

## Analogy between dislocation creep and relativistic cosmology

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A formal, physical analogy between plastic deformation, mainly dislocation creep, and Relativistic Cosmology is presented. The physical analogy between eight expressions for dislocation creep and Relativistic Cosmology have been obtained. By comparing the mathematical expressions and by using a physical analysis, two new equations have been obtained for dislocation creep. Also, four new expressions have been obtained for Relativistic Cosmology. From these four new equations, one may determine the neutron energy,  $u_N$ , by knowing of the present value of the universe radius and the Einsteinian gravitational constant. Another new expression gives the neutron radius,  $r_N$ , as the present value of the universe radius,  $R_{OU}$  divided by  $10^{40}$ .

*Keywords:* Dislocation creep; plastic deformation; relativistic cosmology.

Se presenta una analogía formal y física entre la deformación plástica, principalmente termofluencia por dislocaciones y cosmología relativista. Se muestran la analogía física entre ocho expresiones de termofluencia por dislocaciones y cosmología relativista. Con la comparación entre las expresiones matemáticas y el uso del análisis físico se encuentran dos expresiones nuevas para termofluencia por dislocaciones y cuatro nuevas expresiones para Cosmología Relativista. De entre las cuatro nuevas ecuaciones una permite determinar la energía del neutrón,  $u_N$ , a través del conocimiento del radio actual del Universo y de la constante Einsteiniana de gravitación. Otra expresión define el radio del neutrón  $r_N$  como el radio actual del universo  $R_{OU}$  dividido entre  $10^{40}$ .

*Descriptores:* Deformación plástica; termofluencia por dislocación; cosmología relativista.

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### 1. Introduction

Since time immemorial human kind has been watching the sky and wondering about its meaning and origin. In those days the information obtained from the skies was used for practical purposes of orientation: in terrestrial space, or for the cultivation of crops. For most human history it has been thought that stars were fixed to some sort of sphere or were pinholes in that sphere, permitting glimpses of a universal fire burned beyond. Whatever the model of the universe (Plato, Aristotle, Ptolemy... models), man was at the center (quite natural in view of the fact that the celestial spheres seems equidistant in all directions). The concept of our central position in the cosmos has been tenacious.

Only after the work of Nicolai Copernicus on a hypothetical scheme of the movements, of the heavens the Earth was theoretically displaced from the center of the universe, with the sun as the center of the planetary system and, of course of the universe. Later, Galileo's observations of outer space by using a telescope for the first time caused an immense stir that led to an unavoidable conflict with the geocentric dogma upheld by Catholic theologians. Galileo saw sunspots, the moons of Jupiter and the phases of Venus. All this factual information established, once and for all, that the heliocentric system proposed by Copernicus was a reality, and could no longer be considered to be merely a computationally convenient hypothesis. Newton, after identifying

the Keplerian orbit of the moon with the Galilean trajectory of a ballistic projectile, reached a theoretical understanding of Kepler's phenomenological laws by establishing the universal law of gravitation. Newton also considered the universe infinite in extension and in duration on time (as considered before by Lucretius in "De Rerum Naturae"). At the beginning of the twentieth century the general vision among astronomers was that everything visible in the heavens belonged to "our" galaxy [1].

In 1917 Einstein, starting from the equality between the inertial and gravitational mass of material objects, constructed his general theory of gravitation, which for the first time allows one to deal with the whole universe [2]. Einstein himself developed a model in which he postulated a cosmic repulsion force, "the cosmological constant" term in the gravitational field equation. The role of such a repulsive force was to balance gravity and yield a static model for the universe. After Hubble's discovery of the expansion of the universe, Einstein considered that the introduction of the cosmological constant was his "biggest blunder" [3]. With Friedmann's [4] and Lemaitre's [5] works published in 1922 and 1927 respectively, the first relativistic cosmological models describing the expansion of the universe began to appear (both models use the cosmological constant term in their field equations). The next big step in cosmology was crystallized in the Big Bang theory, which is usually associated with Gamow. He

provided cosmology with a link between the science of the universe and nuclear physics [6]. There are three different types of experimental data which support the Big Bang theory. The expansion of the universe according to Hubble's law, which describes the radial velocity of galaxies and other cosmic systems as a function of the distance from any observer, provides direct evidence for an explosive beginning [7-10]. The Nucleosynthesis prediction for the light elements in the first minutes of the Big Bang have been verified by observation of the abundance light elements [11-16]. And also the observation of the cosmic microwave background gives a fundamental support for the Big Bang theory [17-23]

Also a long time ago, the standard Big Bang Nucleosynthesis theory was complemented by the theory of stellar element formation [24], which solves the problem of generating elements heavier than helium, by explaining in a general way the abundance of practically all the isotopes or elements from helium through uranium by synthesis in stars and supernovae.

From 1965 to the fall of 1997, research in cosmology evolved as expected, without surprises, until Professor Saul Perlmutter, at the beginning of 1998, announced that he and an international team of observers of supernovae, uses supernovae as beacons to judge how the cosmic expansion rate has changed over time. Not only did the results support the earlier evidence that the expansion rate has slowed too little for gravity ever to bring it to a stop; they also hinted that something is nudging the expansion along. This discovery introduced important evidence that there is a cosmological constant [25]. The first detailed publications on the subject support the acceleration of the expansion of the universe [26,27].

A review of recent observations suggests a universe that is lightweight (matter density about one - third of the critical value), spatially flat and accelerating [28,29].

The acceleration of the expansion of the universe requires the existence of an energy to overcome the gravitational self-attraction of matter. The cosmological constant - also called lambda (written as  $\lambda$  or  $\Lambda$ ) has long time been a candidate for serving as this energy reservoir. In 1967, Zel'dovich [30] showed that the energy density of the vacuum should act precisely as the energy associated with the cosmological constant. Lately, theorists have been dusting it off again and speculating about sources for the energy based on the fleeting particles that wink in and out of existence in vacuum space, according to quantum mechanics. But calculations based on that idea lead to lambda's that are 120 orders of magnitude greater than the energy contained in all the matter in the universe [31-35]. So theorists are playing with alternatives. For instance, some workers consider that the cosmological constant arises from different possibilities: local voids or non-homogeneities in the expansion of the universe [36,37], a true Casimir effect on a scalar field filling the universe [38], the acceleration of the universe [39]; or give alternative scenarios to a pure cosmological constant provided by a classical scalar field known as quintessence [29,40-44], also the self-tuning bran scenario in an attempt to solve the cosmological constant have been used [45], and some people use the

anthropic principle trying to explain the small value of the cosmological constant [46,47]. As far as we know, there are no experimental data which give conclusive support to any of the previous models for the cosmological constant problem. In other words, this problem and also associated with the acceleration of the expansion of the universe still remains unsolved.

The main purpose of this work is to give an isomorphic analogy between the main results of the deformation of crystalline materials and the relativistic cosmology theory for the expansion of the observable universe; after that, some calculations and predictions will be made. This is done by considering with Zel'dovich (1962), [48] our deep conviction that it would be naive to expect from astronomy new rules or theories about nuclear reactions, the creation of the elementary particles and the laws of general theory of relativity. The point is to use existing theories correctly, not to introduce new ones. In other words, we restrict ourselves to work strictly in the theoretical and experimental frame work established by the work of human kind in our terrestrial facilities, knowledge corroborated now and then, or hundreds, thousand or millions of times in the every day experience of humanity, trying to follow Occam razor very closely in order to reduce to a minimum the use of speculative ideas without experimental support of some type.

Before we make the comparison between the deformation of crystalline materials and Cosmology, and before we analyze its global implications and in order to make this paper self-contained, let us make a brief synthesis of both subjects.

## 2. A brief synthesis of the deformation of crystalline materials

It is well known that Hooke's law describes the elastic linear deformation of materials under the action of an external force  $F$ .

Expressed as an axial applied stress  $\sigma$ , such relationship reads,

$$\sigma = Ee \quad (1)$$

where  $e$  is the axial engineering strain,

$$e \equiv (L - L_0) / L_0 \quad (2)$$

$L$  the length of the deformation sample at time  $t$ , and  $L_0$  the initial length before deformation, and  $E$  is Young's Modulus, usually with values on the order of  $10^9$  Pa [49]; Eq. (1) is commonly obeyed for all crystalline materials up to an upper limit of about  $e = 0.1\%$  [50].

A simple microscopic explanation about Hooke's law is the following: If we take the harmonic oscillator

$$V(r) = -\frac{1}{2}k_{at}(r - r_0)^2,$$

it describes in a first approximation (for small oscillations) the potential energy of first neighbors inside a crystalline, cubic lattice of a solid. Here  $k_{at}$  is the coefficient which characterizes the potential strength, and  $r_o$  is the equilibrium lattice parameter. With these conditions, the force between two neighbors  $F_{at}$  is given by,

$$F_{at} = -\nabla V(r) = k_{at}(r - r_o) \tag{3}$$

and, if the elastic deformation of the solid is homogeneous, then  $e_{micro} = e_{macro}$ . Therefore Eq. (3) and (1) imply that,

$$E = \frac{k_{at}}{r_o} \tag{4}$$

This equations give the stiffness of the crystalline lattice or the stiffness of the crystalline space as a function of the strength of the interaction potential between adjacent lattice points,  $k_{at}$  and the inverse of the lattice parameter. As  $k_{at}$  grows and  $r_o$  diminish,  $E$  tends to grow.

Also, it is possible to show that the stored strain energy per unit volume,  $U_\sigma$ , due to an axial stress  $\sigma$  acting on a material is given by [51],

$$U_\sigma = \sigma^2 / (2E) \tag{5}$$

where E is related to the shear modulus  $\mu_m$  by the standard relation

$$\mu_m = E / [2(1 + \gamma)] \tag{6}$$

with  $\gamma$  as Poisson's ratio.

Note that in the following paragraphs and sections a slight change has been made in the usual notations for physical parameters of common fields here mentioned to avoid or reduce misunderstandings.

In 1926, Frenkel [52] developed a model to explain the yield stress for metals (the beginning of plastic deformation, which has an irreversible character). Essentially, he considered that plastic deformation occurs in the sliding of one atomic layer over its layer immediately below. The theoretical values (of the order of magnitude of the shear modulus of the material  $\mu_m$ ) were too big when compared with experimental data (a thousand or one hundred thousand times greater than the real values), and the model does not take into account the strain hardening [52,54]. In 1934, Polanyi [55], Taylor [56], and Orowan [57] propose, independently, the concept of edge dislocation. In 1939, Burgers [58] proposed the concept of screw dislocation. Both types of crystalline defects are required to explain the main and fundamental features of plastic deformation in crystalline structures.

In the following paragraph the main topological characteristics of a straight edge dislocation, and their implications for gliding movement are described. In Fig. 1 a schematic arrangement of the atoms in a normal plane of a straight edge dislocation is exhibited. The characteristics of this crystalline defect produce compressive stresses above the glide plane and tension stresses below it. The inferior ending edge of

the extra-semi- plane is denoted by the symbol ( $\perp$ ). Under the action of a shear stress  $\tau_\sigma$ , the dislocation moves in the direction of the Burgers' vector,  $b$ ; this movement has a transient phase shown in Fig. 2. Under the action of the shear stress above the glide plane, the atoms (above the glide plane) move slightly to the right of their equilibrium positions. Simultaneously, below the glide plane, the shear stresses move the atoms (below the glide plane) slightly to the left of their equilibrium positions. Events occur in such a way that the extra-semi-plane 5 displaces to the right, because during deformation the plane named 6 - 5' is transformed into the plane 5 - 5' with a "rupture" of continuity of the plane 6 - 5' giving place to an extra - semi plane 6. As the atoms around the extra - semi plane move very little around their equilibrium positions, the interaction potential between them can be considered harmonic, and because of the symmetry of the distortions of the atom's positions by the semi - extra plane, as a first approximation during the gliding of an edge dislocation there are no net atomic forces acting on the dislocation (at low gliding velocities as compared with the velocity of transverse waves of sound in the material) [59-61].

This type of crystalline defect, which is necessary to explain the plastic deformation of materials, is not in thermodynamic equilibrium. Therefore, when a pseudo-particle (edge dislocation) and its pseudo-anti-particle (edge dislocation of the opposite sign) are very close in position, an annihilation process develops between them. During this process, the stored elastic energy of the field of each pseudo particle is transformed into incoherent sound waves, and the restoration of the local perfection of the lattice and the local relaxation of elastic stresses occur, [see Fig. 3].

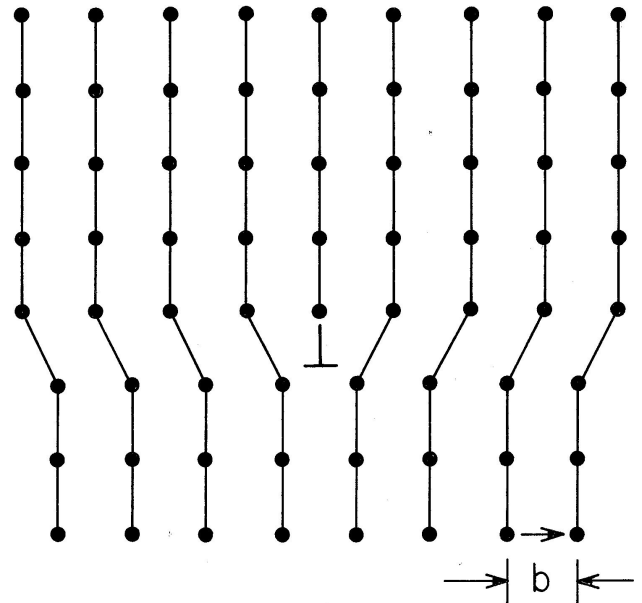


FIGURE 1. Atomic arrangement in a perpendicular plane of a straight edge dislocation (Schematic), [61].

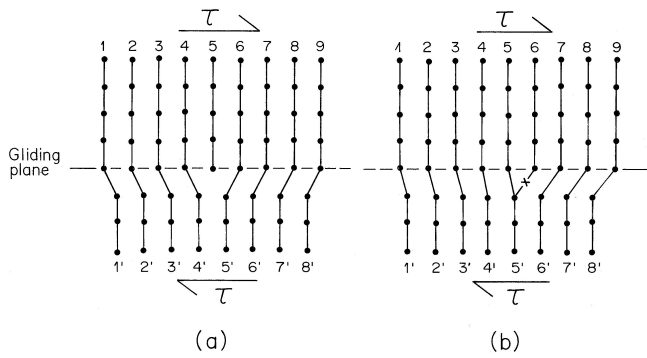


FIGURE 2. (a) Atoms near an edge dislocation. (b) Edge dislocation movement to the right, under the action of a shear stress  $\sigma$ . The magnitude of the displacement of the edge dislocation is  $b$ , [61].

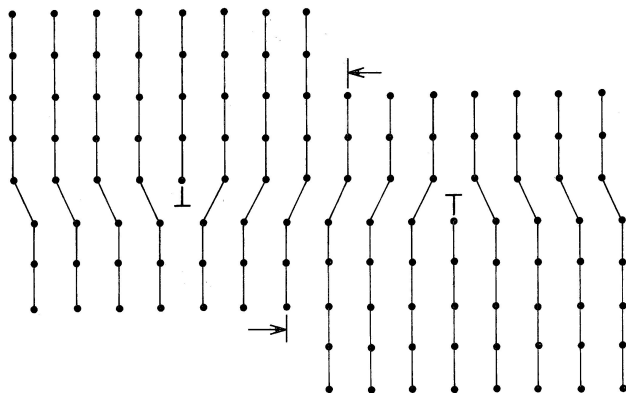


FIGURE 3. A schematic view of the geometrical aspects of an annihilation event between one edge dislocation  $\perp$ , and one edge dislocation of the opposite sign ( $\top$ ). In this figure it is clear that the spatial superposition of both crystalline defects restore the crystalline perfection ( and relaxes the local internal stresses); dissipating, by phonon emissions, the stored elastic energy of both defects. Note: for simplicity has been supposed that diffusion phenomena occurred before the situation here described.

Usually the external power which causes deformation increases the length of dislocations per unit volume  $\rho$ . The increase in  $\rho$  causes a decrease in the mean velocity,  $v_g$ , of the mobile dislocation density  $\rho_m$ . The strain rate  $\dot{\epsilon}$  is given by [62,63].

$$\dot{\epsilon} = \frac{b}{M} \rho_m v_g \tag{7}$$

where  $M$  is a geometric factor (the Taylor factor) and  $b$  is the modulus of the Burger's vector. When  $\rho_m$  and  $v_g$ , change very quickly, in order to describe the plastic deformation it is required to use an equation for  $d\dot{\epsilon}/dt$  which reads,

$$\frac{d\dot{\epsilon}}{dt} = \frac{b}{M} \left( \rho_m \frac{dv_g}{dt} + v_g \frac{d\rho_m}{dt} \right) \tag{8}$$

and is also known as Fuchs and Ilschner's equation [64,65].

According to the statistical mechanical analysis of plastic deformation by dislocations [65-67], the Fuchs and Ilschner's equation (Eq. (8)) is related to the volumetric net force,  $f$ , acting in the center of mass of the mobile dislocation density,

and Eq. (7) describes the volumetric linear momentum,  $p$ , of the mobile dislocation density. In other words, one has

$$p = \frac{m}{Mb} \dot{\epsilon} \tag{9}$$

$$f = \frac{dp}{dt} = \frac{m}{Mb} \frac{d\dot{\epsilon}}{dt} \tag{10}$$

where  $m$  is the inertial mass per unit length of dislocation. Until now, there have been no trustable experiments to determine the inertial mass of dislocations. However, by analogy with the theory of general relativity in which the inertial mass is identical to the gravitational mass of the object, some authors [60,68] have considered that the inertial mass of dislocations is identical to the mass arising from the elastic field expressions for the self-energy of dislocations per unit length,  $u_d$ . The inertial mass per unit length of dislocations is obtained from,

$$u_d = m_o v_s^2 \tag{11}$$

where  $m_o$  is the mean value of the rest mass per unit length of dislocation in an homogenous material. And the velocity of transverse sound waves is given by [69],

$$v_s = \sqrt{\mu_m / \rho_{gr}} \tag{12}$$

with  $\rho_{gr}$  as the density of the material in grams per cubic centimeter.

Usually  $u_d$  is expressed [60, 68] as,

$$u_d = \frac{1}{4\pi} \frac{(2 - \gamma)}{[2(1 - \gamma)]} \mu_m b^2 [\ln(x/r_o) + 0.15] \tag{13}$$

where  $x$  is a distance which characterizes the more distant dislocation interactions, and  $r_o \cong 5b$  is an effective core radius. Eq. (13) implies that the mass per unit length of dislocation is not a local quantity. The term with 0.15 is due to a non-linear contribution arising from the core of the dislocation.

The geometrical laws of the continuous linear elasticity theory for static dislocations have by now been established in their classical form and one should not expect them to experience any fundamental change [70]. Also specific expressions for the static-elastic fields for deformation and stresses due to dislocations are available in many classical texts [49,71-73]. And for the case of the dynamical linear elasticity theories of dislocations the most complete version is due to Kosevich [74-76]. Equations (9) and (10), are based on quasi-particle concepts which are usually alien to the elastic-field theories of dislocations mentioned in this paragraph.

The quasi-particle concepts for dislocations are based on an old analysis made by Frenkel and Kontorova in 1939 [77], they to study the propagation of one unit of displacement in a unidimensional infinite chain of atoms elastically bonded between them, which stay above another chain (or similar

atoms) which remain still. To describe this physical situation they obtain the following equation (expressed in present notation),

$$C_o^2 \frac{\partial^2 \phi}{\partial x^2} - \frac{\partial^2 \phi}{\partial t^2} = w_o^2 \sin \phi \quad (14)$$

where  $C_o$  and  $w_o$  are the velocity and characteristic frequency of the system. In this equation, from left to right, the first term represents the elastic interaction energy between neighboring atoms, the second one their kinetic energy and the last term the potential energy due to the existence of a lower chain of atoms which is at rest. This non-linear field equation, well-known as the Sine-Gordon equation, is invariant under Lorenz transformations with the transverse wave sound velocity playing the role of the speed of light [78,79].

The solution of this field equation, which does not need to depend on time, has two parts: one for large amplitudes where the solution is highly localized in space; and also solutions arising from small amplitudes which are spatially very extended. The solutions of this field equation called solitons have the dynamic characteristics of particles, and under the effect of disturbances behaves like deformable particles. These solutions are topologically stable entities due to a dynamical equilibrium between two opposite potential energy tendencies: the dispersive effects, which tend to spread the wave package, are balanced by the non-linear terms of the solution that promote their agglutination. This last effect is due to a physical process arising from the different velocities of the Fourier components.

Under non-quantum or relativistic conditions, the center of mass of one soliton obeys Newton's Second Law, as actually occurs for one dislocation [68]; and the movement of the center of mass of a system of solitons also obeys Newton's Second Law, as actually occurs for dislocation creep [65,66].

When the stresses applied to a crystalline material are high enough, plastic deformation occurs through the creation, motion and interaction of dislocations. The dislocation density can only increase as long as the number of new dislocations and also the length of the previous existing dislocations grow. The only mechanical way for a dislocation to increase its length is by gliding in its glide plane. Therefore, from the nine components of the applied stress tensor, in principle, only six of them (the shear stresses) are acting on the gliding planes and are able to create dislocations; the hydrostatic or pressure components of the applied stress can only produce vacancies or volumetric defects mechanically. By applying these ideas for the creation of dislocations recently for the first time, a new theory appeared for the creation rate of mobile dislocations considered as quasi-particles [60].

This theory is based on the principle of conservation of energy. For the purpose of creation events, dislocations are considered as quasi-particles obeying an effective relativistic equation for the self-energy of dislocation per unit length (Eq. (11), where  $\bar{m}_o$  is the average dislocation mass per unit length which takes into account that screw and edge dislocations in metals are created in equal number, and also in equal

quantity of both sign of dislocations (dislocations and anti-dislocations) in both types of dislocations. This is in order that crystalline deformation can occur in an homogeneous. The authors also consider that in dislocation creep deformation, the gliding of mobile dislocations occurs at low glide velocity as compared with  $v_s$ , and then the dissipate forces acting during gliding are absent or negligible [59,80]. With the use of these main considerations, the authors arrive at the following equation for the creation rate of the mobile dislocation density,  $\dot{\rho}_m^+$ ,

$$\dot{\rho}_m^+ = \frac{\sigma \dot{\epsilon}}{M \bar{u}} \quad (15)$$

As far as we know, Eq. (15) is the only case of an expression for the creation rate of dislocations without any free parameter. This equation has been used to explain many problems of plastic deformation [60,66,81-84]. For instance, for *Al-11Zn* in the region where  $\dot{\epsilon}$  shows an exponential dependence on stress [85], the theoretical prediction for the creation rate of the mobile dislocation density and the experimental data for different stresses are in full agreement with a ratio of theoretical to experimental data equal to  $1.00 \pm 0.03$  in the full range of the applied stress [60].

For power-law creep, the subgrain formation process proceeds as follow [83]: at the beginning of plastic deformation, rapid multiplication of gliding dislocations at a rate  $\dot{\rho}_m^+$  occurs and causes the density of mobile dislocations  $\rho_m$  to rise. The new dislocations glide over a mean free path,  $L_{fp}$ .

$$L_{tp} = v_g t_{coll} \quad (16)$$

before they are held up by dislocations of the opposite sign with an edge component which glides in the opposite direction under the action of the same applied shear stress.  $t_{coll}$  is the meantime of collision between opposite signed gliding dislocation. Their mutual internal stress field promotes local dislocation movements; and eventually, if the temperature is high enough to facilitate atomic diffusion, a steady state condition is attained in which the rate of increase of dislocations in such a region is equal to the annihilation rate of dislocations in the sub grain walls structure. Therefore the creation of dislocations and anti-dislocations occurs at the center of sub grains (dislocations and anti-dislocations glide in opposite directions form the dislocation sources), and annihilation events occur in the subgrain wall structure where gliding dislocations and anti-dislocations collide (coming from different contiguous subgrains).

The subgrain diameter at steady state is given by [83],

$$d_{sg} = \frac{2}{\phi_M^2} \left( \frac{\bar{u}}{\sigma b} \right) \quad (17)$$

where  $\phi_M \equiv 1/M$ . Equation (17) relates the subgrain diameter with the mean energy per unit length of dislocation  $\bar{u}$ . And for the usual case where  $\phi_M = 1/3$  and  $\bar{u} = \mu_m b^2$  [60]

this expression resembles the phenomenological expression,

$$d_{sg} = K \left( \frac{\mu_m b}{\sigma} \right) \tag{18}$$

with  $K_{theo} = 20$  for metals and ceramics. These theoretical value for  $K$  has a difference of 10% and 20% with the experimental data of  $K$  for metals and ceramics respectively [67,83]. It is the first time, that a theory of subgrain formation has provided an expression which gives the numerical prediction of  $K$  as a function of basic parameters of plastic deformation.

The elastic interaction between dislocations decelerates the mobile ones. For a long time ago [56], we have known that the internal stress,  $\sigma_i$ , which opposes to glide (due to the elastic interaction with other dislocations) is given by

$$\sigma_i = \alpha M \mu_m b \sqrt{\rho}$$

with  $\alpha$  as a constant that characterizes the interaction of dislocations and depends on the geometrical arrangement of the dislocation structure as a whole ( $\alpha \cong 0.5$ ) [86]. As a first approximation it remains constant during deformation [87].

This stress  $\sigma_i$  represents some sort of special average value of the actual positions of dislocation segments, and the law that relates the glide dislocation velocity  $v_g$  to the effective applied shear stress to the gliding dislocation (this average is determined in the region existing between the source of dislocation at the center of the subgrain and the subgrain wall) [82].

Obviously, the actual internal stress within the whole subgrain interior must vary with the position and must have a spatial average that is equal to zero. This is because of the symmetry of the situation in which in half of the subgrain volume the actual internal stress points in one direction, and in the other half of the volume it points in the opposite direction [88].

Through plastic deformation, a heterogeneous dislocation structure, (which very often appears in many creep theories, starting from the pioneer work of Weertman [89-90] on power-law creep and viscous glide at steady state) is created (see Figs. 4 and 4b). Many other authors [67,92-99] consider similar arrangements to those which, schematically, could be represented by Fig. 5, provided that in addition to the dipolar character of the subgrain wall structure, we add their sources that lead to this arrangement [83]. From Fig. 5, it is clear that angles are added (caused by the elastic bending of the lattice in a region with one signed dislocations, see Fig. 1 in Ref. 100. In each half of the subgrain, we have that the total angle  $\theta_T$ , caused by the elastic bending on the lattice is,

$$\pm\theta_T = \pm\theta_m \pm \theta_w \tag{19}$$

where the sign (+) is for the right hand region of the source of dislocations and the sign (-) is for the left hand region of the source until the middle region of the subgrain wall structure is reached. And, because the misorientation angles  $\theta_m$  and  $\theta_w$  are related to the spacing of dislocations within the subgrain interior  $d_m = 1/\sqrt{\rho_m}$ , and within the subgrain wall

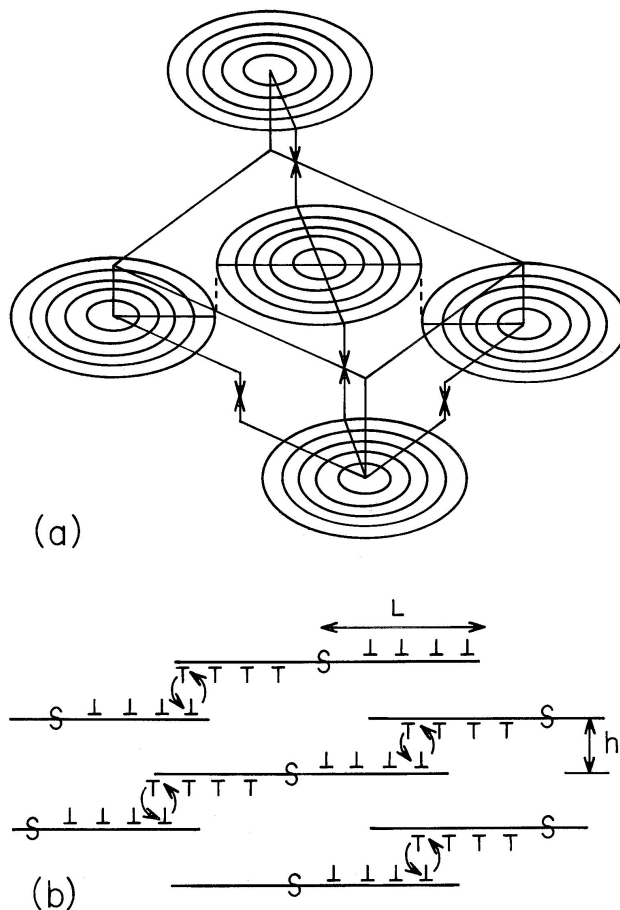


FIGURE 4. Emission of dislocations loops from sources(s) and subsequent annihilation through climb (diffusion processes associated to edge dislocations by Weertman’s model. (a) Three dimensions and (b) two dimension’s model, after Weertman’s [89-90] work.

structure  $d_w = 1/\sqrt{\rho_w}$  we have  $d_m = b/(tg\theta_m)$ ;  $d_w = b/(tg\theta_w)$  [50]. For  $\theta_m$  and  $\theta_w \ll 1$ , then  $d_m = b/\theta_m$  and  $d_w = b/\theta_w$ . Therefore, by using these relations in Eq. (19)

$$\theta_T = b\sqrt{\rho_m} + b\sqrt{\rho_w}. \tag{20}$$

And by multiplying both sides of Eq. (20), by  $(\alpha M \mu_m)$  the following equation for the mean long range internal stress is obtained:

$$\sigma_i = \alpha M \mu_m b (\sqrt{\rho_m}) + \alpha M \mu_m b (\sqrt{\rho_w}) \tag{21}$$

where  $\sigma_i$  have been defined as  $\rho_i \equiv \alpha M \mu_m b \theta_T$ . Eq. (21) is equal to the simplified version of the soft and hard region theory [67,82-84].

We note that  $\sigma_i$  as given by Eq. (21) arise from an analysis of the lattice deformation due to the existence of regions of high elastic energy fields in the form of defects of the crystalline lattice. Dislocations deform crystalline space in a more or less static way.

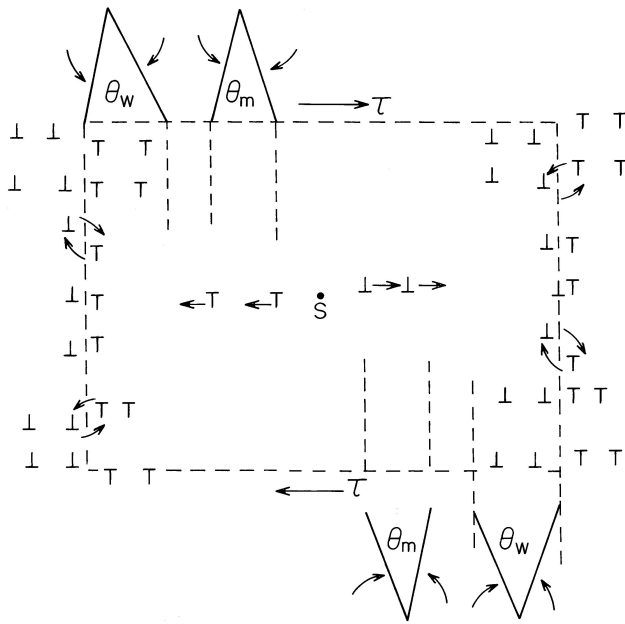


FIGURE 5. Schematic model for creep substructure as formed during high temperature plastic deformation; and the topological characteristics of the dipolar subgrain wall created through collision and interactions during the movement of mobile dislocations. Only primary dislocations are shown [104].

With the knowledge of the topological meaning of  $\sigma_i$  and its relation with the associated parameter  $d_{sg}$ , its now possible to express  $d_{sg}$  as a function of the dislocation density  $\rho$ , and also to understand that  $d_{sg}$  separates regions with dislocations of opposite sign.

By considering that  $\rho_w \gg \rho_m$  (in general) [82,101] and setting  $\rho_w = \rho$ , then with the use of Eqs. (21) and (17), we arrive at,

$$d_{sg} = \frac{2}{\alpha M \phi_M^2} \left( \frac{u}{\mu_m b^2 \sqrt{\rho}} \right) \quad (22)$$

and by using as before  $\phi_M = \frac{1}{M} = \frac{1}{3}$ ,  $\bar{u} = 1\mu_m b^2$ ,  $\alpha \cong 0.5$ ,

$$d_{sg} = \frac{12.4}{\sqrt{\rho}}. \quad (23)$$

This last equations explains experimental data on *Al-11 pctZn* [85] very well.

With the use of the same condition used before, it is possible to show that,

$$\rho = \frac{4(1+\gamma)}{\alpha^2 M^2 \mu_m b^2} \left( \frac{\sigma_i}{2E} \right)^2 \equiv \frac{4(1+\gamma)}{\alpha^2 M^2 \mu_m b^2} U_{\sigma_i} \quad (24)$$

where Eqs. (5) and (21) have been used. Eq. (24) shows that the dislocation density is a function of the volumetric density of elastic energy in the crystalline solid.

Finally, a new technique for scanning electron microscopy which provides a mesoscopic coordinates system inscribed in the center of a tension test specimen of  $371 \mu_m$  of gauge length has recently been proposed [102-104].

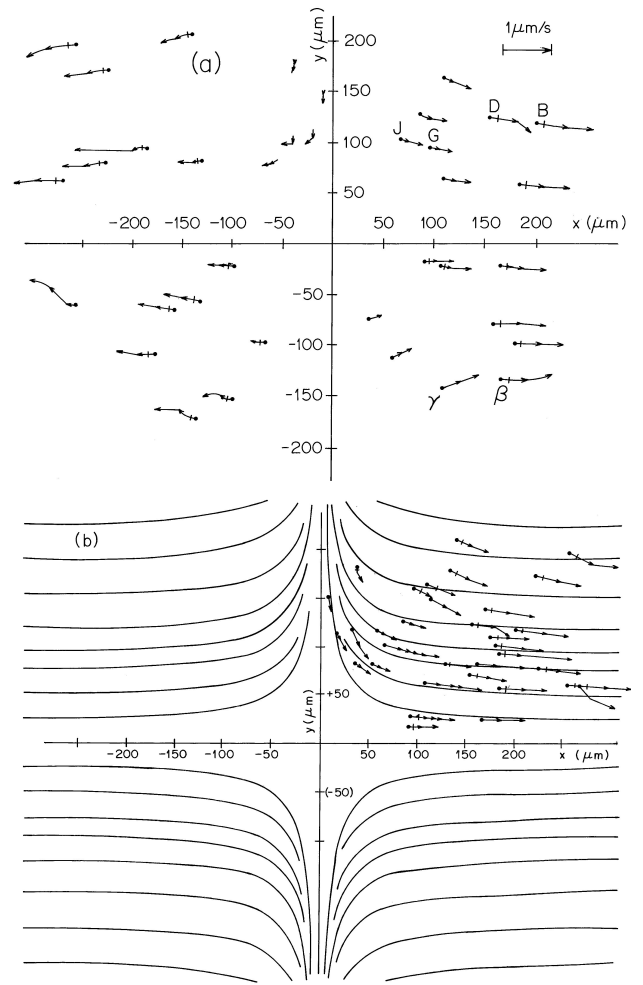


FIGURE 6. Quantitative flow map of velocities on specimen through deformation. (a) Real flow in the four (x, y) regions of sample under deformation. (b) Flow lines obtained by using symmetry operations on real data on (a), denoted by arrows. Also is shown the idealized velocity flow map by which the experimental data can be described; denoted by the continuous curves. Data on *Zn/20.2% Al/1.8% Cu* alloy at room temperature [102-104].

Among other studies, with this technique it is possible, for the first time, to build up the mapping of the granular flow during superplastic deformation as can be seen for instance in Fig. 6. From the analysis of these data in terms of the second law of Newton, it is clear that grains flow along the force lines; the aleatory deviations around the force lines are due to the finite size of the elements of the flowing material and to the mechanical interaction between the flowing grains. From an elastic point of view, the tension acting along the *x-axis* induces a compression lateral stress in the deforming solid. Together such stresses are responsible for the flow of matter. In other words, in the plane of Fig. 6, the grains are flowing along the force lines corresponding to a transverse sound wave travelling along the *z-axis*.

One we have presented the main ideas behind crystalline deformation, we now move on to our next topic. Relativistic cosmology, like plastic deformation, is a very broad subject

to study, so in the next section we restrict ourselves to describing only the results that will be compared with the material previously described in this section.

### 3. A brief synthesis of relativistic cosmology

In Newtonian physics, space-time is an infinitely rigid conceptual grid. Gravitational waves cannot exist in this theory. They would have infinite velocity and infinite energy density because in Newtonian gravitation the metrical elastic modulus of space is infinite. Conversely general relativity introduces a finite coupling coefficient between the curvature of space time, described by the Einstein curvature tensor, and the stress energy tensor which describes the mass-energy which gives rise to the curvature. This coupling is expressed by the Einstein equation.

$$T = \frac{C^4}{8\pi G} R \quad (25)$$

where  $T$  is the stress-energy tensor,  $R$  is the Einstein curvature tensor,  $C$  is the speed of light and  $G$  is Newton's gravitational constant. The coupling coefficient  $C^4/(8\pi G)$  is an enormous number, on the order of  $10^{43}$ . This expresses the extremely high stiffness of space which is the reason that the Newtonian law of gravitation is an excellent approximation in normal circumstances, and why gravitational waves have a small amplitude, even when their energy density is very high. The existence of gravitational waves is intuitively obvious as soon as one recognizes that space-time is an elastic medium [105].

The gravitational wave equation [106] for a gravitational plane wave propagating along the  $z$ -axis in the positive direction is

$$\left( \frac{\partial^2}{\partial z^2} - \frac{1}{C^2} \frac{\partial^2}{\partial t^2} \right) h_i^k = 0 \quad (26)$$

where all the  $h_i^k$  are functions of  $(t - z/c)$ , with the gravitational waves propagating at the speed of light to within a fractional accuracy of  $5 * 10^{-11}$  [107].

Einstein, takes an experimental fact from classical mechanics namely that the gravitational mass of a body is equal to its inertial mass and, by making a Gedanken experiment with a local elevator, shows that it is not possible to distinguish by simple mechanical observations whether a system is in a local uniform gravitational field or whether it is in a gravity-free region, but subjected to a constant acceleration of the appropriate magnitude and direction.

With this analysis, he shows the conceptual necessity of the law of the equality of inertial and gravitational mass. He also proposed that this equivalence be a fundamental principle of nature, and that all laws of physics be in agreement with it [108].

To develop his general theory of gravitation, which for the first time deals with the whole universe Einstein in principle needed to compute the motions of discrete masses (galaxies, etc.) separated by vast empty spaces under their mutual

interactions, which is still a problem bristling with extreme difficulties. Instead of this mathematical situation, he considered the actual discrete matter distribution as a homogeneous, continuous distribution, one like a fluid or a tenuous gas. This is no doubt an extreme idealization, but it yields what can be regarded as a first approximation to the actual problem.

Einstein's equation obtained in his 1915 formulation:

$$R_{mn} - \frac{1}{2} g_{mn} R = - \frac{8\pi G}{C^4} T_{mn} \quad (27)$$

where  $R_{mn}$  denotes the Ricci curvature tensor, and  $R$  is a invariant curvature derived from  $R_{mn}$ . The components of the metric tensor  $g_{mn}$  are functions of the coordinates in the sense that they specify the space-time geometry, the invariant distance  $ds$  between two neighboring points in space-time being  $ds^2 = g_{mn} dx^m dx^n$ , where the Einstein convention about repeated indices has been used [109].

Usually the quantity

$$8\pi G/C^4 \equiv K, \quad (28)$$

is called the Einsteinian gravitational constant. Finally,  $T_{mn}$  is the energy momentum (or energy-stress) tensor. The physical meaning of Eq. (27) is that it relates the tensors describing the geometry of space-time (left-hand side) with the energy-momentum tensor arising from the physical content of the universe. These equations tell us in quantitative terms how the physical content of the universe (sources of energy, matter and momentum) stresses the space-time structure and causes its geometrical deformation. Conservation of energy and momentum is guaranteed by the zero divergence of the left-hand side of Eq. (27).

These equations of general relativity, permitted models for homogeneous and isotropic universe which could not be static. In order to ensure a universe that was static in time, Einstein was led to an important change in his original equations. According to Einstein [110], on the left-hand side of the field equations we may add the fundamental tensor  $g_{\mu\nu}$  (in our case  $g_{mn}$ ) multiplied by a universal constant, -  $\lambda$ , at the present unknown, without destroying the general covariance. In place of the field equation Eq. (27) we write:

$$R_{mn} - \frac{1}{2} g_{mn} R - \lambda g_{mn} = -K T_{mn} \quad (29)$$

where the dimension of the universal cosmological constant is that of the inverse of the square of a distance. This field equation, with  $\lambda$  sufficiently small, is in any case compatible with the fact of experience derived from the solar system, and it also satisfies the laws of momentum and energy conservation [110]. The  $\lambda$  - term introduces a force of repulsion between two bodies that increases in proportion to the distance between them. In this scheme, the evolution of the universe is determined by the competition between the cosmic repulsive  $\lambda$  - force and the attractive Newtonian gravitational force. In Einstein's static model of the universe ( $\lambda > 0$ ), the two forces are in balance, in unstable equilibrium.



Most of the work on the standard model in cosmology today is based on the early work of Friedmann [4], which was not fully appreciated at first, because it lacked information about observational consequences and his entire discussion was limited to mathematics, with no attempt to incorporate physics or astronomy. These characteristics meant that scientists in both areas failed to pay attention to the points that were later seen to be the essential message of the paper [111]. From that time until the present, different space-time line elements have been used to analyze gravitational field equations [2,4,5,112].

After the independent work of Robertson [113] and Walker [114], finally a rigorous approach to cosmological models emerged. Based on two well-defined considerations, viz, Weyl's postulate and the cosmological principle of homogeneity and isotropy, they were able to obtain the most general line element as

$$ds^2 = C^2 dt^2 - a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right] \quad (30)$$

where  $a(t)$  is called the radius of the universe, with  $k$ , according to Perlmutter and coworkers [115] taking values  $k = -1, 0$  or  $1$ , represents the three possible geometries for the Universe: open, flat or closed. Here  $(r, \theta, \phi)$  are the constant comoving coordinates of a typical galaxy. Let us to specify that the fact that such coordinates can be defined rest on the assumption that the worldliness of galaxies form a bundle of non-intersecting geodesics diverging from a space-time point in a remote past. Thus a unique member of the bundle passes through each space-time point. The time coordinate is that measured by a galaxy as its proper time. This is Weyl's postulate. The cosmological principle tells us that the hypersurfaces  $t = \text{constant}$  are homogenous and isotropic. Thus Weyl's postulate and the cosmological principle single out a global coordinate system. The time coordinate  $t$ , commonly called cosmic time, arises in this way. There is no contradiction between this global symmetrical coordinate system and the local covariance of general relativity [116].

According to Einstein's general theory of relativity, the geometrical properties of space are determined by the density of energy-matter in the universe. The most obvious energy sources that come to mind are ordinary matter and radiation. A much less obvious source of energy, which potentially can have an enormous impact on the structure of the universe as we figured out is the quantum empty space itself: the quantum vacuum.

Quantum fields theory predicts a huge energy density for the vacuum, and this high value of energy density should have very large gravitational effects. A conservative calculation estimates that space-time geometry appears with severe geometrical distortions over distances of one kilometer or less; so according to Quantum field theory then we can not see our own fingers before our open eyes [117]. However these effects are not observed (even at a distance of about  $10^{23}$  kilometers), and the discrepancy between theory and observation of the energy density for the vacuum is of about 120 orders

of magnitude. There is no generally accepted explanation for this huge discrepancy, although numerous papers have been written about it [31].

Despite the possible conceptual difficulties in accepting the reality of the energy density of the vacuum, there is ample experimental evidence for it; all the experiments on the Casimir effect [118] measure the inward force existing between two closely spaced conducting plates due to the shift in local vacuum energy density [119].

The notion of a vacuum energy is also unfamiliar because that energy cannot be detected by normal techniques. Energies, as in the case of the Casimir effect, are usually determined by measuring the change in the energy of a system when it is modified in some way, or by measuring a difference in energy between two systems. By convention, energies are often measured in relation to the vacuum. When it is defined in this way, the vacuum automatically has zero energy in relation to itself. The only way to establish an absolute measure of energy is by using gravity. As mentioned before, in general relativity, energy is the source of the gravitational fields in the same way that electric charge is the source of electric fields in the Maxwell theory of electromagnetism. Energy density of any kind, including that produced by fluctuation in the vacuum, generates a gravitational field that reveals itself as a change in the geometry of space-time.

In the standard model, as in any quantum field theory, the vacuum is defined as the state of lowest energy, or more properly as the state of least energy density. However, this does not imply that the energy density of the vacuum is zero. The energy density can in fact be positive, negative or zero depending on the values of various parameters in the theory. There are three different terms which contribute to the total vacuum energy density. First there is the bare cosmological constant. That is, the value the cosmological constant would have if none of the known particles existed and if the only force in the universe were gravity. The bare cosmological constant is a free parameter that can be determined only by experimentally measuring the true value of the cosmological constant. Secondly, we have the contribution arising from quantum fluctuations (this term usually is due to the local fluctuations in energy allowed by the Heisenberg's uncertainty principle). The third term represents the contributions due to additional particles and interactions that may exist but we do not yet know about. The value of this term is of course unknown [31,117].

In the Big Bang theory, the effect of ordinary matter on the expansion of the universe is to decelerate this expansion at an ever decreasing rate. However, a positive cosmological constant,  $\lambda > 0$ , would tend to make the galaxies accelerate away from one another and increase the expansion rate of the universe, as has recently have been observed [26,27]. This is the reason behind the renewed interest in the cosmological constant problem, and all related subjects.

In the following paragraphs, some known expressions involving  $\lambda$ , are presented for purposes of comparison, also

with expressions from the plastic deformation of crystalline materials.

As mentioned before, in general relativity the energy density of the vacuum has an absolute meaning, and it can be determined by measuring the gravitational field produced not by matter but by the vacuum itself. According to Einstein’s theory [2,117], the cosmological constant can be related with the vacuum energy density,  $U_V$ , through the following expression:

$$\lambda = \frac{8\pi G}{C^4} * U_V \tag{31}$$

also, the cosmological constant can be related with the radius,  $R_U$ , of the universe,  $\lambda = 1/R_U^2$  [2]. Or in other way,

$$R_U = \frac{1}{\sqrt{\lambda}}. \tag{32}$$

So, the square root of the reciprocal of the cosmological constant is a distance, but not any distance. According to Abbott [117], this distance has a direct physical meaning: it is the length scale over which the gravitational effects of a nonzero vacuum energy density would have an obvious and highly visible effect on the geometry of space an time. So, in principle, by studying the geometrical properties of the universe over length scales on the order of that distance, the value of the cosmological constant can be determined.

All galactic surveys agree that there is no evidence for any global (non local) space-time distortion to the furthest distance accessible to astronomers, namely 15,000 million light-years, or about  $1.5 \cdot 10^{23}$  kilometers. According to this last expression, this implies that the magnitude of the cosmological constant must be smaller than:  $1/(1.5 \cdot 10^{23} \text{ kilometers})^2$ .

In another vein, more recently, mainly in connection with inflationary universe scenarios, cosmological applications of vacuum decay have been extensively investigated [120-126], and many authors have also proposed phenomenological models with a slowly decaying vacuum energy density, considering that the vacuum energy density is a dynamic variable [127-138].

According to Lima in their study about the thermodynamics of decaying vacuum cosmologies [124], vacuum is regarded as a second fluid component transferring energy continuously to the material component. In other words, the slow decay of vacuum energy density provides the source term for matter and radiation. Without going into the details of Lima’s analysis, we can say that their relevant results (for our purpose) are as follows: Lima considers that, in order to have a complete fluid description, besides its vacuum energy-momentum tensor, it is necessary to define the particle current  $N^\alpha$  and the entropy current  $S^\alpha$  in terms of the fluid variables. The current  $N^\alpha$  is given by

$$N^\alpha = nu^\alpha \tag{33}$$

where  $n$  is the particle number density of the fluid component, and  $u^\alpha$  as usual is the four-velocity of the particle,

$u^\alpha = dx^\alpha/ds; \alpha = 1, 2, 3, 4$ , where the vector  $dx^\alpha$  transforms with the following law  $\partial \bar{x}^\alpha = (\partial \bar{x}^\alpha / \partial x^\beta) dx^\beta$ , and  $ds$  is the line element. Since material constituents are continuously generated by the decaying vacuum, the above four-vector (Eq. (32)) satisfies a balance equation  $N_{;\alpha}^\alpha = \varphi$  or, equivalently,

$$\dot{n} + n\theta = \varphi \tag{34}$$

where the over dot denotes covariant derivative along the world line (for instance,  $\gamma := u^a \gamma_{;a}$ , with  $\gamma$  as the fluid energy density) and  $\theta = u^\alpha_{;\alpha}$  is the scalar of the expansion. And  $\varphi$  is the particle source ( $\varphi > 0$ ) or sink ( $\varphi < 0$ ) term. For decaying vacuum models,  $\varphi$  is positive, and must be related with the variation of  $\Lambda$  as follows:

$$-\frac{\dot{\Lambda} C^4}{8\pi G} = \beta \varphi \tag{35}$$

where  $\beta$  is a positive-definite parameter in order to guarantee that for  $\varphi > 0$ , we have  $\Lambda < 0$ .

According to a recent analysis due to Chao-Guang Huang, Lia Liu and Bobo Wang [139] for a static de Sitter universe where its line element of universe is

$$ds^2 = - (1 - H^2 r^2) dt^2 - (1 - H^2 r^2)^{-1} dr^2 + r^2 d\Omega^2 \tag{36}$$

$r$  being the radial coordinate:

$$r \leq H^{-1} = \sqrt{3/\Lambda} \tag{37}$$

(Here Hubble’s parameter which is fixed by the rate of change of the expansion parameter, is independent of time).

Several other expressions have been obtained, some of which are of interest in our synthesis.

From the line element Eq. (36), they can define the “3-volume” of the spatial hypersurface at  $t=constant$  inside the event horizon by:

$$V_{vac} = \frac{4\pi}{3H^3}. \tag{38}$$

The energy density contribute from the cosmological constant is

$$U_V = \frac{3H^2 C^4}{8\pi G}. \tag{39}$$

The pressure of the Sitter universe coming from the cosmological constant is:

$$P_v = -U_V = \frac{3H^2 C^4}{8\pi G} \tag{40}$$

From the thermodynamically quantities of the de Sitter universe [139], the area entropy of the horizon satisfies

$$s_{vac} = \frac{2\pi}{H} E_{vac} \tag{41}$$

where  $E_{vac}$  is the vacuum energy of the Sitter universe within the event horizon.

Also in general relativity, the stress-energy tensor has to satisfy the conservation law expressed by the covariant derivative of  $T_i^k$ ,

$$T_{i;k}^k = O. \tag{42}$$

That is,

$$\frac{\partial T_i^k}{\partial x_k} - \Gamma_{i\beta}^\alpha T_\alpha^\beta = O \tag{43}$$

where  $\Gamma_{i\beta}^\alpha$  is the Cristoffel symbol. According to Weyl [140], the first term is the “real” total force per unit volume:

$$F_i = -\frac{\partial T_i^k}{\partial x_k} \tag{44}$$

which, in general, does not vanish but must be counter balanced by the “pseudo force” which has its origin in the metrical field, namely:

$$\bar{F}_i = \Gamma_{i\beta}^\alpha T_\alpha^\beta = \frac{1}{2} \frac{\partial g_{\alpha\beta}}{\partial x_i} T^{\alpha\beta} \tag{45}$$

As we know, two closely related circumstances characterize the “pseudo forces” arising from the metrical field: Firstly, the acceleration which they impart to a point-mass situated at a definite space-time point (or, more exactly, one passing through this point with a definite velocity) is independent of its mass, *i.e.* the force itself is proportional to the inertial mass of the point-mass at which it acts. Secondly, if we use an appropriate coordinate system, namely, a geodesic one, at a definite space-time point, these forces vanish.

Now we have provided all the background required for our subsequent discussion. In the following section a comparison between the results of the two previous sections will be carried out.

#### 4. A comparison between deformation of crystalline materials and relativistic cosmology

Equation (1) and equation (25) have a formal analogy provided that we identify the following parameters: the axial applied stress  $\sigma$  with the stress-energy tensor  $T$ , the Young Modulus of the Crystalline material with the “stiffness” of vacuum space ( $C^4/8\pi G$ ), and the axial engineering strain  $\epsilon$  with the Einstein curvature tensor  $R$ . Therefore, an elastic Young modulus of the vacuum space,  $E'_{YVM}$  can be defined by:

$$E'_{YVM} \equiv \frac{C^4}{8\pi G} \tag{46}$$

where we note that  $E'_{YVM}$  has dimensions of an elastic modulus time the square of a standard distance. This is due to the fact that the tensor  $R$ , has dimensions of the inverse of a squared distance.

The current  $N^\alpha$  given by Eq. (32) is equivalent to Eq. (7) if we identify the four-velocity of the particle  $u^\alpha$  with the

mean value of the gliding velocity of mobile dislocations  $v_g$ , and  $n$ , the particle number density of the fluid component with the mobile dislocation density of dislocations  $\rho_m$  (whose dimensions are: number of centimeters of dislocations per cubic centimeter). We note, that  $\dot{\epsilon}/b^2$  has the dimensions of  $N^\alpha$ , and an equivalent physical meaning; therefore  $N^\alpha$  is proportional to the four-momentum density of particle of the fluid.

By comparing Eq. (24) and Eq. (31), it is clear that  $U_{\sigma_i}$  in dislocation creep plays the role of  $U_V$  in Relativistic cosmology. In fact, by using Eqs. (12), (2), and (6) in Eq. (24) it is possible that  $\rho$  can be written as:

$$\rho = \frac{2 \left[ \frac{2}{3} \frac{(1+\nu)}{(1-\nu)} \right]^2}{\alpha^2 M^2 b^2 \rho_{gr}^2} * \frac{E}{v_s^4} * U_{\sigma_i} \tag{47}$$

which clearly exhibits the resemblance respectively between respective the role of  $\rho$  and  $\lambda$ .

Also, from Eq. (23) which describes the length at which the presence of the dislocation density  $\rho$  causes an observable elastic distortion of the crystal lattice it is clear that this equation plays an analogous role to equation (32), which describes the distance at which the presence of a volumetric energy of vacuum (proportional to  $\lambda$ , Eq. (40)) causes an observable gravitational distortion of the cosmic space-time geometry.

It is important to note that, also equation (23) and its equivalent Eq. (17) for dislocation creep indicate that this distance  $d_{sg}$  corresponds to the geometrical region where this dislocations and opposite-signed dislocations collide after gliding along their mean free path (see Eq. (16)) from their respective dislocation sources. In these collision regions eventually annihilation events eventually occur.

Equation (15), which describes the creation rate of dislocations per unit volume, has the classical form of expressions for particle-antiparticle creation events of matter in nuclear physics. This expression (Eq. (15) could be rewritten by using Eq. (24). In this case:

$$\frac{d\rho^+}{dt} = \frac{4(1+\gamma)}{\alpha^2 M^2 \mu_m b^2} \frac{dU_{\sigma_i}^+}{dt} \tag{48}$$

where the (+) symbols are used to denote creation events.

After comparing between Eq. (48) and Eq. (15), it is clear that

$$\frac{dU_{\sigma_i}^+}{dt} = \frac{\alpha^2 M^2}{4(1+\gamma)} * \sigma \dot{\epsilon}. \tag{49}$$

This analysis suggests that for Relativistic Cosmology the equation corresponding to Eq. (48), will be,

$$\frac{d\lambda^+}{dt} = \frac{8\pi G}{C^4} \frac{dU_V^+}{dt}. \tag{50}$$

If  $\lambda$  is associated to the dislocation density  $\rho$ , then the meaning of  $d\lambda^+/dt$  could be found by multiplying this equation times  $(10^{80}/R_{OU})$ . Then we have,

$$\frac{dn_B^+}{dt} \equiv \frac{10^{80}}{R_{OU}} \frac{d\lambda^+}{dt} = \frac{10^{80}}{R_{OU}} \frac{8\pi G}{C^4} * \frac{dU_V^+}{dt} \tag{51}$$

and  $(10^{80}/R_{OU})(d\lambda^+/dt)$  could be considered to be proportional to the creation rate of matter per unit volume,  $dn_B^+/dt$ .

After comparing between Eq. (48) with Eq. (51), we note that in Eq. (48) the inverse of an energy per unit length of dislocation appears ( $\sim \mu_m b^2$ ). Therefore it is possible to suggest that in Eq. (51) in some way the inverse of the energy required to form a neutron,  $u_N$ , is described by the factor facing  $U_V^+$ . In other words without taking into account some minor numerical factors,

$$u_N \equiv m_N C^2 \cong \frac{R_{OU}}{10^{80}} * \left( \frac{C^4}{8\pi G} \right) \tag{52}$$

or, in terms of the Einsteinian gravitational constant

$$u_N \cong \frac{R_{OU}}{K} * \frac{1}{10^{80}}. \tag{53}$$

We note that, if the formal analogy here presented has some deep physical meaning, then Eq. (53) has the following interpretation: the self energy of a neutron is directly related to the actual universe radius and to the inverse of the Einsteinian constant of gravitation. In other words, the value of the self energy of a quantum particle arising, in principle, from a quantum field theory, is linked to a universe scale parameter,  $R_{OU}$  and to a general relativistic gravitational constant  $K$ .

Obviously Eqs. (34) to (35) from some cosmology model previously described are related to Eqs. (48) to (51).

From Eq. (5) applied for  $U_{\sigma_i}$  is clear that:

$$\sigma_i = 2E \sqrt{\left( \frac{U_{\sigma_i}}{2E} \right)}. \tag{54}$$

This expression shows how an elastic volumetric density of energy inside a crystal causes an internal stress. This fact is in analogy to Eq. (40), where a volumetric energy arising from vacuum cosmic space,  $U_V$ , causes a gravitational pressure (or stress) in the de Sitter model of the universe.

Also, in a crystalline solid the longitudinal and transverse sound waves obey wave equations [141], which have the same mathematical structure of the gravitational wave equation given by Eq. (26).

It is also very interesting that, following in the vein suggested by the previous results, it is possible to obtain some complementary expressions involving  $\lambda$  in the cosmology of the de Sitter universe. If we use Eq. (41) and (38) an area entropy of the horizon per “3-volume” of the spatial hypersurface at  $t = \text{constant}$  inside the event horizon,  $S_v$ , can be defined,

$$S_V \equiv \frac{s_{vach}}{V_{vac}} = \frac{2\pi E_{vac}}{H V_{vac}}. \tag{55}$$

Obviously, in this scheme  $U_V$ , is given by,

$$U_V = \frac{E_{vac}}{V_{vac}} \tag{56}$$

and  $S_V$  can be written as:

$$S_V = \frac{2\pi}{H} U_V. \tag{57}$$

And, if for short we rename  $S_V$  as the entropy per unit volume of the Sitter universe, then we have that the change with time of the entropy density in the de Sitter universe is proportional to the time derivative of the density of the vacuum energy of cosmic space,

$$\frac{dS_V}{dt} = \frac{2\pi}{H} \frac{dU_V}{dt}. \tag{58}$$

Therefore, by using Eq. (31) in Eq. (58) the following relationship for  $dS_V/dt$  can be obtained,

$$\frac{dS_V}{dt} = \frac{2\pi}{H} \frac{1}{K} \frac{d\lambda}{dt} \tag{59}$$

or

$$\frac{dS_V}{dt} = \frac{C^4}{4HG} \frac{d\lambda}{dt} \tag{60}$$

Finally, by taking Eq. (17), it is possible to find another interesting result for cosmology. Equation (17) could be written as

$$\sigma b R_{sg} = M^2 u_d \tag{61}$$

where  $R_{sg} = d_{sg}/2$ , and  $\phi_M = 1/M$  has been used. Equation (61) admits the following physical interpretation: as we know, Ref. 49,  $\sigma b$  is the force per unit length acting on a dislocation. Therefore if such force per unit length acts through a length  $R_{sg}$  (corresponding to the subgrain radius), work per unit length with magnitude  $M^2 u_d$  is developed. In this case, the swept area is  $b$  times  $R_{sg}$ . Equation (61) relates stresses and distances through which stresses perform work in order to lead to the creation energy of the unit length of dislocations.

In analogy to the previous case, for cosmology it is obvious that we have

$$U_V * \frac{4\pi}{3} (R_{OU})^3 = 10^{80} * u_N \tag{62}$$

where  $10^{80}$  corresponds to the baryon number of this universe [48], and  $u_N$  is used for denoting the creation energy of one neutron. Eq. (62) can be rewritten as follow:

$$\frac{4\pi}{3} U_V \left( \frac{R_{OU}}{10^{40}} \right)^2 R_{OU} = u_N. \tag{63}$$

This equation, resembles Eq. (61) from dislocation creep, but in Eq. (61) we have a distance of magnitude  $b$  which swept an area of magnitude  $bR_{sg}$ , and here, we have the square of a distance  $(R_{OU}/10^{40})^2$ , an area which swept a volume of magnitude  $(4\pi/3) (R_{OU}/10^{40})^2 R_{OU}$ . By taking into account this situation Eq. (63) may have the following physical interpretation:  $U_V (R_{OU}/10^{40})^2$  gives the force arising from the energy momentum tensor  $T$  times an

area  $(R_{OU}/10^{40})^2$ , and this force times  $R_{OU}$  gives the energy required to create a neutron. If we define  $r_N$  as,

$$r_N \equiv \frac{R_{OU}}{10^{40}} \quad (64)$$

the Eq. (63) may be rewritten as:

$$R_{OU} = \frac{3}{4\pi} \frac{u_N}{U_V r_N^2}. \quad (65)$$

In this regard from Eq. (51) it is clear that it can be integrated over volume, so:

$$\int_{V_{ou}} \frac{10^{80}}{R_{OU}} d\lambda^+ dv = \frac{10^{80}}{R_{OU}} \frac{8\pi G}{C^4} \int_{V_{OU}} dU_V^+ dv \quad (66)$$

which is integrated and leads to:

$$10^{80} = \frac{10^{80}}{R_{OU}} \frac{8\pi G}{C^4} U_V \left( \frac{4}{3} \pi R_{OU}^3 \right) \quad (67)$$

and by using Eq. (52), the following equation is obtained:

$$R_{OU} = \frac{3}{4\pi} \frac{u_N}{\left[ U_V \left( \frac{R_{OU}}{10^{40}} \right)^2 \right]}. \quad (68)$$

Now it is clear that Eq. (51) leads to Eq. (68) or to Eq. (65) [by using Eq. (64)]. This analysis proves that, as proposed above,  $(10^{80}/R_{OU})(d\lambda^+/dt)$  is the creation rate of matter per unit volume, and also shows internal compatibility between Equation (51) and Eq. (62). The physical analysis following Eq. (63), and the comparison between Eq. (68) and Eq. (17) suggest that  $r_N = (R_{OU}/10^{40})$ , possibly plays the same role in Eq. (68) that  $b$  plays in Eq. (17). This fact suggests the possibility that vacuum cosmic space has a constant characteristic distance called  $r_N$ . In other words, there is a possibility that vacuum cosmic space has a crystalline structure with a lattice parameter  $r_N = R_{OU}/10^{40}$ .

The general possibility that vacuum space could have a crystalline character has been suggested before. See for instance Ref. 142 for the case of a 4 dimensional electromagnetic space.

## 5. Discussion and conclusions

As far as we know, a formal, and physical analogy has never before been established with Relativistic Cosmology in the almost exhaustive way presented here. Eight mathematical expressions for dislocation creep resembling cosmological expressions have been presented.

From the comparison between mathematical expressions, and by using a physical analysis, two new equations have been obtained for dislocation creep. Also, as far as we know, five new expressions have been obtained for Relativistic Cosmology. In principle, two of these equations for Cosmology allow us to determine physical parameters never determined before. One of them, Eq. (53), makes it possible to calculate the neutron energy,  $u_N$ , from the knowledge of the universe radius (present) and the Einsteinian gravitational constant.

The other expression, Eq. (64), defines the radius of the neutron,  $r_n$ , as the present universe radius divided by  $10^{40}$ . Elementary calculations give the right order of magnitude for each  $u_N$  and  $r_N$ . Reluctant to follow the analogy between dislocation creep and Relativistic cosmology to the limit, we feel obliged by the above results to search for the ultimate implications concerning the possibility that vacuum cosmic space could have a crystalline structure with lattice parameter  $r_N = R_{OU}/10^{40}$ . This investigation into the general implications of a possible vacuum cosmic space with crystalline structure will be analyzed in other papers.

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