

The Gordon and Zarmi model for convective atmospheric cells under the ecological criterion applied to the planets of the solar system.

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In the present paper we calculate the surface temperatures of the nine planets of the Solar System by means of the Gordon and Zarmi model for dealing with the Earth's wind energy as a solar-driven Carnot-like heat engine, incorporating the role of the greenhouse effect and internal irreversibilities in the performance of this heat engine model. This oversimplified Carnot-like engine corresponds very approximately to the global scale motion of wind in convective cells. Our numerical results for the surface temperatures are in good agreement with the observed temperatures reported in the literature. Our calculations were made by means of two regimes of performance of the model: the maximum power regime and the maximum ecological function regime. In particular, Venus and Earth temperatures were calculated with a new approach by using the role of the tropopauses on the convective cells.

Keywords: Convective cells; finite-time thermodynamics; solar system.

En este artículo calculamos las temperaturas superficiales de los nueve planetas del sistema solar mediante el modelo de Gordon y Zarmi que se usa para tratar con la energía de los vientos como una máquina tipo Carnot manejada por el Sol, incluyendo el papel del efecto invernadero y de las irreversibilidades internas en el modo de operación del modelo de máquina térmica. Esta máquina tipo Carnot sobresimplificada corresponde en buena aproximación con el movimiento a escala global de los vientos en celdas de convección. Nuestros resultados numéricos para las temperaturas superficiales están en buen acuerdo con las temperaturas observadas reportadas en la literatura. Nuestros cálculos fueron realizados mediante dos regímenes de operación: el régimen de potencia máxima y el régimen de función ecológica máxima. En particular, las temperaturas de Venus y de la Tierra fueron calculadas mediante un nuevo enfoque utilizando el papel de las tropopausas sobre las celdas de convección.

Descriptores: Celdas de convección; termodinámica de tiempos finitos; sistema solar.

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1. Introduction

In a recent paper [1], Fischer and Hoffmann have shown that a simple endoreversible model (the so-called Novikov engine) can reproduce the complex engine behavior of a quantitative dynamical simulation of an Otto engine including, but not limited to, effects from losses due to heat conduction, exhaust losses and frictional losses. In that article, the spirit of finite-time thermodynamics (FTT) is illustrated emphasizing the virtues and limitations of FTT-methodology. However, the usefulness of FTT-models is shown beyond any doubt. In fact, we can assert that the FTT-spirit is consistent with the spirit of a Carnotian thermodynamics in the sense of the search for certain kind of limits for thermodynamic

variables and functionals. The problem of thermal balance between the planets of the solar system and the Sun under an FTT approach has been treated by several authors [2-8]. In some of these articles the question of the conversion of solar energy into wind energy is also discussed. When only the global thermal balance between the Sun and a planet is considered, one can roughly obtain the planet's surface temperature T_p . If the conversion of solar energy into wind energy is to be modeled, it is necessary to involve at least two representative atmospheric temperatures in order to make the creation of work possible, that is, to take the planet's atmosphere as a working fluid that converts heat into mechanical work. This permits us to introduce the concept of atmospheric "heat engine" in a natural way. In this context, pro-

cess variables such as work rate, heat fluxes and efficiency for instance [8] find a simple theoretical framework where thermodynamical restrictions play an important role. This is in contrast with disciplines such as non-equilibrium thermodynamics and hydrodynamics based on local differential equations where the transition from local to global variables is not a trivial task [9]. In 1989, Gordon and Zarmi (GZ) [2] introduced an FTT model taking the Sun-Earth-Wind system as an FTT-cyclic heat engine where the heat input is solar radiation, the working fluid is the Earth's atmosphere, the energy of the winds is the work produced and the cold reservoir to which the engine rejects heat is the surrounding 3K universe. By means of this oversimplified model, Gordon and Zarmi were able to obtain reasonable values for the annual average power in the Earth winds and for the average maximum and minimum temperatures of the atmosphere, without resorting to detailed dynamic models of the Earth's atmosphere, and without other effects (such as the Earth's rotation, the Earth's translation around the Sun, ocean currents, etc.). Later, several authors [3-6] extended the GZ model to take into account the wind energy dissipation and other such effects as the inclination angle of the Earth's axis with respect to the ecliptic. Another extension of the GZ model was presented in Refs. 7 and 8. In Ref. 7, the GZ model was used for atmospheric cycles with internal entropy production and under a new criterion of merit named the ecological criterion [10], which consists in the maximization of a function E that represents a good compromise between high power output and low-entropy production. The function E is given by

$$E = P - T_{ex}\Sigma, \quad (1)$$

where P is the power output of the cycle, Σ is the total entropy production (system plus surroundings), and T_{ex} is the temperature of the cold reservoir. This optimization criterion for the case of the so-called Curzon-Ahlborn cycle [11], for instance, leads to a cycle configuration such that for maximum E it produces around 75% of the maximum power and only about 25% of the entropy produced in the maximum power regime [12]. By means of this optimization criterion in Ref. 7, reasonable values for the annual average power of the winds and for extreme temperatures of the Earth's atmosphere were also found. In Ref. 8, an additional extension of the GZ model was proposed by means of the inclusion of a coefficient γ that takes into account the greenhouse effect. This coefficient was introduced by De Vos [4] to consider the attenuation of the heat flux emitted by the Earth in the far-infrared spectrum. This coefficient can be taken as the normalized greenhouse effect introduced by Raval and Ramanathan in Ref. 13, which is defined as the infrared radiation trapped by atmospheric gases and clouds. In the present paper, we apply the extended GZ model used in Ref. 8 to calculate the surface temperature of nine planets of the solar system. The article is organized as follows: In Sec. 2, we briefly discuss the GZ model under two performance regimes (maximum power regime (MPR) and maximum ecological regime (MER)). In Sec. 3, we introduce the generalization

of the GZ model including the albedo effect, the greenhouse effect and the internal irreversibilities of the cycle by means of a parameter arising from the Clausius inequality [14, 15]. In this section, we apply the extended GZ model to calculate the surface temperature of the nine planets of the solar system. The model is used under both the maximum power and the maximum ecological regimes. We also present an alternative calculation of the surface temperature of Venus by considering the very high greenhouse effect of this planet, which has an atmosphere with a great concentration of CO_2 . This calculation uses a cold reservoir different from the surrounding 3K universe, consisting in an atmospheric structure of the tropopause type. Finally, in Sec. 4, we present the conclusions.

2. Endoreversible GZ model for atmospheric convection

As is well known [4], cosmic radiation, starlight and moonlight can be neglected for the thermal balance of any of the planets of the solar system and only the following quantities have an influence: the incident solar influx or solar constant I_{sc} , the planet's albedo ρ , and the greenhouse effect of the planet's atmosphere crudely evaluated by means of a coefficient γ [4, 13]. The endoreversible GZ model is based on the annual average quantities and thus it does not represent actual convective cells, but a kind of annual virtual cell that takes into account the global thermodynamic restrictions over the convection as a dominant energy transfer mechanism in the air (which has a large Rayleigh number). Besides, this kind of model must only be taken as that producing better upper bounds than those calculated by means of classical equilibrium thermodynamics, which is the main purpose of FTT.

2.1. Maximum power regime

In Fig. 1, a schematic view of a simplified Sun-Earth-Winds system as a heat engine cycle is depicted. This cycle consists of four branches:

1. two isothermal branches, one in which the atmosphere absorbs solar radiation at low altitudes and one in which the atmosphere rejects heat at high altitudes to the universe, and
2. two intermediate instantaneous adiabats [16] with rising and falling currents. In Ref. 17, it was shown that a Curzon-Ahlborn FTT cycle in the endoreversible limit with instantaneous adiabats is reached for large compression ratios. In the GZ virtual cells, it is feasible to consider that this condition is fulfilled.

According to GZ, this oversimplified Carnot-like engine corresponds very approximately to the global scale motion of wind in convective cells. Below, we use all of GZ model's assumptions. For example, the work performed by the

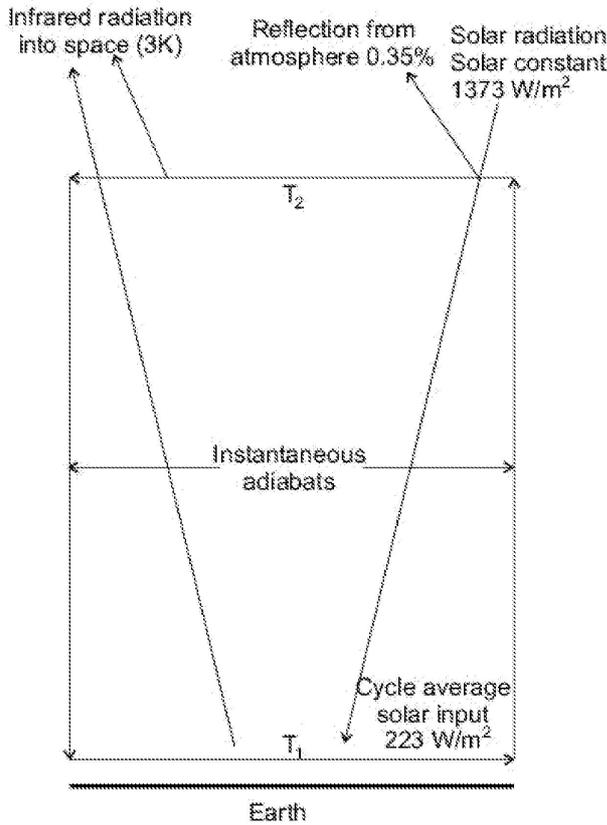


FIGURE 1. Scheme of a simplified solar-driven heat engine (taken from Ref. 2).

working fluid in one cycle W , the internal energy of the working fluid U , and the yearly average solar radiation flux q_s are expressed per unit area of the Earth's surface. The temperatures of the four-branch cycle are taken as follows: T_1 is the working fluid temperature in the isothermal branch at the lowest altitude, where the working fluid absorbs solar radiation for half of the cycle. During the second half of the cycle, heat is rejected via black-body radiation from the working fluid at temperature T_2 (highest altitude of the cell) to the cold reservoir at temperature T_{ex} (the surrounding 3K universe). In the GZ model, the objective is to maximize the work per cycle (average power) subject to the endoreversibility constraint [16], that is,

$$\Delta S_{int} = \int_0^{t_0} \left(\frac{q_s(t) - \sigma [T^4(t) - T_{ex}^4(t)]}{T(t)} \right) dt = 0, \quad (2)$$

where ΔS_{int} is the change of entropy per unit area, t_0 is the time of one cycle, σ is the Stefan-Boltzmann constant ($5.67 \times 10^{-8} \text{ W/m}^2\text{k}^4$), and q_s , and T are functions of time

t , taken as [2]

$$T(t) = \begin{cases} T_1; & 0 \leq t \leq \frac{t_0}{2} \\ T_2; & \frac{t_0}{2} \leq t \leq t_0 \end{cases}$$

$$q_s(t) = \begin{cases} 0; & \frac{t_0}{2} \leq t \leq t_0 \\ \frac{I_{sc}(1-\rho)}{2}; & 0 \leq t \leq \frac{t_0}{2} \end{cases}; \quad (3)$$

in the same way, $T_{ex} = 3 \text{ K}$ for $0 \leq t \leq t_0$, with I_{sc} the yearly average solar constant (1373 W/m^2) and $\rho = 0.35$ [2], the effective average albedo of the Earth's atmosphere. The GZ model maximizes the work per cycle W , taken from the first law of thermodynamics:

$$\Delta U = -W + \int_0^{t_0} \{q_s(t) - \sigma [T^4(t) - T_{ex}^4(t)]\} dt = 0, \quad (4)$$

by means of the Euler-Lagrange formalism and denoting average values as

$$\bar{T} = \frac{T_1 + T_2}{2}, \quad \bar{T}^n = \frac{T_1^n + T_2^n}{2}, \quad \bar{q}_s = I_{sc} \frac{(1-\rho)}{4}, \quad (5)$$

where n is an integer with values $n = 3$ or 4 . The factor of $1/4$ arises from a factor of $1/2$ to account for the day/night difference, and a geometric factor of $1/2$ to account for the Earth's cross section, which is intercepted by solar radiation, as opposed to the corresponding hemispherical surface area of the Earth. From Eqs.(4) and (5), and taking into account the constraint given by Eq.(2), GZ construct the following Lagrangian L :

$$L = T^4(t) + \lambda \left[\frac{q_s(t)}{T(t)} - \sigma T^3(t) \right], \quad (6)$$

where λ is a Lagrange multiplier. By using $\partial L(t)/\partial T(t) = 0$, GZ found the following values for the Earth's atmosphere: $T_1 = 277 \text{ K}$, $T_2 = 192 \text{ K}$ and $P_{max} = W_{max}/t_0 = 17.1 \text{ W/m}^2$. These numerical values are not so far from "actual" values, which are $P \approx 7 \text{ W/m}^2$ [18], $T_1 = 290 \text{ K}$ (at ground level) and $T_2 \approx 195 \text{ K}$ (at an altitude of around 75-90 Km). However, as GZ assert, their power calculation must be taken as an upper bound due to several idealizations in their model. In Ref. 7, another endoreversible case was analyzed, but using as a cold reservoir the tropopause shell with $T_{ex} = 200 \text{ K}$. In this case, the following Lagrangean was used:

$$L = \bar{q}_s + \sigma \bar{T}_{ex}^4 - \sigma \bar{T}^4 - \alpha \left[\frac{\bar{q}_s}{\bar{T}_1} - \frac{\sigma(T_1^3 + T_2^3)}{2} - \sigma T_{ex}^4 \left(\frac{1}{T_1} + \frac{1}{T_2} \right) \right], \quad (7)$$

with α a Lagrange multiplier; and by numerically solving $\partial L(t)/\partial T(t) = 0$, we obtained

$$T_1 = 293.387 \text{ K}, \quad T_2 = 239.267 \text{ K},$$

which are excellent values for convective cells restricted to the troposphere. If these temperature values are substituted in the average power [7]

$$P = \bar{q}_s + \sigma \bar{T}_{ex}^4 - \sigma \bar{T}^4, \quad (8)$$

the following value is immediately obtained:

$$P = 10.758 \text{ W/m}^2,$$

which is a good value for the wind power [18].

2.2. Ecological criterion

As De Vos and Flater [3] state, no mechanism guarantees that the atmosphere maximizes the wind power. In fact, some authors [19–21] have recognized that the Earth’s atmosphere operates at nearly its maximum efficiency; thus, from an FTT point of view, an ecological-type criterion seems feasible. This is due to the properties of the E -function, which at its maximum value represents an austere compromise between power and entropy production, additionally leading to a high efficiency [10, 12]. This ecological criterion, as previously occurred with the concepts of power output and efficiency [22], has also been used in the context of irreversible thermodynamics [23,24]. In particular, in Ref. 7 the so-called ecological criterion was applied to the GZ-model. This criterion consists in maximizing Eq. (1). By means of the second law of thermodynamics, first, we calculate ΔS_u , the total entropy change per cycle (system plus surroundings),

$$\Delta S_u = \int_0^{t_0} \left(\frac{-q_s(t) + \sigma [T^4(t) - T_{ex}^4]}{T(t)} \right) dt; \quad (9)$$

from Eqs. (3), we obtain

$$\begin{aligned} \Delta S_u = & \int_0^{\frac{t_0}{2}} \left[-\frac{q_s(t)}{T_1} + \sigma \left(T_1^3 - \frac{T_{ex}^3}{T_1} \right) \right] dt \\ & - \int_{\frac{t_0}{2}}^{t_0} \frac{\sigma (T_2^4 - T_{ex}^4)}{T_{ex}}; \end{aligned} \quad (10)$$

Thus, the total entropy production is given by [7],

$$\Sigma = \frac{\Delta S_u}{t_0} \approx -\frac{\bar{q}_s}{T_1} + \frac{\sigma}{2} \left[T_1^3 + \frac{T_2^4}{T_{ex}} \right], \quad (11)$$

here, we have used the approximation $\bar{q}_s \gg \sigma T_{ex}^4$ ($223 \text{ W/m}^2 \gg 4.59 \times 10^{-6} \text{ W/m}^2$) with $T_{ex} = 3 \text{ K}$. So, the ecological function E for this case is

$$E = \bar{q}_s - \sigma \bar{T}^4 + \frac{T_{ex} \bar{q}_s}{T_1} - \frac{\sigma T_{ex}}{2} \left[T_1^3 + \frac{T_2^4}{T_{ex}} \right]. \quad (12)$$

By using Eq. (12) and the constraint given by Eq. (2), we proposed the following Lagrangean function L_E :

$$\begin{aligned} L_E = & \bar{q}_s - \sigma \bar{T}^4 + \frac{T_{ex} \bar{q}_s}{T_1} \\ & - \frac{\sigma T_{ex}}{2} \left[T_1^3 + \frac{T_2^4}{T_{ex}} \right] - \alpha \left[\frac{\bar{q}_s}{T_1} - \sigma \bar{T}^3 \right], \end{aligned} \quad (13)$$

with α being the Lagrange multiplier. By substituting the values of \bar{q}_s , σ and T_{ex} and numerically solving $\partial L_E(t)/\partial T(t) = 0$, we find

$$T_1 = 294.08 \text{ K}, \quad T_2 = 109.54 \text{ K}, \quad P = 6.89 \frac{\text{W}}{\text{m}^2},$$

which are reasonable values for T_1 and P , but not for T_2 . However, if we use as a cold reservoir, the tropopause shell [28] with $T_{ex} \approx 200 \text{ K}$, we can now use the Lagrangean function,

$$\begin{aligned} L = & \overline{q_s + \sigma T_{ex}^4} - \sigma \bar{T}^4 + \left(\bar{q}_s + \frac{\sigma T_{ex}^4}{2} \right) \frac{T_{ex}}{T_1} \\ & - \frac{\sigma T_{ex}}{2} \left(T_1^3 + \frac{T_2^4}{T_{ex}} \right) - \frac{\sigma T_{ex}^4}{2} \\ & - \beta \left[\frac{\bar{q}_s}{T_1} - \frac{\sigma (T_1^3 + T_2^3)}{2} + \sigma T_{ex}^4 \left(\frac{1}{T_1} + \frac{1}{T_2} \right) \right], \end{aligned} \quad (14)$$

with β a Lagrange multiplier. By using again the Euler-Lagrange formalism, we numerically obtain

$$T_1 = 303 \text{ K}, \quad T_2 = 219 \text{ K}, \quad P = 7 \frac{\text{W}}{\text{m}^2},$$

which are very good values, for T_1 , T_2 and P . Besides, these values are restricted to typical values in the troposphere, where the climatic phenomena occur. It is important to note that the power values (6.89 W/m^2) and (7 W/m^2) calculated by means of the ecological function in this section did not consider the greenhouse effect (γ - coefficient). When this quantity is taken into account, the values of P are greater than (7 W/m^2) [8]. These scenarios lead to greater upper bounds for the wind’s power, permitting an energy excess for other relevant dissipative processes such as ocean currents and biological structuring.

3. Nonendoreversible GZ model including greenhouse effect for the planets of the solar system

In this section we include the greenhouse effect in the GZ model, performing in both maximum power and maximum ecological function regimes. Internal irreversibilities are also included through a parameter measuring the degree of departure from the reversible regime. This extended GZ model is applied to the nine planets of the solar system.

3.1. Maximum power regime

Within the context of FTT thermal engine models, some authors [14, 15], [25–27] have included the role of internal irreversibilities in thermal cycles through a lumped parameter R arising from the Clausius inequality,

$$\Delta S_{w1} + \Delta S_{w2} \leq 0, \quad (15)$$

with ΔS_{w1} being the entropy change of the working fluid along the upper isothermal branch and ΔS_{w2} the corresponding entropy change along the lower isothermal branch. For the endoreversible case, the sum of ΔS_{w1} and ΔS_{w2} is zero; but when internal irreversibilities are taken into account,

$$\Delta S_{w1} + \Delta S_{w2} < 0. \tag{16}$$

To convert Eq. (16) to a constraint, the inequality is transformed into an equality by means of

$$\Delta S_{w1} + R\Delta S_{w2} = 0, \tag{17}$$

where R is given by [14, 15]

$$R = \frac{\Delta S_{w1}}{|\Delta S_{w2}|} < 1; \tag{18}$$

thus, in terms of R , the thermodynamic restriction Eq. (2) becomes

$$\begin{aligned} \Delta S_R &= \int_0^{t_0} \left(\frac{q_s(t) - \sigma(1-\gamma)R[T^4(t) - T_{ex}^4(t)]}{T(t)} \right) dt \\ &= 0, \end{aligned} \tag{19}$$

where we have included the greenhouse coefficient γ [4] through the factor $(1-\gamma)$ affecting the flow released into space. From Eqs. (3) and (5), Eq. (19) can be written as

$$\frac{\bar{q}_s}{T_1} - \frac{\sigma R}{2} [(1-\gamma)T_1^3 + T_2^3] = 0. \tag{20}$$

By means of Eq. (20) as the new constraint we propose the following Lagrangean:

$$\begin{aligned} L'_R &= \bar{q}_s - \frac{\sigma}{2} [(1-\gamma)T_1^4 + T_2^4] \\ &\quad - \lambda' \left\{ \frac{\bar{q}_s}{T_1} - \frac{\sigma R}{2} [(1-\gamma)T_1^3 + T_2^3] \right\}, \end{aligned} \tag{21}$$

which λ' being a Lagrangean multiplier. By means of the extremal conditions, we obtain the following equations:

$$T_1^5 - T_2 T_1^4 - \frac{2\bar{q}_s}{\sigma R(1-\gamma)} T_2 = 0, \tag{22}$$

$$T_1^4 - \frac{1}{(1-\gamma)} T_1 T_2^3 - \frac{2\bar{q}_s}{\sigma R(1-\gamma)} = 0 \tag{23}$$

By numerically solving equations (22) and (23), we obtain, for T_1 (the surface temperature), the values shown in Table I for the nine planets of the solar system. The values for ρ and γ , I_{sc} and T_{exp} , were taken from Refs. 4 and 19. In column T_{cal} , we show the values for T_1 obtained by means of the extended GZ model operating under the maximum power regime. In all of the cases of Table I, $R = 1$ was considered, that is, all the T_1 -values are for an endoreversible model. It is important to note that, for these calculations, the greenhouse effect was only considered in the first half of the GZ-cycle, that is, in the lowest isothermal branch. In Ref. 8, it was shown that this scenario is more appropriate for the inclusion of the greenhouse effect. In Table I, we also include some cases (Venus, Earth, Saturn, Uranus and Pluto) where calculations for $R < 1$ are also shown to improve T_{cal} (T_1)

TABLE I. Surface temperatures of the nine planets under a maximum power regime, taking the greenhouse effect only in the lower part of the atmosphere.

Planets	ρ	γ	I_{SC} ($\frac{W}{m^2}$)	R	T_{cal} (K)	T_{exp} (K)
Mercury	0.058	0.0	9200	1	489	450
Venus	0.71	0.997	2600	1	727.4	
				0.97	732.9	740
Earth	0.35	0.2	1373	0.93	740.7	
				1	289.5	
				0.97	290.2	288
Mars	0.17	0.0	600	0.95	293.2	
				1	239.6	220-240
Jupiter	0.5	0.4	50	1	125.2	120
Saturn	0.6	0.4	15	1	87.6	
				0.94	88.9	
				0.90	89.4	95
Uranus	0.7	0.4	4	0.88	90.5	
				0.75	95.1	
				1	158.6	
Neptune	0.7	0.4	2	0.98	58.89	59
				0.95	59.2	
Pluto	0.5	0.1	1	1	43.5	
				0.9	44.8	
				0.8	46.1	50
				0.7	47.6	
				0.65	49.2	

in comparison with T_{exp} . In those cases, a certain degree of internal irreversibility is considered through the lumped parameter R . As can be seen in Table I, through the GZ-model the surface temperatures of the solar system planets can be recuperated in an appropriate way.

3.2. Maximum ecological function regime

We now use the ecological function given by Eq. (1) as an objective function. In this case, by means of the considerations used in Sect. 3.1, we used the following Lagrangean function [8]:

$$\begin{aligned} L'_{ER} &= \bar{q}_s - \frac{\sigma}{2} [(1-\gamma)T_1^4 + T_2^4] + \frac{T_{ex}\bar{q}_s}{T_1} \\ &\quad - \frac{\sigma R T_{ex}}{2} \left[(1-\gamma)T_1^3 + \frac{T_2^4}{T_{ex}} \right] \\ &\quad - \alpha' \left\{ \frac{\bar{q}_s}{T_1} - \frac{\sigma R}{2} [(1-\gamma)T_1^3 + T_2^3] \right\}, \end{aligned} \tag{24}$$

which under Euler-Lagrange formalism $\partial L(t)/\partial T(t) = 0$ leads to the equations

$$T_1^5 + \left[\frac{3T_{ex}}{4}R - (1 + R)T_2 \right] T_1^4 - \frac{2\bar{q}_s}{3\sigma} \left(\frac{1+R}{R} \right) \left(\frac{1}{1-\gamma} \right) T_2 + \frac{T_{ex}\bar{q}_s}{2\sigma} \left(\frac{1}{1-\gamma} \right) = 0, \quad (25)$$

$$T_1^4 + \frac{1}{(1-\gamma)}T_1T_2^3 - \frac{2\bar{q}_s}{\sigma R} \left(\frac{1}{1-\gamma} \right) = 0. \quad (26)$$

In Table II, we show the Earth’s surface temperature without the inclusion of the greenhouse effect [$\gamma = 0$ in Eqs.(25) and (26)] and the surface temperatures for the rest of the solar system planets, by considering some cases with $R < 1$. In Tables III and IV, we show as particular cases the calculations of surface temperatures of the Earth and Venus for both maximization regimes MPR and MER, respectively. In both cases, $T_{ex} = 3K$ and $R = 1$ were used, and several values of γ were considered. In the cases of the Earth and Venus, their atmospheres have atmospheric shells known as tropopauses with temperatures approximately constant (for the Earth $T_{tp} \approx 200K$ and for Venus $T_{tp} \approx 165K$ [28]) which are the limit of their tropospheres where climatic phenomena occur. The Earth’s tropopause has a width of approximately 2 Km and is at an altitude of 12 Km on the average. The Venus tropopause is at an altitude of 85 – 110 Km. If the approximation $\bar{q}_s \gg \sigma T_{ex}^4$ is not used, then the corresponding Lagrangean function for the MPR is given by Eq. (14) and, by using the Euler-Lagrangean formalism, we obtain the following equations:

TABLE II. Surface temperatures of the nine planets under a maximum ecological function regime, taking the greenhouse effect only in the lower part of the atmosphere.

Planets	ρ	γ	I_{SC} (W/m ²)	R	T_{cal} (K)	T_{exp} (K)
Mercury	0.058	0.0	9200	1	519	450
Venus				0.86	740	
	0.71	0.992	2600	1	725.8	740
		0.993		0.95	742.1	
Earth	0.35	0.0	1373	1	294.0	288
Mars	0.17	0.0	600	1	254.0	220-240
Jupiter	0.5	0.4	50	1	135.0	120
Saturn	0.6	0.4	15	1	94.0	95
Uranus	0.7	0.4	4	1	63.6	59
Neptune	0.7	0.4	2	1	53.2	48
Pluto				1	46.2	50
	0.5	0.1	1	0.95	46.8	
				0.75	49.4	
				0.71	50	

$$T_1^5T_2^4 - T_1^4T_2^5 + \frac{T_{ex}^4}{3} (T_1^5 - T_2^5) - \frac{2\bar{q}_s}{3\sigma R(1-\gamma)}T_2^5 = 0, \quad (27)$$

$$T_1^4T_2 + \frac{T_1T_2^4}{(1-\gamma)} - \left[T_{ex}^4 + \frac{2\bar{q}_s}{\sigma R(1-\gamma)} \right] T_2 - \frac{T_{ex}^4}{(1-\gamma)}T_1 = 0. \quad (28)$$

In Table V, we show the surface temperatures of the Earth and Venus for several scenarios of the greenhouse effect, taking as cold reservoirs the tropopause shells at $T_{ex} \approx 200K$ and $T_{ex} \approx 165K$, respectively. It is remarkable how, for high values of γ in the case of Venus, the surface temperature is practically recuperated by the GZ model. For the same cases, that is, $T_{ex} \neq 3K$, but maximizing the following Lagrangean

TABLE III. Surface temperatures of Earth and Venus under a maximum power regime, taking the greenhouse effect only in the lower part of the atmosphere.

EARTH	$T_{ex}(K)$	γ	$T_{calc}(K)$	$T_{exp}(K)$
		0	277.2	
$\rho = 0, 35$		0.20	289.3	
	3	0.25	293.3	≈ 288
$\bar{q}_s = 223 \frac{W}{m^2}$		0.30	297.2	
		0.35	301.4	
		0.40	306.1	
VENUS		γ	$T_{calc}(K)$	$T_{exp}(K)$
		0.0	378	
$\rho = 0.59$		0.96	665	
	3	0.97	698	≈ 740
$\bar{q}_s = 195 \frac{W}{m^2}$		0.98	748	
		0.99	841	

TABLE IV. Surface temperatures of the Earth and Venus under a maximum ecological function regime, taking the greenhouse effect only in the lower part of the atmosphere.

EARTH	$T_{ex}(K)$	γ	$T_{calc}(K)$	$T_{exp}(K)$
		0	294	
$\rho = 0.35$		0.20	309	
	3	0.25	314	≈ 288
$\bar{q}_s = 223 \frac{W}{m^2}$		0.30	319	
		0.35	324	
VENUS		γ	$T_{calc}(K)$	$T_{exp}(K)$
		0	401	
$\rho = 0.59$	3	0.94	731	≈ 740
		0.95	756	
$\bar{q}_s = 195 \frac{W}{m^2}$		0.96	787	

TABLE V. Surface temperatures of the Earth and Venus under a maximum power regime, taking the greenhouse effect only in the lower part of the atmosphere with $T_{ex} \neq 3K$.

EARTH	$T_{ex}(K)$	γ	$T_{calc}(K)$	$T_{exp}(K)$
		0	293.4	
$\rho = 0.35$		0.20	303.8	
	200	0.25	307.8	≈ 288
$\bar{q}_s = 223 \frac{W}{m^2}$		0.30	310.4	
VENUS		γ	$T_{calc}(K)$	$T_{exp}(K)$
		0	296	
$\rho = 0.59$		0.97	697	
	165	0.975	728	≈ 740
$\bar{q}_s = 195 \frac{W}{m^2}$		0.976	738	

TABLE VI. Surface temperatures of the Earth and Venus under a maximum ecological function regime, taking the greenhouse effect only in the lower part of the atmosphere with $T_{ex} \neq 3K$.

EARTH	$T_{ex}(K)$	γ	$T_{calc}(K)$	$T_{exp}(K)$
		0	301.5	
$\rho = 0.35$		0.10	308.1	
	200	0.15	311.8	≈ 288
$\bar{q}_s = 223 \frac{W}{m^2}$		0.20	315.9	
VENUS		γ	$T_{calc}(K)$	$T_{exp}(K)$
		0	306	
$\rho = 0.59$		0.99	704	
	165	0.992	730.9	≈ 740
$\bar{q}_s = 195 \frac{W}{m^2}$		0.993	747	

$$L_{ER} = \bar{q}_s + \sigma T_{ex}^4 - \frac{\sigma}{2} [(1 - \gamma)T_1^4 + T_2^4] + \sigma(1 - \gamma)T_{ex}^4 + \left(\frac{\bar{q}_s + \frac{\sigma(1 - \gamma)RT_{ex}^4}{2}}{2} \right) \frac{T_{ex}}{T_1} - \frac{\sigma R}{2} (T_{ex}^4 + 2T_2^4) - \sigma R \left[\frac{(1 - \gamma)T_1^3}{2} - \omega_E T_{ex}^4 \left(\frac{1 - \gamma}{T_1} + \frac{1}{T_2} \right) \right] - \omega_E \left[\frac{\bar{q}_s}{T_1} - \frac{\sigma R((1 - \gamma)T_1^3 + T_2^3)}{2} \right], \quad (29)$$

we get the following equations:

$$T_1^5 + \left[\frac{3R}{4} T_1^4 + \frac{RT_{ex}^4}{4} - \frac{\bar{q}_s}{2\sigma(1 - \gamma)} \right] (T_{ex} - \omega_E) = 0, \quad (30)$$

$$T_2^5 + RT_2^5 - \frac{3R}{4} \omega_E T_2^4 - \frac{RT_{ex}^4}{4} \omega_E = 0 \quad (31)$$

$$(1 - \gamma)T_1^4 T_2 + T_1 T_2^4 - \left(\frac{2\bar{q}_s}{\sigma R} + (1 - \gamma)T_{ex}^4 \right) T_2 - T_{ex}^4 T_1 = 0, \quad (32)$$

In Table VI, we show the numerical results for the Earth and Venus under the MER for several γ -values.

4. Conclusions

Although real heat engines are complex devices, realistic upper bounds can be placed on their performance via relatively simple thermodynamic models, as is the case of FTT-models [29]. This fact has been recently emphasized by Fischer and Hoffmann through a very illustrative case [1]. Simple FTT-models have been used to describe some global atmospheric properties, such as surface temperatures and wind power of the planets of solar system. A simple and elegant

model for atmospheric convective cells was proposed by Gordon and Zarmi [2]. In this model, an atmospheric “heat engine” of the Carnot-type performing at maximum power regime was used. However, as De Vos and Flater state [3], no mechanism guarantees that the atmosphere will maximize the wind power. In fact, some authors [19–21] have recognized that the Earth’s atmosphere operates at close to its maximum efficiency; thus, from an FTT point of view, an ecological-type criterion seems feasible. In this article, we have used the so-called ecological optimization criterion. In addition, we have extended the GZ model to involve the greenhouse effect and internal irreversibilities through a lumped parameter arising from the Clausius inequality. With this GZ extended model, we have calculated the surface temperatures of the nine planets of the solar system under both maximization regimes, the MPR and the MER, respectively. Another change we proposed in the GZ model was the use of the tropopause shells of the Earth and Venus as alternative cold reservoirs. With this proposal, the surface temperatures of these planets were considerably improved.

In summary, with the extended GZ model we have reasonably reproduced the surface temperatures of the nine planets of the solar system. It is remarkable that surface temperatures strongly depend on the greenhouse coefficient, as can be seen in Tables III-VI. This result is consistent with other of our results reported in Ref. [8], where we show that wind power increases with the size of the greenhouse effect. This effect has been recently calculated by Emmanuel [30] for hurricanes over the last thirty years.

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