Modulation of spatial coherence of optical field by means of liquid crystal light modulator

A.S. Ostrovsky and E. Hernández García
Facultad de Ciencias Físico Matemáticas, Universidad Autónoma de Puebla,
72570 Puebla, Pue., México,
e-mail: andreyo@fcfm.buap.mx

Recibido el 22 de noviembre de 2004; aceptado el 24 de agosto de 2005

The theory of modulation of spatial coherence of optical field with the aid of a dynamic diffusion screen is given. Some useful models of a random phase screen are considered. The possibility of modulation of spatial coherence by means of liquid crystal light modulator is demonstrated with a physical experiment.

Keywords: Cross-spectral density; spatial coherence; random phase screen; liquid crystal light modulator.

Se da la teoría de la modulación de coherencia espacial de un campo óptico por medio de una pantalla de difusión dinámica. Se consideran algunos modelos útiles de la pantalla de fase aleatoria. La posibilidad de la modulacón de coherencia espacial se demuestra con un experimento físico.

Descriptores: Densidad espectral cruzada; coherencia espacial; pantalla de fase aleatoria; modulador de luz de cristal líquido.

PACS: 42.25.Kb; 42.79.Kr

1. Introduction

When using the laser illumination, it is frequently necesary to destroy completely or partially the spatial coherence of an optical field. Such a situation arises, for example, in the coherent imagery when it is necessary to eliminate the undesirable speckles in the image (see, for example, [1]) or in the problem of generating the propagation-invariant fields, when it is necessary to create a secondary source with a special structure of spatial coherence [2–4]. Two decades ago it was shown that the spatial coherence may be considerably reduced by transmitting the optical field through a rotating diffuser [5]. Recently, the general theory of such a transformation of coherence has been presented [6]. Nevertheless, the moving diffuser does not solve a considered problem in its total extent. Firstly, the moving diffuser does not permit the generation of the secondary source with an arbitrary space structure of coherence; secondly the presence in an optical device of some moving units is frequently undesirable. In the present paper we propose an alternative solution to the considered problem.

2. Transmission of an optical field through a diffusion screen

Let us consider a fluctuating, statistically stationary optical field of any state of coherence incident onto a thin diffusion screen located in the plane $z = 0$. Let $U_-(\mathbf{x},\nu)$ be the optical signal associated with this field just in front of the diffusion screen at a point $\mathbf{x} = (x, y)$ and at frequency ν. The statistical description of the corresponding field is given by the cross-spectral density $W_-(\mathbf{x}_1, \mathbf{x}_2,\nu)$ defined as [7]

$$W_-(\mathbf{x}_1, \mathbf{x}_2,\nu) = \left\langle U_-^*(\mathbf{x}_1, \nu)\, U_-(\mathbf{x}_2, \nu)\right\rangle, \quad (1)$$

where the asterisk denotes the complex conjugate and the angular brackets denote the average over the ensemble of field realizations. The normalized cross-spectral density (1),

$$\mu_-(\mathbf{x}_1, \mathbf{x}_2, \nu) = \frac{W_-(\mathbf{x}_1, \mathbf{x}_2,\nu)}{[W_-(\mathbf{x}_1, \mathbf{x}_1,\nu)]^{\frac{1}{2}}\,[W_-(\mathbf{x}_2, \mathbf{x}_2,\nu)]^{\frac{1}{2}}}, \quad (2)$$

is known as the spectral degree of coherence. The absolute value $|\mu_-(\mathbf{x}_1, \mathbf{x}_2,\nu)|$ satisfies, for all values of its arguments, the relation

$$0 \leqslant |\mu_-(\mathbf{x}_1, \mathbf{x}_2,\nu)| \leqslant 1, \quad (3)$$

and hence may serve as the quantitative measure of spatial coherence of the optical field at frequency ν. The thin diffusion screen at a point $\mathbf{x}$ and at frequency ν may be represented by the complex amplitude transmittance

$$T(\mathbf{x},\nu) = |T(\mathbf{x},\nu)| \exp\{i \arg[T(\mathbf{x},\nu)]\}, \quad (4)$$

where $|T(\mathbf{x},\nu)|$ describes the absorption and $\arg[T(\mathbf{x},\nu)]$ describes the phase delay of the wave introduced by the screen. We shall examine the influence of the diffusion screen on the spatial coherence of the transmitted optical field.

At first we consider the case when the diffusion screen is a static one, *i.e.* when its complex amplitude transmittance $T(\mathbf{x},\nu)$ represents the time-independent deterministic function of spatial coordinates. In this case, the field just behind the diffusion screen may be represented by the optical signal

$$U_+(\mathbf{x},\nu) = T(\mathbf{x},\nu)\, U_-(\mathbf{x},\nu), \quad (5)$$

and hence may be statistically described by the cross-spectral density

$$\begin{aligned} W_+(\mathbf{x}_1, \mathbf{x}_2,\nu) &= \left\langle U_+^*(\mathbf{x}_1,\nu)\, U_+(\mathbf{x}_2,\nu)\right\rangle \\ &= T^*(\mathbf{x}_1,\nu)\, T(\mathbf{x}_2,\nu)\, W_-(\mathbf{x}_1, \mathbf{x}_2,\nu). \quad (6) \end{aligned}$$

As may be readily shown, the spectral degree of coherence of the field emerging from the diffusion screen appears to be

$$\mu_+ (\mathbf{x}_1, \mathbf{x}_2,\nu) = \frac{T^* (\mathbf{x}_1,\nu)\, T (\mathbf{x}_2,\nu)}{|T (\mathbf{x}_1,\nu)|\, |T (\mathbf{x}_2,\nu)|} \mu_- (\mathbf{x}_1, \mathbf{x}_2,\nu) . \quad (7)$$

It follows directly from Eq. (7) that

$$|\mu_+ (\mathbf{x}_1, \mathbf{x}_2,\nu)| = |\mu_- (\mathbf{x}_1, \mathbf{x}_2,\nu)| . \quad (8)$$

This result shows that the static diffusion screen does not change the state of the spatial coherence of the transmitted field.

Now we consider the case when the complex amplitude transmittance of the diffusion screen represents the function of spacial coordinate stochastically depending on time t. Such a screen may be referred to as the random dynamic diffusion screen. As is customarily done in such situations, the random dynamic diffusion screen may be represented by an ensemble of spatial realizations $T_j (\mathbf{x},\nu)$, taken at fixed moments t_j, and hence may be statistically described by the correlation function

$$K_T (\mathbf{x}_1, \mathbf{x}_2,\nu) = \left\langle T_j^* (\mathbf{x}_1,\nu)\, T_j (\mathbf{x}_2,\nu) \right\rangle . \quad (9)$$

In this case, taking into account the statistical independence of processes $U_- (\mathbf{x},\nu)$ and $T (\mathbf{x},\nu)$, the cross-spectral density of the field emerging from the diffusion screen takes the form [compare with Eq. (6)]

$$W_+ (\mathbf{x}_1, \mathbf{x}_2,\nu) = K_T (\mathbf{x}_1, \mathbf{x}_2,\nu)\, W_- (\mathbf{x}_1, \mathbf{x}_2,\nu) . \quad (10)$$

The corresponding spectral degree of coherence now appears to be

$$\mu_+ (\mathbf{x}_1, \mathbf{x}_2,\nu) = \frac{K_T (\mathbf{x}_1, \mathbf{x}_2,\nu)}{[K_T (\mathbf{x}_1, \mathbf{x}_1,\nu)]^{\frac{1}{2}}\, [K_T (\mathbf{x}_2, \mathbf{x}_2,\nu)]^{\frac{1}{2}}} \times \mu_- (\mathbf{x}_1, \mathbf{x}_2,\nu) . \quad (11)$$

In view of the well-known property of the correlation function,

$$|K_T (\mathbf{x}_1, \mathbf{x}_2,\nu)| \leqslant [K_T (\mathbf{x}_1, \mathbf{x}_1,\nu)]^{\frac{1}{2}}\, [K_T (\mathbf{x}_2, \mathbf{x}_2,\nu)]^{\frac{1}{2}} , \quad (12)$$

it follows at once from Eq. (11) that, this time,

$$|\mu_+ (\mathbf{x}_1, \mathbf{x}_2,\nu)| \leqslant |\mu_- (\mathbf{x}_1, \mathbf{x}_2,\nu)| . \quad (13)$$

The result (13) shows that, with the certain choice of the random dynamic screen, it is possible to reduce the absolute value of the spectral degree of coherence of the incident field. We refer to such a reduction of $|\mu_- (\mathbf{x}_1, \mathbf{x}_2,\nu)|$ as the *modulation of spatial coherence* of the field.

3. Some models of a random phase screen

Here we consider a few models of a random dynamic screen which may find an application in practical problems. Taking into account that the absorption results in the inevitable energy loss, we limit our consideration to purely phase screens. For the sake of simplicity from now on we shall omit an explicit dependence of the functions considered on frequency ν.

3.1. Homogeneously uncorrelated random screen

The complex amplitude transmittance of such a screen is given by

$$T (\mathbf{x}) = \exp[i\Phi(\mathbf{x})], \quad (14)$$

where $\Phi (\mathbf{x})$ represents, at every point $\mathbf{x}$, a random variable with the uniform probability density

$$p(\Phi) = \begin{cases} 1/2\pi & \text{when } \Phi \in [0, 2\pi] \\ 0 & \text{when } \Phi \notin [0, 2\pi] \end{cases} . \quad (15)$$

Besides, for every two different points $\mathbf{x}_m$ and $\mathbf{x}_n$ the random variables $\Phi (\mathbf{x}_m)$ and $\Phi (\mathbf{x}_n)$ are statistically independent, *i.e.*

$$\langle \Phi (\mathbf{x}_m)\, \Phi (\mathbf{x}_n) \rangle = \delta_{mn,} \quad (16)$$

where δ_{mn} is the Kronecker symbol. On making use of the definition (9) with due regard for Eq. (14), we may write the correlation function of this screen in the form

$$K_T (\mathbf{x}_m, \mathbf{x}_n) = \langle \exp(i\Psi_{mn}) \rangle , \quad (17)$$

where $\Psi_{mn} = [\Phi (\mathbf{x}_n) - \Phi (\mathbf{x}_m)]$ is a new random variable which, in view of relations (15) and (16), has the probability density

$$p(\Psi_{mn}) = \begin{cases} 1/4\pi & \text{when } \Psi_{mn} \in [-2\pi, 2\pi] \\ 0 & \text{when } \Psi_{mn} \notin [-2\pi, 2\pi] \end{cases} . \quad (18)$$

Calculating an average on the right-hand side of Eq. (17) according to the formula

$$\langle \exp(i\Psi_{mn}) \rangle = \int_{-\infty}^{\infty} \exp(i\Psi_{mn}) p(\Psi_{mn}) \mathrm{d}\Psi_{mn}, \quad (19)$$

we find

$$K_T (\mathbf{x}_m, \mathbf{x}_n) = \delta_{mn}. \quad (20)$$

Finally, substituting from Eq. (20) into Eq. (10) and writing the result in the usual form, we obtain

$$W_+ (\mathbf{x}_1, \mathbf{x}_2) = W_- (\mathbf{x}_1, \mathbf{x}_1)\, \delta (\mathbf{x}_2 - \mathbf{x}_1) , \quad (21)$$

where $\delta (\mathbf{x})$ is the two-dimensional Dirac function. As is well known, the cross-spectral density of the form (21) describes the completely spatially incoherent optical field. Hence, the dynamic phase screen with the transmittance given by Eqs. (14) to (16) achives the complete destruction of the spatial coherence of incident field. Such a screen may be used, for example, in laser imaging systems for eliminating the undesired speckle structure of the image [1].

3.2. Radially uncorrelated screen with circular symmetry

The complex amplitude transmittance of such a screen at some point of the plane $z = 0$, written in polar coordinates, has the form

$$T(r,\theta) = T(r) = \exp[i\Phi(r)], \tag{22}$$

where $\Phi(r)$ has the same meaning as in Eq. (14). Manipulating by analogy with the foregoing, we obtain that, in this case,

$$W_+(r_1,\theta_1,r_2,\theta_2) = W_-(r_1,r_1)\,\delta(r_1 - r_2), \tag{23}$$

where, this time, $\delta(r_2 - r_1)$ is a two-dimensional circular Dirac function. The cross-spectral density of the form (23) describes an optical field which is completely spatially coherent in the azimuthal direction, but is completely spatially incoherent in the radial direction. Such a screen is required, for example, in the problem of generating the "light string beam" [3,4].

3.3. Azimuthally uncorrelated screen

This screen is the counterpart of previous model and has the complex amplitude transmittance of the form

$$T(r,\theta) = T(\theta) = \exp[i\Phi(\theta)], \tag{24}$$

where $\Phi(\theta)$ has the same meaning as in Eq. (14). The cross-spectral density of the field emerged from this screen is given by

$$W_+(r_1,\theta_1,r_2,\theta_2) = W_-(\theta_1,\theta_1)\,\delta(\theta_2 - \theta_1), \tag{25}$$

where, this time $\delta(\theta_2 - \theta_1)$ is a two-dimensional angular Dirac function. The cross-spectral density of the form (25) describes an optical field which is completely spatially coherent in the radial direction, while being completely spatially incoherent in the azimuthal direction. Unfortunately we do not know the possible applications of such a screen but we suppose that they exist.

3.4. Azimuthally cosine-correlated and radially uncorrelated screen

Firstly we consider a phase screen with the complex amplitude transmittance given by

$$T''(r,\theta) = \exp[i\Psi(r,\theta)], \tag{26}$$

where

$$\Psi(r,\theta) = \begin{cases} 2\pi + \theta - \Phi(r) & \text{when } 0 \leqslant \theta \leqslant \Phi(r) \\ 0 & \text{when } \Phi(r) < \theta \leqslant 2\pi \end{cases}, \tag{27}$$

and $\Phi(r)$ has the same meaning as in Eq. (14). Then we locate such a screen at the input of a Mach-Zehnder interferometer with a Dove prism in one of its arms and the phase compensator in the other. The Dove prism carries out the mirror mapping of the incident field, providing thereby the phase conjugation of the distribution (26), so that the entire interferometric system may be considered to be an equivalent diffusion screen with complex amplitude transmittance of the form

$$T(r,\theta) = \cos[\theta - \Phi(r)]. \tag{28}$$

Taking into account that

$$\langle \cos[\theta_1 + \Phi(r_m)] \cos[\theta_2 + \Phi(r_n)] \rangle = \delta_{nm} \cos(\theta_1 - \theta_2), \tag{29}$$

it may be readily shown that the cross-spectral density of the field at the output of the equivalent screen is as follows:

$$W_+(r_1,\theta_1,r_2,\theta_2) = W_-(r_1,\theta_1,r_1,\theta_1)\,\delta(r_1 - r_2)\cos(\theta_1 - \theta_2). \tag{30}$$

The cross-spectral density of the form (30) describes an optical field which is completely spatially incoherent in the radial direction, while being partially spatially coherent in accordance with the cosine law in the azimuthal direction. Such a screen is required, for example, in the problem of generating the "light capillary beam" [3,4].

4. Coherence modulation using a liquid crystal light modulator

As follows from Sec. 2, the modulation of spatial coherence may be achieved by transmitting the optical field through a thin diffusion screen with the complex amplitude transmittance which is randomly changed in time. Such a screen may be created in practice by means of the electrically controlled spatial-time light modulator on the basis of a twisted nematic liquid crystal placed between two polarizers (see, for example [8]). As has been shown in Ref. 9, depending on the mutual orientation of the main axes of polarizers, this modulator may produce to a good approximation either intensity-only or phase-only modulation of optical field. To demonstrate the practical possibility of the modulation of spatial coherence of an optical field by the use of liquid crystal light modulator, we conducted the physical simulation experiment which is discussed below.

In the experiment we used the commercial computer-controlled liquid crystal light modulator HoloEye LC2002. This device has an active area of 26.2×20.0 mm, a spatial resolution of 832×624 pixels, and a frame frequency of 60 Hz in the SVGA mode. The configuration and control of the modulator was carried out with the aid of the attached software. To provide the phase mode of modulation, we placed the liquid crystal modulator between two polarizers with the orientation of the main axes recommended by the manufacturer, for a light wave length of 532 nm, as 44° and -54°, respectively. Accordingly, as a primary light source in our experiment we used the argon laser. By doing this we achieved approximately phase-only modulation of the optical field with a good linearity over the dynamic range of 1.8π. As can be seen from the analysis given in Sec. 3.1, the slight deviation of this range from the 2π value needed for full-scaled phase modulation does not affect the expected results. At the same time the intensity distortions coupled with the phase modulation may violate the purity of experiment. To reduce the factor of this undesirable intensity modulation to the least possible value of 5%, we chose values for the brightness and contrast parameters of 100 and 255, respectively. The modulation part of the experiment is sketched schematically in Fig. 1.

The spectral degree of coherence was evaluated by calculating the visibility of the interference pattern registered in the output plane of the modified Young interferometer shown in Fig. 2. In this interferometer, the V-shaped aperture mask is imaged onto a translating slit to make two Young's pinholes with an adjustable separation. The registration of interference pattern was realized with the aid of the CCD camera. To provide a solid statistical averaging, we used the accumulation of the recorded data during several seconds.

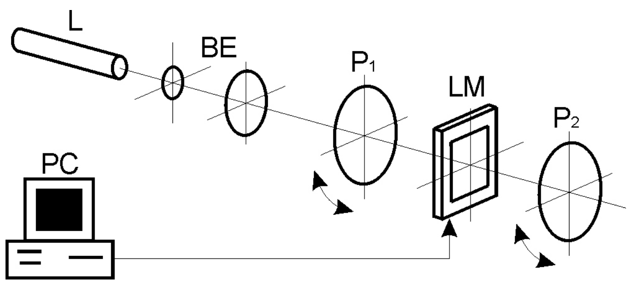

FIGURE 1. Experimental setup: L - Laser; BE - beam expander; P_1, P_2 - polarizers; LM - liquid crystal light modulator.

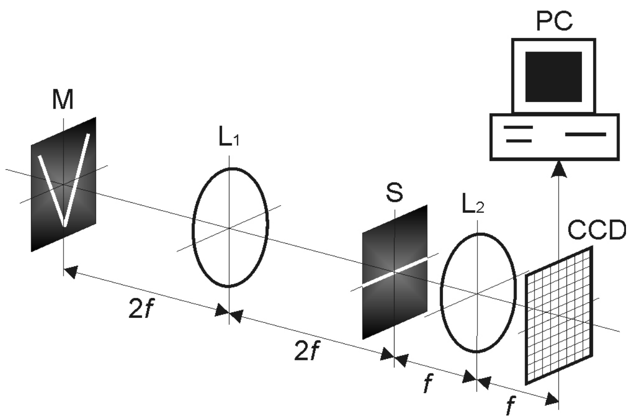

FIGURE 2. Modified Young interferometer: M-V - shaped mask; L_1, L_2 - lenses; S - translating slit.

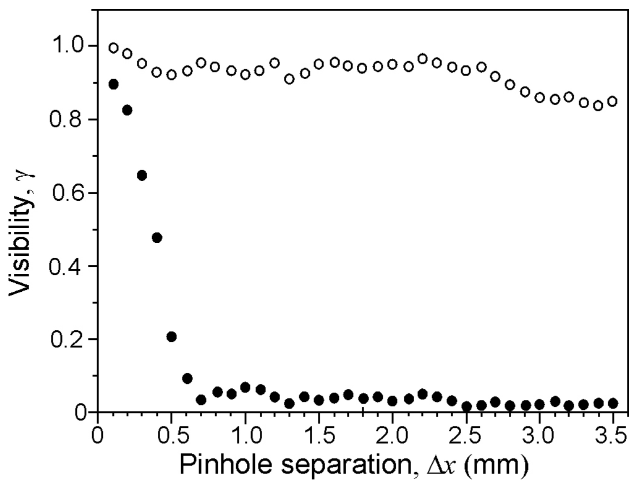

FIGURE 3. Visibility measured as a function of pinhole separation: • with modulation; ○ without modulation.

In our experiment, using the program generator of random numbers, we created at the liquid crystal the phase profile corresponding approximately to the one-dimensional version (along the coordinate x) of the expression (14). In this way we simulated the random screen, which is completely uncorrelated in its x-section. In doing this, we expected to achieve the complete destruction, in the direction x, of the spatial coherence of the incident laser field which may be considered to be approximately completely coherent. The measured visibility of the interference fringes γ as a function of the pinhole separation $\triangle x$ is plotted in Fig. 3. In the same figure, for comparison, the visibility γ measured in the absence of liquid crystal modulator is shown. The results obtained show that in our experiment we have achieved an approximately complete destruction of the spatial coherence of incident laser radiation.

5. Conclusions

It has been shown that the dynamic diffusion screen may reduce the spatial coherence of the incident optical field. Some useful models of a random dynamic screen have been proposed and their possible practical applications have been discussed. The possibility of modulation of spatial coherence by means of a liquid crystal light modulator has been discussed and illustrated with the experiment.

Acknowledgements

This work was supported by the National Council for Science and Technology of Mexico (CONACyT) under the project 36875-E.

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