Failure probabilities associated with failure regions containing the origin: application to corroded pressurized pipelines

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This work develops expressions to calculate failure probabilities associated with failure regions containing the expected value of random variables in the standard space (origin). These expressions are an extension based on the classical case that calculates failure probabilities associated with non-origin-containing failure regions. A simple form is established to know whether the failure region is origin- o non-origin-containing and to calculate the failure probability associated with the region in question. It is shown through an example of corroded pressurized pipelines that such an extension may be necessary to calculate failure probabilities in practical conditions. Reliability methods analyzed are FORM and directional simulation.

Keywords: Origin; reliability; failure probability; failure region; limit state function; failure pressure; corrosion.

En este trabajo se desarrollan expresiones para calcular probabilidades de falla asociadas con regiones de falla que contienen al valor esperado de las variables aleatorias en el espacio estándar (origen). Estas expresiones son una extensión basada en el caso clásico que calcula probabilidades de falla asociadas con regiones de falla que no contienen al origen. Se establece una forma simple para conocer si la región de falla contiene ó no al origen y para calcular la probabilidad de falla asociada con la región en cuestión. Se muestra por medio de un ejemplo de ductos presurizados corroídos que tal extensión puede ser necesaria para el cálculo de probabilidades de falla en condiciones prácticas. Se analiza la confiabilidad del sistema con los métodos FORM y de simulación direccionada.

Descriptores: Origen; confiabilidad; probabilidad de falla; región de falla; función de estado límite; presión de falla; corrosión.

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1. Introduction

The failure probability of structural systems is estimated by well-known methods, such as FORM (First Order Reliability Method) and SORM (Second Order Reliability Method) [1,2], which are based on the ideas of Cornell [3] and Hasofer and Lind [4]; they developed their approaches keeping in mind a design scheme with failure probabilities associated with non-origin-containing failure regions, *i.e.*, when the origin belongs to the safe region. Subsequent works have continued along the same line [1,2,5,6]. However, some applications require the calculation of failure probabilities associated with origin-containing failure regions. For instance, this is the case with applications where conditional failure probabilities need to be calculated in relation to a random variable or parameter of interest, as may be the level of damage associated with seismic intensities in a given site, with long recurrence periods, that may mean that the structure is in near-to-collapse conditions [7,8]. This can also happen to corrosion degraded, pressurized pipelines, whose degraded ligament approaches zero.

This work develops expressions that can be used in FORM and directional simulation to calculate failure probabilities associated with origin-containing failure regions. A simple form is established to know whether the failure region is origin- o non-origin-containing and to calculate the failure probability associated with the region in question. For this purpose, three cases are analyzed. The first deals with the case when the origin is in the safe region, the second, when it is completely contained in the failure region and the third, when the origin is in the limit-state function. The ideas developed herein are applied to a problem of corroded pressurized pipelines, which requires calculating failure probabilities associated with origin-containing failure regions.

2. Failure probability estimated by geometrical reliability index

It is known that failure probability of a structural system can be expressed as:

$$P_F = \int\limits_{W(X)\leq 0} f_X\left(x\right)dx, \tag{1}$$

where $W\left(X\right) \leq 0$ is the event denoting the failure region and X is a vector of basic random variables of order n, with joint probability density function $f_X(\cdot)$. The failure region is limited by safety margin or limit state surface $W(x) = 0$, which defines the boundary between the failure and safe regions. For convenience, the limit state surface is transformed to $G(U) = T[W(X)]$, a function where U is a vector of independent random variables with a standard normal probability distribution function and joint probability density function$f_U(u)$. Thus, the failure probability is expressed as:

$$P_F = \int\limits_{G(U)\leq 0} f_U(u)\ du \tag{2}$$

In the theory of reliability the index is defined:

$$\beta = \min_{u\in G(u)=0}\left(\sqrt{\sum_{k=1}^{n} u_k^2}\right) = \min_{G(u)=0} \|u\|, \tag{3}$$

known as the geometrical reliability index [5] (Fig. 1). In the case in which the limit state surface corresponds to a hyperplane, the relationship between β and P_F is given by:

$$P_F = \Phi[-\beta], \tag{4}$$

where $\Phi[\cdot]$ is the standard normal distribution function. In the FORM method, the limit state surface is substituted by a first-order approximation in Taylor's series (hyperplane), around the point u^*satisfying $\|u^*\| = \beta$, and $\Phi[-\beta]$ is considered to be an approximation of P_F.

The relation $P_F = \Phi[-\beta]$ is valid when the origin is located in the safe region, that is to say, this relation is valid when $G(U=0) > 0$ is satisfied. Reliability methods reported in the literature have been developed taking into account this condition.

The index β will here be related to P_F, associated with origin-containing failure regions. These regions satisfy the condition $G(U=0) \leq 0$.

In the case where the origin is completely contained within the failure region, *i.e.*, $G(U=0) < 0$, as is shown in Fig. 2, β is defined, as in the above case, by Eq. (3). How-

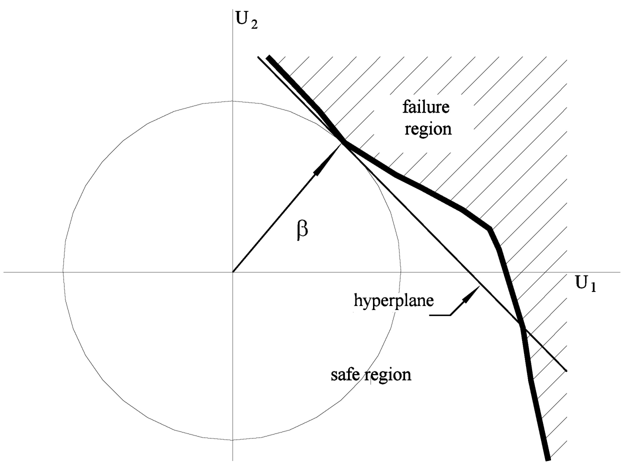

FIGURE 1. Failure region: the origin belongs to the safe region.

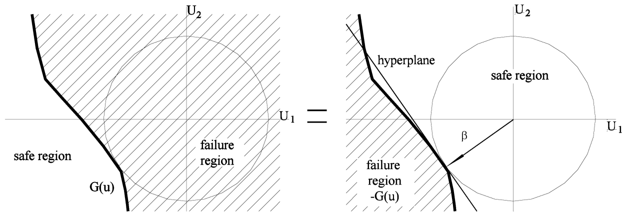

FIGURE 2. Failure region: the origin belongs to the failure region. Two equivalent figures are shown.

ever, when the limit state function corresponds to a hyperplane, the relation between β and P_F is not given by $P_F = \Phi[-\beta]$, as in the case mentioned above, but by:

$$P_F = 1 - \Phi[-\beta] = \ \Phi[\beta], \tag{5}$$

obtained by defining $H(u) = -G(u)$ and relating β to the failure region$H(u) \leq 0$. If FORM is applied to a nonlinear limit state function, $\Phi[\beta]$ can be considered to be an approximation of P_F.

When the origin belongs to the limit state surface, that is $G(U=0) = 0$, it turns out that $\beta = 0$, and when the limit state surface is a hyperplane, it turns out that $P_F = \Phi[0] = 1/2$.

If the limit state surface corresponds to a hyperplane, the difference between the first two cases consists of associating β with failure probability through the Eq. (4) or (5), respectively. These two cases are complementary.

3. Failure probability estimated by directional simulation

In most practical applications the FORM method gives good results in reliability estimations when the structural system has one failure mode and the radius of curvature of the limit state surface is not large. In the case of many failure modes, the FORM method is not directly applicable. Below, generalized mathematical expressions to estimate failure probabilities by directional simulation are developed. These expressions are applicable to limit state surfaces of structural systems with one or more failure modes, and are independent of the radius of curvature. In addition, the limit state surfaces can have folds. The expressions can be used to perform multiple integrals of irregular functions with several variables.

According to Bjerager [9], the failure probability of a system can be expressed as:

$$\begin{aligned} P_F &= \int\limits_{G(U)\leq 0} f_U(u)\, du \\ &= \int P[G(RA) \leq 0\, | A = a]\, f_A(a)\, da, \end{aligned} \tag{6}$$

where the operator $P[\cdot]$ denotes the probability of the event within square brackets and $U = R\ \ A$, in which R is a random variable that describes the distance from the origin to the limit state surface $G(u) = 0$, and A is a vector of directions of dimension $n - 1$ that describes a unit hypersphere. To calculate (6), the directional simulation technique is used [2,10,11].

It is here assumed that $(A = a) \neq 0$ and that the straight line $l_a = \{ra : r \geq 0\}$ intercepts the limit state function in one point at most. If the origin is in the safe region, *i.e.*, $G(0) > 0$, the failure probability for any direction $A = a$ is given by [9] as:

$$P\left[G(RA) \leq 0 \,|A = a\right] = P\left[R > r_a\right] = 1 - \chi^2\left(r_a^2\right), \quad r_a \geq 0 \qquad (7)$$

where $\chi^2(\cdot)$ is the Chi-Square probability function with n degrees of freedom and r_a satisfies the conditions that $G(r_a a) = 0$. In the case when the straight line l_a and the limit state surface do not intercept, the following expression follows from Eq. (7):

$$P\left[G(RA) \leq 0 \,|A = a\right] \;\; = 0 \qquad (8)$$

If the origin is inside the failure region, *i.e.*, $G(0) < 0$, the failure probability for any direction $A = a$ is given by:

$$P\left[G(RA) \leq 0 \mid A = a\right] = P\left[R \leq r_a\right] = \chi^2\left(r_a^2\right), \quad r_a \geq 0. \qquad (9)$$

If the line l_a and the limit state surface do not intercept, the following expression follows from Eq. (9):

$$P\left[G(RA) \leq 0 \,|A = a\right] \;\; = 1. \qquad (10)$$

For the case when the origin belongs to the limit state surface, as shown in Fig. 3, *i.e.*, $G(0) = 0$ there are directions $A = a$ in which $G(r_a a) < 0$ and others, in which $G(r_a a) > 0$.

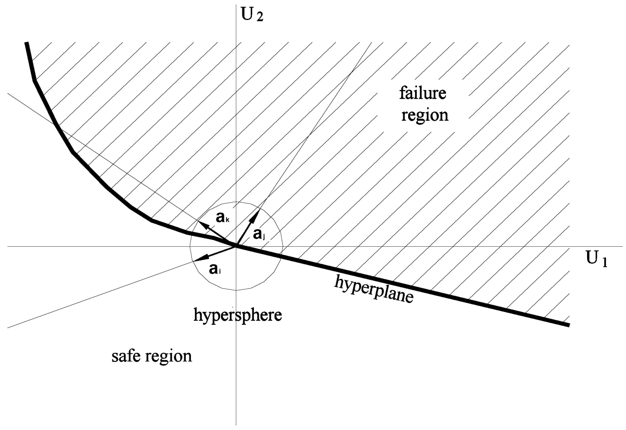

FIGURE 3. Failure region: the origin belongs to the limit state surface. It is shown that some directions are outside or inside of the failure region.

If for a given direction

1. $G(ra) > 0$, for all $0 \leq r \leq r_a$, then Eq. (7) or (8) is applied.

2. $G(ra) \leq 0$, for all $0 \leq r \leq r_a$, then Eq. (9) or (10) is applied.

3. $G(ra) = 0$, for all $r > 0$, then $P\left[G(RA) \leq 0 \,|A = a\right] = 0$; this occurs when the limit state surface in a given direction coincides with a hyperplane.

On the other hand, if the line l_a intercepts the limit state surface at the points $r_1 a, \;\; r_2 a, ..., \;\; r_N a$, where $N > 1$ denotes the number of roots with $r_k < r_{k+1}$ and if $G(0) > 0$, then the failure probability for any direction $A = a$, is given by:

$$P\left[G(RA) \leq 0 \,|A = a\right] \;\; = \begin{cases} \displaystyle\sum_{k=1}^{N/2} \chi^2\left(r_{2k}^2\right) - \chi^2\left(r_{2k-1}^2\right) & N \quad \text{even} \\ \left(1 - \chi^2\left(r_N^2\right)\right) + \displaystyle\sum_{k=1}^{(N-1)/2} \chi^2\left(r_{2k}^2\right) - \chi^2\left(r_{2k-1}^2\right) & N \quad \text{odd.} \end{cases} \qquad (11)$$

This equation corresponds to the case where the limit state surface has folds and the interception points of the failure surface with the line l_a are $r_1 a, \;\; r_2 a, \ldots, \;\; r_N a$, as is shown in Fig. 4. When$N = 1$, the above equation corresponds to Eq. (7). In Eq. (11), for$N > 1$, and odd, from origin to the first point of intersection, the line l_a belongs to the safe region. From the first point to the second point, l_a belongs to the failure region, and from the second point to the third point, l_a belongs to the safe region, and this behavior continues point r_{N-1}. The region from the last point to infinity corresponds to the failure region. From this we have the result that the failure region intersects the line in $(N - 1)/2$ segments and the last segment $\{ra : r > r_N\}$ is given by the first term of Eq. (11). For the others terms we have the result that the intervals$[r_1, r_2]$, $[r_3, r_4], \ldots, [r_{N-2}, r_{N-1}]$ correspond to the failure region, and the contribution to failure probability of each one is given by:

$$(1 - \chi^2(r_{2k-1}^2)) - (1 - \chi^2(r_{2k}^2)) = \chi^2(r_{2k}^2) - \chi^2(r_{2k-1}^2).$$

A similar approach can be made, to obtain Eq. (11) when $N > 1$, and even.

If $G(0) < 0$, then the failure probability for any direction $A = a$ is given by Eq. (12) which is complementary to Eq. (11).

$$P\left[G\left(RA\right)\leq 0\left|A=a\right.\right] = \begin{cases} \chi^2\left(r_1^2\right)+\left(1-\chi^2\left(r_N^2\right)\right)+\sum\limits_{k=1}^{(N-2)/2}\chi^2\left(r_{2k+1}^2\right)-\chi^2\left(r_{2k}^2\right) & N \quad \text{even} \\ \chi^2\left(r_1^2\right)+\sum\limits_{k=1}^{(N-1)/2}\chi^2\left(r_{2k+1}^2\right)-\chi^2\left(r_{2k}^2\right) & N \quad \text{odd.} \end{cases} \tag{12}$$

Similar expressions can be found for the case when $G(0)=0$.

On the other hand, the failure of a series system can be expressed as the union of all events that may cause its failure, *i.e.*, $\bigcup\limits_k (G_k(U)\leq 0)$. The failure probability of the system can be expressed as:

$$P_F = \int\limits_{\bigcup\limits_k (G_k(U)\leq 0)} f_U\left(u\right)\ du$$

$$= \int P\left[\bigcup_k (G_k(R_kA)\leq 0 \mid A=a)\right] f_A\left(a\right)\ da \tag{13}$$

and for a parallel system, the event within the operator $P\left[\cdot\right]$ in Eq. (13) is defined as $\bigcap\limits_k (G_k\left(R_kA\right)\leq 0\left|A=a\right.)$.

For a given direction in series systems, if $G(0)>0$, then the failure probability is calculated as [9,10]:

$$1-\chi^2\left(\min\left(r_a^2\right)\right) \tag{14}$$

This last expression also allows us to calculate the failure probability for a given direction of parallel systems, when $G(0)<0$.

For series systems, when $G(0)<0$ and the number of limit state surface is finite, the failure probability in any direction is given by:

$$\chi^2\left(\max\left(r_a^2\right)\right) \tag{15}$$

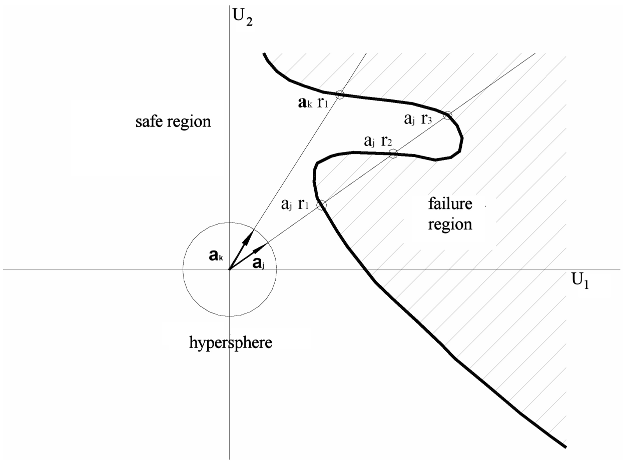

FIGURE 4. Failure region has folds and the interceptions with the straight line occur in several points. The origin belongs to the limit state surface. It is shown that some directions are outside or inside of the failure region.

This same expression serves to calculate the failure probability for a given direction of parallel systems, when $G(0)>0$.

Equations (14) and (15) are valid when the straight line l_a intercepts at least one limit state surface. Otherwise, Eq. (8) or (10) is used, as applicable.

4. Failure probabilities obtained from reliability functions and conditional probability functions

This section develops expressions that allow us calculating failure probabilities as a function of a fixed variable (here: time). These probabilities are obtained here from a reliability function defined in the space of random variables that control the system failure or deterioration, and from the probability distribution of random variables, conditioned by a fixed variable. The reliability function is represented by reliability indices for given values of random variables.

When $W\left(Z\right)=0$ is the limit state surface for given values of $Z=z$, the failure probability $P\left[W\left(Z\right)\leq 0\left|Z=z\right.\right]$ for given values of this variable can be represented in terms of the reliability function:

$$\tilde{\beta}\left(z\right) = -\Phi^{-1}\left[P\left[W\left(Z\right)\leq 0\left|Z=z\right.\right]\right] \tag{16}$$

On the other hand, failure probability for given values of the variable t, can be obtained from the probability density function $f_{Z|t}\left(\cdot\right)$ as:

$$P_F\left(t\right) = \int\limits_0^\infty P\left[W\left(Z\right)\leq 0\left|Z=z\right.\right]\ f_{Z|t}\left(z\mid t\right)\ dz$$

$$= \int\limits_0^\infty \Phi\left[-\tilde{\beta}\left(z\right)\right]\ f_{Z|t}\left(z\mid t\right)\ dz. \tag{17}$$

Integrating by parts and making proper changes of the variable, the following is obtained:

$$P_F\left(t\right) = 1-\int\limits_{-\tilde{\beta}(0)}^\infty F_{Z|t}\left(\tilde{\beta}^{-1}\left(-y\right)\mid t\right)\ \varphi\left(y\right)\ dy \tag{18}$$

where $\varphi\left(\cdot\right)$ is the derivative of Φ, while $\tilde{\beta}^{-1}\left(\cdot\right)$ denotes the inverse function of $\tilde{\beta}$, which leads to the assumption that this function is injective (one to one). In problems where $\tilde{\beta}(0) \to \infty$, Eq. (18) can be represented as:

$$P_F(t) = 1 - \underset{Z}{E}\left[F_{Z|t}\left(\tilde{\beta}^{-1}(-z)\right)\right] \tag{19}$$

The approximations made by point estimates [12] gives us:

$$P_F(t) \approx 1 - \frac{1}{2}\left[F_{Z|t}\left(\tilde{\beta}^{-1}(-1)\right) + F_{Z|t}\left(\tilde{\beta}^{-1}(1)\right)\right] \tag{20}$$

5. Application example

The concepts described in the previous section are applied to the reliability analysis of a pipeline with a corrosion defect with parabolic geometry. The pipeline performance is measured in terms of the resistant pressure of the pipe and estimated by the mechanical model proposed by Cronin and Pick [13], and modified by Oliveros *et al.* [14]. In this model, the resistant pressure at a point x_0 with corrosion depth $d(x_0)$is estimated using the expression:

$$p_R^d(x_0) = p^d_{LongGroove} + \left(p^d_{PlainPipe} - p^d_{LongGroove}\right) g_d(x_0), \tag{21}$$

where x_0 is in an appropriate Cartesian system that, in the longitudinal direction of the pipe, defines the position of x_0, and in the perpendicular direction, the corresponding depth $d(x_0)$. $p_{PlainPipe} = p_{PlainPipe}(z)$ is the resistant pressure of a pipe without a corrosion defect and $z = (z_g, z_m)$ is a vector integrated by a vector of geometrical properties of the pipe z_g and mechanical properties of steel z_m, whereas $p_{LongGroove} = p_{LongGroove}(z, d_{\max})$ is the resistant pressure of a pipe with a corrosion defect whose geometry corresponds to a groove of infinite length and depth $d_{\max} = \max\limits_{x\in[a,b]}\{d(x)\}$, where $[a,b]$ is the interval within which the corroded material exists. The function g_d takes on values in the interval $[0,1]$ and takes into account the influence of the corrosion defect geometry. The resistant pressure of the pipe is given by $p^d_{R\min} = \min\limits_{x_0[a,b]}\{\, p^d_R(x_0)\,\}$. This resistant pressure is a function of the reduction of the wall thickness, and this reduction is attributed to the corrosion process.

In operating conditions, the resistant pressure of a pipeline will be uncertain, principally due to variability in the constitutive function of steel. In addition, the geometry of the corrosion effect is also uncertain and changes over time, whereas the operating pressure is variable. Based on the above facts, the limit state function associated with failure due to pressure can be specified as:

$$\left(W = \varepsilon\ P^d_{R\min} - P_D\right) \leq 0, \tag{22}$$

where $P^d_{R\min}(\cdot)$ is a random variable that denotes resistant pressure associated with corrosion defect, P_D is a random variable of operating pressure and ε is a random variable that considers the error in the prediction of the mechanical model.

Practically, if the geometry of a corrosion defect is represented by a parabolic geometry with maximum depth of $d_{\max}$ and length $l = b - a$, the resistant pressure of the system will be associated with the point of maximum depth $x_{\max} = (b-a)/2$, and hence the above safety margin can be expressed in extended form as:

$$W = \varepsilon\ \left[P^d_{LongGroove} + \left(P^d_{PlainPipe} - P^d_{LongGroove}\right)\ g_d(x_{\max})\right] - P_D. \tag{23}$$

$P^d_{PlainPipe}$and $P^d_{LongGroove}$ are random variables strongly correlated by the random vector with geometrical and mechanical properties $Z = (Z_g, Z_m)$. These variables can be related through the independent random variable Ψ, as follows:

$$P^d_{LongGroove} = \Psi\ P^d_{PlainPipe}, \tag{24}$$

where $\Psi = \Psi(d_{\max})$ depends upon the maximum depth of corrosion and $P^d_{PlainPipe} = P^d_{PlainPipe}(Z)$. Based on numerical tests, it was found that the variability of Ψ is small and can be considered independent from the steel strength; $P^d_{LongGroove}$ can thereby be represented by:

$$P^d_{LongGroove} \approx \psi\ P^d_{PlainPipe}, \tag{25}$$

where $\psi = E[\Psi]$. Hence, W is expressed as:

$$W = \varepsilon\left[\psi P^d_{PlainPipe} + g_d\left(P^d_{PlainPipe} - \psi P^d_{PlainPipe}\right)\right] - P_D = \varepsilon\ P^d_{PlainPipe}\left[\psi + g_d(1-\psi)\right] - P_D, \tag{26}$$

where the random variable ε is obtained from experimental tests and from the mechanical model represented by Eq. (21), in which the defect geometry is parabolic. According to this $\varepsilon = \varepsilon(l)$ has Lognormal distribution with mean $\bar{\varepsilon} = \exp(-0.21 + 0.34\ l/\bar{r}_0)$ and variance $\sigma^2_\varepsilon = 0.12\ \bar{\varepsilon}^2$, where $\bar{r}_0$ is the mean radius of the pipe. The function $\psi(d_{\max})$ is obtained from Monte Carlo simulations.

Here we obtain, a pipeline reliability function (steel X52) for a single defect. In the case of multiple defects in a given pipe segment the failure probability should be calculated using Eq. (13). The pipeline under study has a mean radius of $\bar{r}_0 = 254\ mm$, a mean wall thickness of $\bar{t}_0 = 8.38$ mm, and a mean yield stress of $\bar{\sigma}_y = 422$ MPa. The joint distribution function of variables that describe the behavior of the steel stress-strain curve is given in detail in Ref. 15. For simplicity, it is assumed that the defect length $l = 0.6\ \bar{r}_0$ does not change with the defect depth or time. Here, the safety margin $W = W(H)$ is expressed as a function of corrosion depths $H = \eta$, where $\eta = d_{\max}/\bar{t}_0$. Hence, the failure probability $P_F(\eta) = P[W(H) \leq 0\,|H = \eta]$ for given values of $H = \eta$ is expressed in terms of the reliability function$\beta(\eta) = \Phi^{-1}[P_F(\eta)]$, as shown in Fig. 5. There it can be seen that the system without degradation corresponds to $\eta = 0$ and to a reliability index of $\beta = 4.1$. The reliability function decreases nonlinearly with the corrosion depth, and as the degradation due to corrosion approximates the thickness of the pipe, the reliability decreases asymptotically. It is important to note that reliabilities associated with corrosion depths above $\eta = 0.87$ correspond to the origin-containing failure regions.

According to the above, the failure of the system essentially depends on the level of degradation due to corrosion and, consequently, on the rate of pipe degradation. This rate depends on environmental conditions and, principally, on the type and concentration of the chemical species of the fluid transported by pipelines. Here, the deterioration due to corrosion is expressed by the Type II Extreme distribution function with mean $\bar{H}(t)\ \bar{t}_0 = \nu_0\ t^{-\gamma}$ (mm) [16] and standard deviation $\sigma_H \bar{t}_0 = \sigma_0\, t$ (mm), where $\sigma_0 = 1/2$ (mm/yr), $\nu_0 = 1$ is an empirical value. This value changes with the type and concentration of the chemical species of the fluid transported by pipelines, and $\gamma \approx 1/2$ is a parameter that characterizes the form of the process over time [16]. Figure 6 shows the reliability function $\beta(t) = -\Phi^{-1}[P_F(t)]$ obtained from Eq. (17). The behavior of this function is observed to be exponential. Note that for the time periods of interest (between 0 and 5 years), the reliability function is associated with non-origin-containing failure regions; however, in order to obtain $P_F(t)$, it was necessary to calculate the reliability function for both origin-containing and non-origin-containing failure regions.

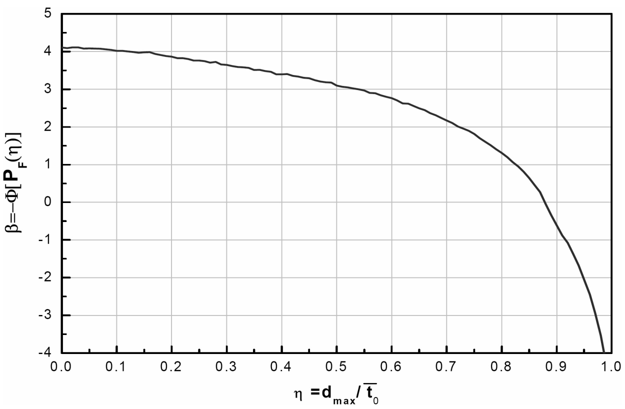

FIGURE 5. Reliability function: reliability indices vs corrosion depths η.

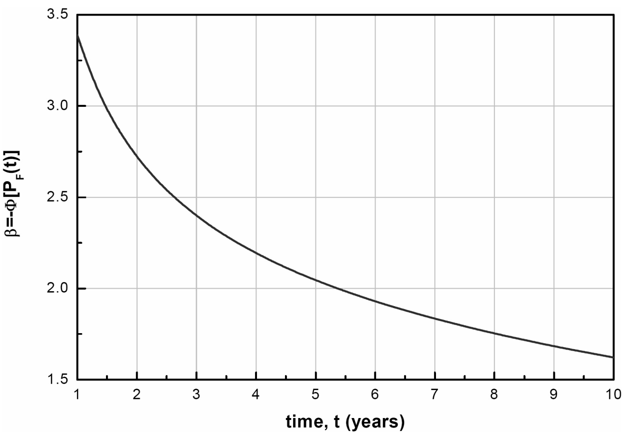

FIGURE 6. Reliability function: reliability indices vs time.

6. Conclusions

In this work, expressions for calculating failure probabilities associated with origin-containing failure regions were obtained, expanding the ideas of the case where the origin is in the safe region. The origin belonging to the failure region was established in terms of the value that the function G assumes at zero. Explicitly:

1. If $G(0) > 0$, the failure region does not contain the origin, *i.e.*, the origin is in the safe region. Equation (4) is applied for FORM and Eqs. (7)-(8) for directional simulation.

2. If $G(0) < 0$ the failure region contains the origin. Equation (5) is applied for FORM and Eqs. (9)-(10) for directional simulation.

3. If $G(0) = 0$, the origin belongs to the failure region. In this case, the failure probability is 1/2 for FORM and for directional simulation there are three possible cases. If for a certain direction:

   a. $G(ra) > 0$, for all $0 \leq r \leq r_a$ then Eq. (7) or (8) is applied.

   b. $G(ra) \leq 0$, for all $0 \leq r \leq r_a$ then Eq. (9) or (10) is applied.

   c. $G(ra) = 0$, for all $r > 0$, then $P[G(RA) \leq 0 \,|\, A = a] = 0$; this occurs when the limit state surface coincides with a hyperplane.

According to the above, it is a simple matter to establish whether the failure region contains the origin or not, as well as to evaluate the failure probability associated with the failure region in question.

The reliability analysis applied to a corroded pressure pipeline showed that, unlike typical applications to structural reliability problems, in the study of corroded pressurized pipelines it is not unusual to find cases where the origin of the U vectors lies in the failure region. Also in professional practice, after a pipeline inspection, it is common to find corrosion defects whose failure regions containing the origin. In those cases, the expression presented in this article should be applied.

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