beam starts to propagate to that one of the surface wave formation, along the propagation axis.

In a further analysis, the dark conductivities will have to be considered, since this physical effect can decrease the photorefractive response speed of the surface wave formation, thus gaining higher relevance over other physical effects produced when the laser beam is propagated along the media interface.

A reason that restricts to a more general solution is that the dark conductivities and the saturation of the impurity photorefractive centers in the crystals are neglected, but for this model, in principle, we could give an initial study so that thereinafter, we can develop the solution in order to take them into account, where these conductivities and the saturation of the impurity sooner or later experimentally tend to exist and are not necessarily negligible.

However this mathematical model is used for two specific beam shapes; it is also used to explain how the surface waves are reproduced by each laser beam propagating along a two medium interface, which allows the beam energy to be confined, and offers us the possibility of manipulating it for technical applications.

With the continuity equations for this specific laser beam it is possible to construct another physical situation of the wave guide, as for example [6], *i.e.* these results are to be used to make the confinement of energy more efficient within the crystal so that it can de easily used and manipulated.

A Appendix

$$f_1 = \frac{1}{2}\exp\left(-\frac{Dx}{2}\right) \tag{A.1}$$

$$f_2 = \frac{1}{2}\left[1 + 3\frac{x-\zeta}{\sqrt{(x-\zeta)^2}}\right]\exp\left(\frac{D\zeta}{2}\right) \tag{A.2}$$

$$f_3 = \exp\left(\frac{D\sqrt{(x-\zeta)^2}}{2}\right)\mathrm{erf}\left[\frac{D}{2}\sqrt{\frac{z}{ib}} + \frac{1}{2}\sqrt{\frac{ib(x-\zeta)^2}{z}}\right] \tag{A.3}$$

$$f_4 = \exp\left[-\frac{D\sqrt{(x-\zeta)^2}}{2}\right]\mathrm{erf}\left[\frac{D}{2}\sqrt{\frac{z}{ib}} - \frac{1}{2}\sqrt{\frac{ib(x-\zeta)^2}{z}}\right] - 2\sinh\left[\frac{D}{2}\sqrt{(x-\zeta)^2}\right] \tag{A.4}$$

$$f_5 = \left[1 + \frac{x+\zeta}{\sqrt{(x+\zeta)^2}}\right]\exp\left(-\frac{D\zeta}{2}\right) \tag{A.5}$$

$$f_6 = \exp\left[\frac{D\sqrt{(x+\zeta)^2}}{2}\right]\mathrm{erf}\left[\frac{D}{2}\sqrt{\frac{z}{ib}} + \frac{1}{2}\sqrt{\frac{ib(x+\zeta)^2}{z}}\right] \tag{A.6}$$

$$f_7 = \exp\left[-\frac{D\sqrt{(x+h)^2}}{2}\right]\mathrm{erf}\left[\frac{D}{2}\sqrt{\frac{z}{ib}} - \frac{1}{2}\sqrt{\frac{ib(x+h)^2}{z}}\right] - 2\sinh\left[\frac{D}{2}\sqrt{(x+h)^2}\right] \tag{A.7}$$

$$f_8 = \frac{1}{2}\left[1 + 3\frac{x-h}{\sqrt{(x-h)^2}}\right]\exp\left(\frac{Dh}{2}\right) \tag{A.8}$$

$$f_9 = \exp\left[\frac{D\sqrt{(x-h)^2}}{2}\right]\mathrm{erf}\left[\frac{D}{2}\sqrt{\frac{z}{ib}} + \frac{1}{2}\sqrt{\frac{ib(x-h)^2}{z}}\right] \tag{A.9}$$

$$f_{10} = \exp\left[-\frac{D\sqrt{(x-h)^2}}{2}\right]\mathrm{erf}\left[\frac{D}{2}\sqrt{\frac{z}{ib}} - \frac{1}{2}\sqrt{\frac{ib(x-h)^2}{z}}\right] - 2\sinh\left[\frac{D}{2}\sqrt{(x-h)^2}\right] \tag{A.10}$$

$$f_{11} = \left[1 + \frac{x+h}{\sqrt{(x+h)^2}}\right]\exp\left(-\frac{Dh}{2}\right) \tag{A.11}$$

$$f_{12} = \exp\left[\frac{D\sqrt{(x+h)^2}}{2}\right]\mathrm{erf}\left[\frac{D}{2}\sqrt{\frac{z}{ib}} + \frac{1}{2}\sqrt{\frac{ib(x+h)^2}{z}}\right] \tag{A.12}$$

$$f_{13} = \exp\left[-\frac{D\sqrt{(x+h)^2}}{2}\right]\mathrm{erf}\left[\frac{D}{2}\sqrt{\frac{z}{ib}} - \frac{1}{2}\sqrt{\frac{ib(x+h)^2}{z}}\right] - 2\sinh\left[\frac{D}{2}\sqrt{(x+h)^2}\right] \tag{A.13}$$

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