A new type of analog cosine converter

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This paper presents the synthesis and the computer-aided design for a new type of analog dynamic converter which carries out the mathematical operation of the cosine function. The lab investigation results of static and dynamic precision of this converter are also included.

Keywords: Analog converters; signal processing.

El artículo presenta la síntesis y diseño computarizado de un nuevo tipo del convertidor analógico dinámico el cual realiza la operación de función de coseno. También se presentan los resultados de investigación en el laboratorio de la exactitud estática y dinámica de este convertidor.

Descriptores: Convertidores analógicos; procesamiento de señales.

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1. Introduction

Though great progress has been made both in technology and the development of digital computer systems, when it comes to industrial applications – industrial instrumentation above all – the use of analog circuits of calculation is more convenient. These circuits are characterized by their great speed and low cost while the precision and complexity of the mathematical operations they carry out are relatively low.

Today the analog static converters (well-known in analog computation as the fixed function generators) which carry out a given mathematical function based on their approximation to the piecewise curve are more frequently used, but then because of their functional principle they already have a calculation error whose value depends on the number of straight line segments that approximate the mathematical function being carried out.

The proposal consists in using a new type of analog converters (called in this paper the analog dynamic converter) based on electronic pulse systems of a special type. The most outstanding advantage of this type of analog converter is a higher degree of precision at a relatively low construction cost.

At this point it is pertinent to mention that, as a result of the investigation related to the analog dynamic converters, the patents for the following converters were obtained: square, square root, sine, arc sine, cosine, arc cosine, cube, cube root. These patents are registered at the Mexican Institute for Industrial Property.

The paper presents the synthesis and the lab investigation results of the analog dynamic converter that carries out the mathematical operation of the cosine function [1].

2. The structure of the dynamic converter

The aim of the analog converter is to convert the analog input signal $U_1(t)$, which is more frequently given in voltage form, into an output analog signal $U_2(t)$ (also given in voltage form) according to the desired mathematical relationship F:

$$U_2(t) = F\left[U_1(t)\right] \tag{1}$$

To carry out this conversion, the use of an electronic pulse circuit without feedback is proposed; its block-diagram is displayed in Fig. 1.

In Fig. 1:

$U_1(t),\ U_2(t)$ $-$input and output voltages respectively, their values varying in a pre-established range ($|U_1(t)| \leq U_{1\max}$, $|U_2(t)| \leq U_{1\max}$). The maximum frequency of the voltage $U_1(t)$ is determined by the width of the admissible band the converter is working with.

$f_1(t),\ f_2(t)$ $-$the periodic voltage waves, having the same period T and the same amplitude. The amplitude equals the maximum admissible value of the voltage $U_1(t)$, meaning:

$|f_1(t)| \leq U_{1\max}\,,\ |f_2(t)| \leq U_{1\max}$ for $t \in [0,T]$.

G_1, G_2 $-$the generators of the periodical voltages $f_1(t)$ and $f_2(t)$, respectively.

Co $-$the comparator.

S $-$the analog switch, controlled by the logic signal α.

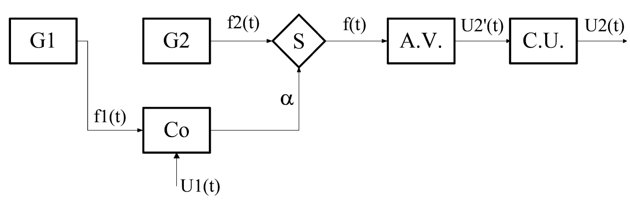

FIGURE 1. The structure of the dynamic converter.

$A.V.$ −the circuit for the calculation of the average value of the input signal $f(t)$ (a low-pass filter).

$C.U.$ −the correction unit.

α −the logic signal, its physical representation depends on the input proprieties of the logic control of analog switch S.

$f(t)$ −the periodic voltage of period T, its form depends on the form of voltage $f_2(t)$ and on the closing and opening times of analog switch S.

$U_2'(t)$ −the output voltage of the average value calculation unit.

During period T, comparator Co generates one or more rectangular pulses, the duration of which depends on the value of voltage $U_1(t)$ and on the form of the periodic voltage$f_1(t)$. The frequency of periodic voltages $f_1(t)$ and $f_2(t)$ should be sufficiently high to be able to consider that the voltage $U_1(t)$ remains practically unchanged during period T. The mathematical function $U_2 = F(U_1)$, carried out by the dynamic converter, depends on the forms of the waves $f_1(t)$ and $f_2(t)$. Therefore the synthesis of the converter consists in determining these waves.

3. The synthesis of the dynamic cosine converter

The synthesis of the converter consists in determining the periodic functions $f_1(t)$ and $f_2(t)$ for which the desired relationship F- between the signals U_2 and U_1- is obtained. For technical reasons, the set of $f_1(t)$ function generators must be confined within limits of the generators that are easy and simple to build as well as having high accuracy in the generated wave. Thus, the problem is reduced to determining function $f_2(t)$ which makes it possible – for a chosen function $f_1(t)$ – to carry out function F.

The cosine converter is requested to carry out the following function:

$$\frac{U_2}{U_{1\,\max}} = \cos\left[\frac{\pi}{2}\left(\frac{U_1}{U_{1\,\max}}\right)\right] \tag{2}$$

It is assumed that generator G_1 generates the triangular voltage (Fig. 2) described by relationship (3).

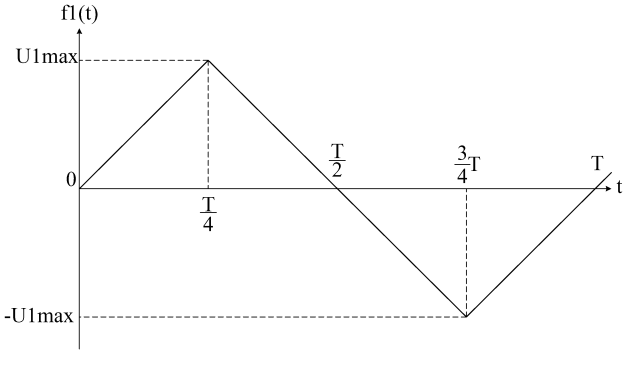

FIGURE 2. The triangular voltage generated by generator G_1

$$f_1(t) = \begin{cases} U_{1\,\max}\frac{4}{T}t & \text{for } 0 \le t \le \frac{T}{4} \\ U_{1\,\max}\left(-\frac{4}{T}t + 2\right) & \text{for } \frac{T}{4} \le t \le \frac{3}{4}T \\ U_{1\,\max}\left(\frac{4}{T}t - 4\right) & \text{for } \frac{3}{4}T \le t \le T \end{cases} \tag{3}$$

It is assumed that comparator Co carries out the function:

$$\alpha = \begin{cases} 1 \;\; if \;\; U_1 - f_1(t) \ge 0 \\ 0 \;\; if \;\; U_1 - f_1(t) < 0 \end{cases} \tag{4}$$

It is assumed that analog switch S, controlled by logic signal α (Fig. 1), passes signal $f_2(t)$ through its analog input when signal α has the logic value of “1”.

Using the synthesis method shown in [2], it can be justified that:
for the intervals of the period T in which the voltage $f_1(t)$ increases:

$$f_2(t) = TU_{1\,\max}\frac{dF\left[f_1(t)/U_{1\,\max}\right]}{dt} \tag{5}$$

for the intervals of the period T in which the voltage $f_1(t)$ decreases:

$$f_2(t) = -TU_{1\,\max}\frac{dF\left[f_1(t)/U_{1\,\max}\right]}{dt} \tag{6}$$

the output voltage of average value unit:

$$\frac{U_2'}{U_{1\,\max}} = KF\left(\frac{U_1}{U_{1\,\max}}\right) + C_1 \tag{7}$$

where: K is the number of the intervals of the period T in which $f_1(t) = U_1$ and the function $f_1(t)$ increases or decreases,

$$C_1 = \sum_{l=1}^{z}\alpha F\left[\frac{f_1(t_{kl})}{U_{1\,\max}}\right] - \sum_{l=1}^{z}\alpha\, F\left[\frac{f_1(t_{pl})}{U_{1\,\max}}\right] - \sum_{j=1}^{q_2}F\left[\frac{f_1(t_{pj})}{U_{1\,\max}}\right] - \sum_{k=1}^{q_3}F\left[\frac{f_1(t_{kk})}{U_{1\,\max}}\right] \tag{8}$$

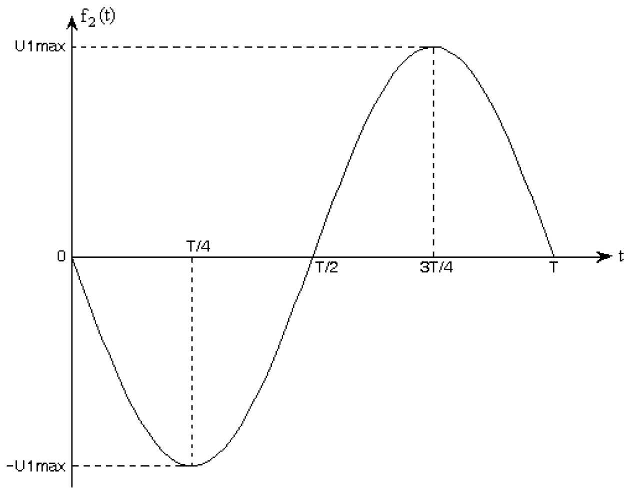

FIGURE 3. The sine voltage generated by generator G_2.

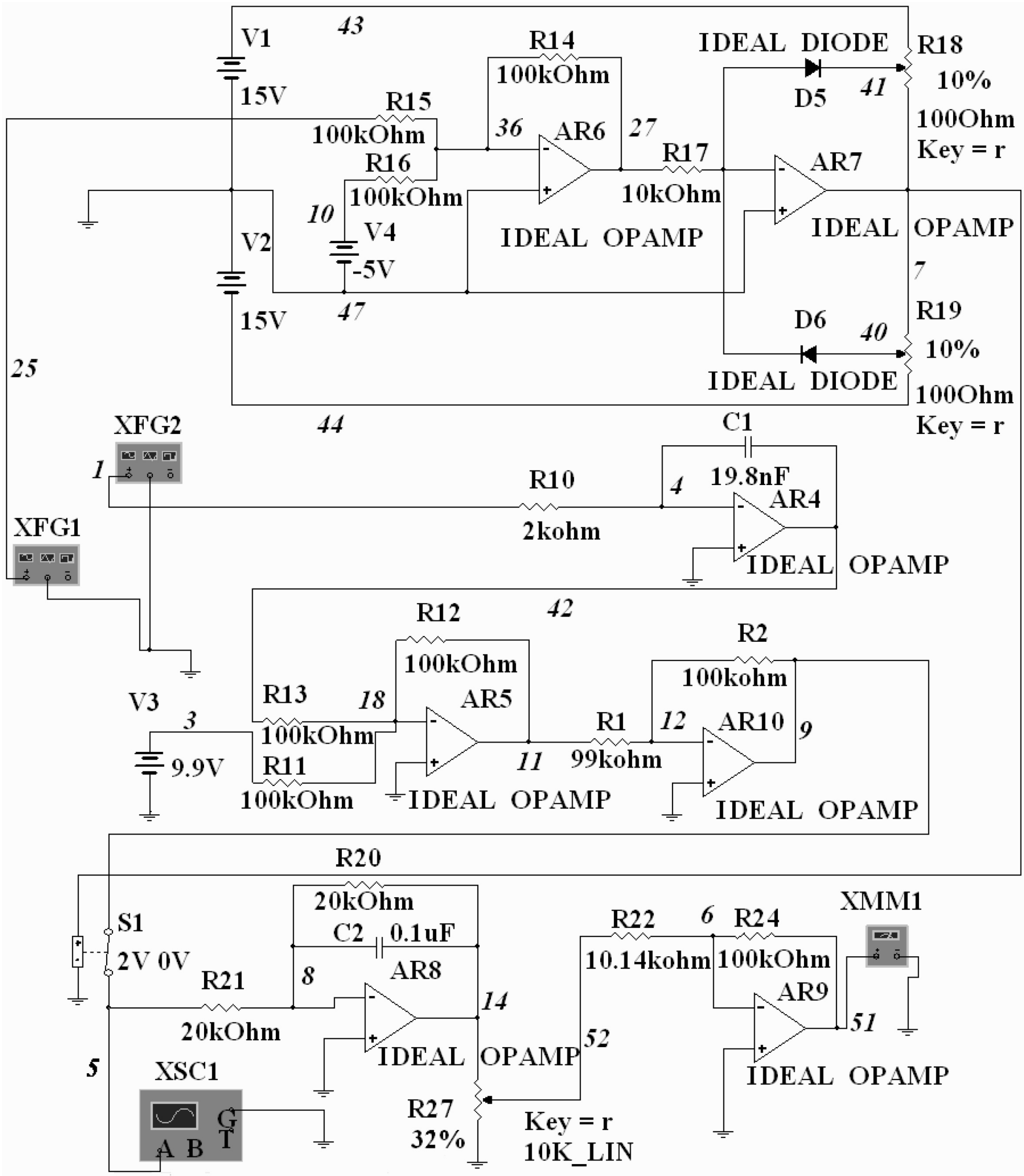

FIGURE 4. Diagram of the cosine converter.

z is the number of the intervals in period T in which the voltage $f_1(t)$, for a given voltage U_1, does not reach value U_1

q_2 is the number of the intervals of the period T in which the function $f_1(t)$ increases and satisfies the equality $f_1(t) = U_1$, for a given voltage U_1

q_3 is the number of the intervals of the period T in which the function $f_1(t)$ decreases and satisfies the equality $f_1(t) = U_1$, for a given voltage U_1

t_p, t_k are the time indicating the beginning and the end of the intervals mentioned above.

Applying the relationships (5) and (6), for triangular wave $f_1(t)$ [Fig. 2, relationship (3)] and for the function F described by (2), the wave$f_2(t)$ must be of the following sine function:

$$f_2(t) = -2\pi U_{1\,\max} \sin\left(\frac{2\pi}{T}t\right) \quad \text{for} \quad 0 \le t \le T \qquad (9)$$

For the technical preferences, it is assumed that the voltage amplitude of the wave $f_2(t)$ equals the maximum admissible value $U_{1\max}$ of the voltage $U_1(t)$:

$$f_2(t) = -U_{1\max} \sin\left(\frac{2\pi}{T} t\right) \quad \text{for} \quad 0 \le t \le T \qquad (10)$$

Figure 3 The sine voltage generated by generator G_2.

Three intervals exist during the time period T in which the function $f_1(t)$ changes monotonically ($K = 3$, Fig. 2):

- one interval in which the voltage $f_1(t)$ increases and satisfies the equality $f_1(t) = U_1$ for a given voltage U_1 ($q_2 = 1$)

- one interval in which the voltage $f_1(t)$ decreases and satisfies the equality $f_1(t) = U_1$ for a given voltage U_1 ($q_3 = 1$)

- one interval in which, for a given value of the voltage U_1 it does not reach the value U_1 ($z = 1$).

The times indicating the beginning and the end of each of these intervals are:

$$t_{p1} = 0, \quad t_{k1} = \frac{T}{4}, \quad t_{p2} = \frac{T}{4},$$

$$t_{k2} = \frac{3}{4}T, \quad t_{p3} = \frac{3}{4}T, \quad t_{k3} = T$$

It can be calculated (7,8) that the converter which uses waves $f_1(t)$ and $f_2(t)$ shown above (Fig. 2, Fig. 3), carries out the mathematical operation of cosine according to the following relationship:

$$\frac{U_2'}{U_{1\max}} = \frac{1}{\pi} \cos\left[\frac{\pi}{2}\left(\frac{U_1}{U_{1\max}}\right)\right]. \qquad (11)$$

Studying relationship (11), it can be observed that in order to carry out function (2) a correction needs to be made which consists in multiplying by the constant, π.

4. The computer-aided design of the cosine converter

To design the electronic circuit of the cosine converter, the electronic system simulator "Electronics Workbench" was used. This is carried out, based on the model of the ideal operational amplifier, accepting that $U_{1\max} = 10V$. We shall use: the generator of the sine wave, the generator of the triangular wave (both with an amplitude of 10V), the analog switch controlled by the rectangular pulses and other components available in the simulator. To measure output voltage U_2, a multimeter is used and to observe the waves an oscilloscope is applied.

The simulation diagram of the cosine converter system is shown in Fig. 4.

The comparator (operational amplifiers AR6 y AR7) compares the voltage of the triangular wave (generated by generator XFG1) to the voltage $U_1(t)$ (voltage source V4), generating at its output (node 7) a series of rectangular pulses which control analog switch S1. At the analog input of this switch (node 9), it enters the cosine wave which is shaped out of the sine wave (generated by the generator XFG2) by an integrator (AR4), a DC shifter (AR5) and an inverter (AR10). A part of the cosine wave, selected by the analog switch, passes through the input (node 5) of the average unit (AR8). The subsystem based on amplifier AR9 is the correction unit.

The main goal of the simulation is to verify whether the proposed structure of the converter is the correct one and to determine the values of some parameters.

The potentiometers R18 and R19 are adjusted, so that top values of the rectangular pulses at the comparator's output (node 7) is adequate to control the analog switch. The potentiometer R27 has to be adjusted so that the gains of the correction unit (AR9) equal π. The value of capacitor C2 (the average unit (AR8)) has to be selected, so that the time constant of the unit mentioned, is much larger than the period of the waves generated by the sine and triangular generators (T = 0.25 ms, f = 4kHz). By increasing the value of this constant, the pulsation in the output voltage of the average unit decreases. The constant mentioned, however, may not will too large either, so that the time-span for establishing the average value be not increase. The time constant of a first order low pass filter was selected to equal 2 ms. In this way the time-span for establishing the average value was obtained at around 25 ms (accordingly, the admissible frequency of voltage U_1 is 40 Hz).

Figure 5 shows the triangular and cosine waves which were obtained during the simulation.

Figure 6 shows the output voltage waves of the comparator (node 5) and of the analog switch (node 7) obtained for voltage $U_1 = 0V$.

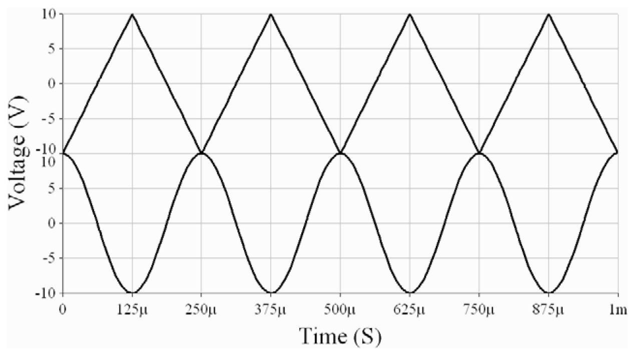

FIGURE 5. Triangular and cosine waves (nodes 25 y 9).

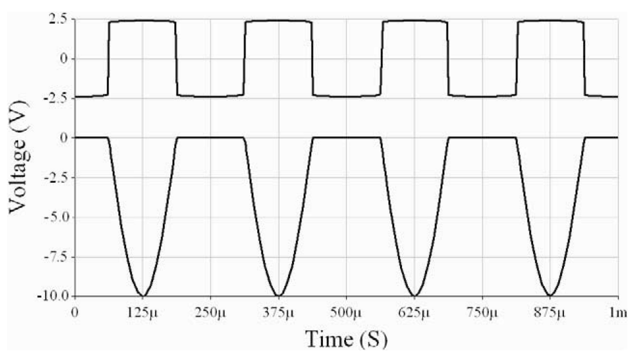

FIGURE 6. Comparator and analog switch output for $U_1 = 0V$.

TABLE I.

U_1[V]	U_2[V] expected	U_2[V] calculated	Relative error[%]
10	0	14mV	—-
9	1.564	1.560	0.3
8	3.090	3.166	2.5
7	4.540	4.670	2.9
6	5.878	5.997	2.0
5	7.071	7.198	1.8
4	8.090	8.277	2.3
3	8.910	9.087	2.0
2	9.511	9.716	2.2
1	9.877	10.073	2.0
0	10.00	10.210	2.1
-1	9.877	10.080	2.1
-2	9.511	9.710	2.1
-3	8.910	9.106	2.2
-4	8.090	8.260	2.1
-5	7.071	7.206	1.9
-6	5.878	6.000	2.1
-7	4.540	4.665	2.8
-8	3.090	3.158	2.2
-9	1.564	1.559	0.3
-10	0	2nV	—-

Table I shows the U_2 voltage values at stationary conditions which were obtained during the simulation for different U_1 voltage values which cover the entire range of admissible variations of this voltage. This same chart also shows the expected U_2 voltage values and the relative calculation error.

5. Lab investigation of converter's exactitude

Due to the satisfactory investigation results for the feasibility of the designed converter by means of a computer, a lab prototype was built in order to evaluate its real precision. Low-cost commercial components such as the operational amplifiers LF353, the analog switch controlled by the logic signal LF133331, the diodes 1N4148, the 5% precision resistors and capacitors as well as 10 rotation potentiometers [3,4,5] are used.

Figure 7 shows a detailed diagram of the cosine converter.

This diagram differs from the one used in the computer-aided design because one single generator is used which is the sine generator, built on the basis of the operational amplifiers 1 and 2. The potentiometers P2, P3 and P1 are employed to adjust the amplitude of the sine wave at the 10V value. The triangular wave is shaped out of the sine wave by using a limiter (operational amplifier 3), an integrator (operational amplifier 4) and a summer (operational amplifier 5). The limiter is used to transform the sine wave into a series of rectangular symmetrical pulses with an amplitude of 10V (the potentiometers P4 and P5 are used to adjust this amplitude). The integrator is used in shaping the triangular wave (potentiometer P6 permits the adjustment of the amplitude of this wave at the value of 10V).

In Figs. 8 and 9, the waves registered by means of a digital oscilloscope correspond to the waves obtained during simulation, shown in Figs. 5 and 6, are presented.

Table II shows the lab investigation results for precision of the dynamic cosine converter.

While investigating the dynamic precision of the cosine converter, the maximum admissible frequency of the variation of the input signal $U_1(t)$ was determined, for which the calculation error practically does not increase. This is a frequency of 150 Hz.

In order to be able to compare the precision of the dynamic converter which was built, to the precision of a static converter, the investigation results for the precision of the sine static converter with 12 straight line segments that approximate this function were used. This static converter was lab built and the same components were used as those used for building the dynamic cosine converter. The maximum error of the static converter mentioned is 10.05%, while its admissible band frequencies are 100 Hz.

TABLE II.

U_1[V]	U_2[V] expected	U_2[V] calculated	Relative error[%]
10	0	0.001	—-
9	1.564	1.560	0.26
8	3.090	3.070	0.65
7	4.540	4.510	0.70
6	5.878	5.865	0.22
5	7.071	7.040	0.44
4	8.090	8.050	0.49
3	8.910	8.900	0.11
2	9.511	9.505	0.06
1	9.877	9.875	0.02
0	10.00	10.06	0.06
-1	9.877	9.898	0.21
-2	9.511	9.520	0.10
-3	8.910	8.922	0.14
-4	8.090	8.114	0.29
-5	7.071	7.097	0.22
-6	5.878	5.900	0.39
-7	4.540	4.550	0.24
-8	3.090	3.104	0.45
-9	1.564	1.562	0.12
-10	0	0.001	—-

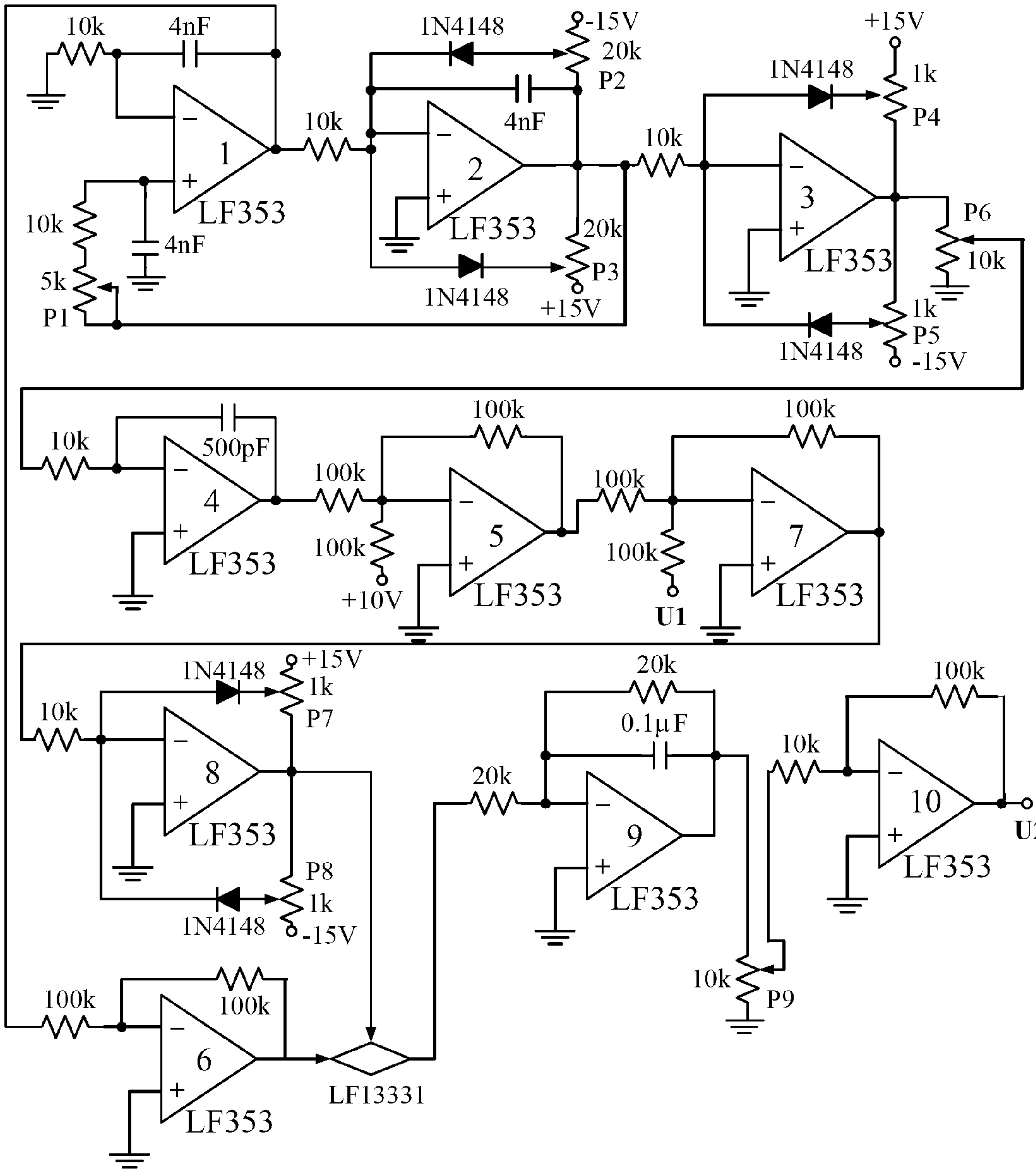

FIGURE 7. Detailed diagram of the cosine converter.

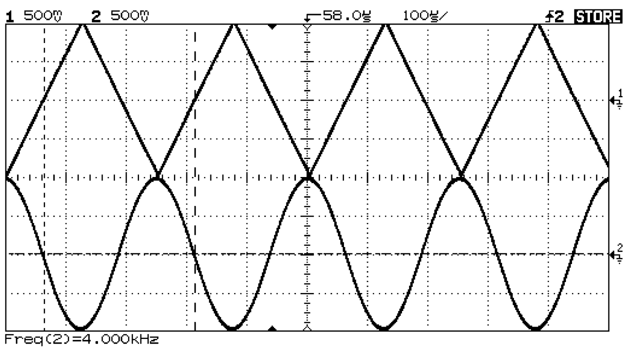

FIGURE 8. Triangular and cosine waves.

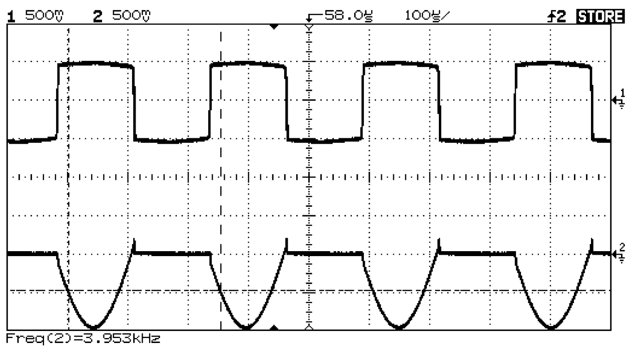

FIGURE 9. Output waves for comparator and analog switch for U_1=0V.

6. Conclusions

The computer-aided design and simulation of the analog dynamic cosine converter's behaviour makes it possible to ensure that the proposed structure of this converter is correct and that it gives satisfactory calculation results. Based on the simulation, the converter's operation could be evaluated qualitatively. The quantitative evaluation is quite inaccurate, due to the inexactness of the electronic component models (most of all of the operational amplifier called "ideal" and the capacitor) used in the computer package Electronic Workbench and also the fact that accuracy of the potentiometer adjustments is limited to 1%, as well as due to the low accuracy with which the mathematical integration operation is carried out in this package. Therefore the evaluation cannot be very much relied on and even less reliable is the dynamic behaviour of the simulated converter. Accordingly, it can be concluded that the quantitative operational evaluation of the converter must be carried out based on experimental lab tests, basing the construction of the designed converter on a chosen type of operational amplifier.

Based on the lab investigation results, the conclusion can be drawn that the dynamic cosine converter's precision is far higher than that of a similar static converter. In the worst of case, the relative calculation error of the dynamic converter equals almost 0.7% while with the static converter, the same error exceeds 10%. The band of admissible frequencies of the time variations in the input voltage $U_1(t)$ of the dynamic cosine converter is even broader than that of the static converter's. It can also be concluded that the dynamic converter, built on the basis of low-cost commercial components, is suitable for industrial applications.

Components of superior quality, such as more sophisticated operational amplifiers, high precision resistors and capacitors, must be used in order to increase the calculation accuracy and the band of admissible frequencies, but this obviously increases the construction costs of the converter. The complete integration of the converter's system by means of chips – each containing one or several converters – would be the decisive step to be taken in order to increase both the static and dynamic precision and to lower construction costs.

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