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Global sensitivity analysis of dispersion maximum position of photonic crystal fibers with circular holes

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We apply global sensitivity analysis technique to investigate the dispersion characteristic maximum behavior of single-mode photonic crystal fiber regarding geometric parameters variation. There have been considered two cases with 5 and 7 parameters. Results demonstrate significant differences in tilt and even the opposite signs slopes of the dependencies when varying different parameters. This fact allows fine tuning the PCF parameters with high flexibility and efficiency. Comparing the 5 and 7-parameters scheme reveals very low differences. However, additional two parameters may be useful for precise tuning of the dispersion characteristic.

Keywords: Photonic band gap materials.

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1. Introduction

Microstructured fibers have recently become popular due to their numerous applications for fiber lasers [1], supercontinuum generation[2] and pulse reshaping [3]. A special class of microstructured fibers called photonic crystal fibers (PCFs) can be separated. A PCF cladding is formed by a periodic array of holes known as photonic crystal (PhC). One of the most important points of PCFs is wide-range dispersion management [4].

Over the last years, there have been presented numerous designs of PCFs possessing specially-designed dispersion [5,6,7]. However, it is not exist a universal technique which would allow tuning the PCF dispersion without using optimization methods.

In our work, we investigate the sensitivity of single-mode PCF dispersion regarding to variation of its basic parameters. This allows to fine-tune the position of local maximum of the PCF dispersion while maintaining other properties unchanged.

The work is organized as follows. First, we discuss the dispersion computation method that is suitable for the global sensitivity analysis. Then we present the global sensitivity analysis for two specific sets of variable parameters.

2. PCF dispersion computation using plane wave expansion method

The dispersion of the PCF at the specific wavelength can be found using following expression:

$$D(\lambda) = -\frac{2\pi c}{\lambda^2}\frac{d^2\beta}{d\omega(\beta)^2} = -\frac{\lambda}{c}\frac{d^2 n_{eff}(\lambda)}{d\lambda^2} \tag{1}$$

Here $\omega(\beta)$ is a dispersion relation. In case of a single-mode PCF β is always the same as the fundamental mode propagation constant k_z due to strong mode field localization within the PCF core [8].

There exist various methods to compute the dispersion relation such as plane wave expansion (PWE) [9], finite difference time-domain (FDTD) [10], finite elements method (FEM) [11,12], multipole method (MP) [13] etc. Each mechod provides different accuracy and takes different computation time.

We have selected the PWE method as it provides the fastest solution giving both eigen-frequency and the mode field distribution which is important since the PCF profile is generated randomly in global sensitivity analysis.

The master-equation solved with PWE method takes following form [9]:

$$\sum_{\vec{G}'} \chi(\vec{G}-\vec{G}') \left|\vec{k}_z + \vec{G}_{x,y}\right| \cdot \left|\vec{k}_z + \vec{G}_{x,y}\right| \cdot \begin{bmatrix} \vec{e}_{2,G}\cdot\vec{e}_{2,G'} & -\vec{e}_{2,G}\cdot\vec{e}_{1,G'} \\ -\vec{e}_{1,G}\cdot\vec{e}_{2,G'} & \vec{e}_{1,G}\cdot\vec{e}_{1,G'} \end{bmatrix} \cdot \begin{bmatrix} h_{1,G'} \\ h_{2,G'} \end{bmatrix} = \begin{bmatrix} h_{1,G} \\ h_{2,G} \end{bmatrix} \tag{2}$$

In order to take into account material dispersion, the permittivity at the specific frequency is found using the Sellmeier's equation with parameters taken for fused silica. Corresponding value is then substituted into unit cell Fourier transform $\chi(\vec{G}-\vec{G}')$.

Solving the equation for specific values of the propagation constant k_z gives both eigen-frequencies of the fiber and the mode field distribution.

Several PCF dispersion characteristics have been computed using the PWE method and compared with FEM results [11]. There are certain defects during numerical precission error (such as peaks around 1800 nm) which can be corrected or eliminated with post-processing. However, the most of the points are very close (less than 1% deviation) to

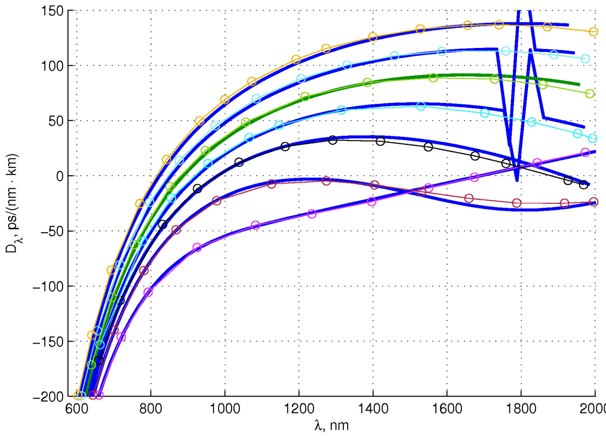


FIGURE 1. PCF dispersion computed with FEM (dotted lines) and with PWE method (solid bold lines).

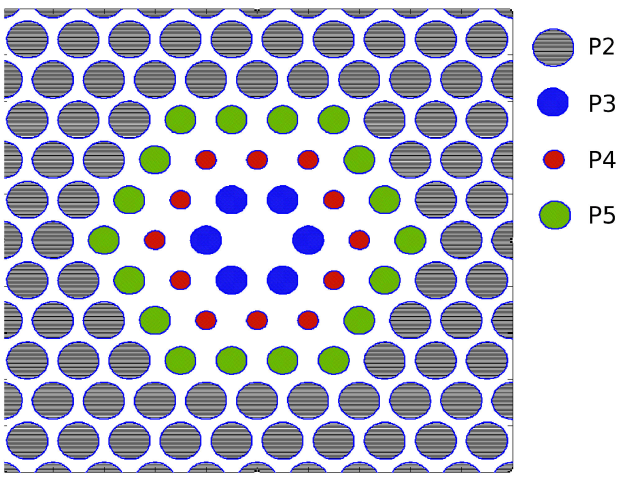


FIGURE 2. Variable parameters for center-symmetric PCF.

the FEM results. Although such accuracy is insufficient for precise PCF characterization, it is enough for optimization purposes as well as for global sensitivity analysis.

3. PCF fiber parameters and objective function definition

One of the most important advantages of the PCF is its non-solid nature which make possible to modify its core and cladding structure in many different ways. Particularly, in this work there has been investigated the PCF dispersion with the near-core area being modified in two different manners:

- Center-symmetric fiber with constant radii within a single ring (see Fig. 2).
- Hexagonal-symmetric fiber with radii varying within a single ring (see Fig. 5).

To perform global sensitivity analysis each parameter is modified randomly within reasonable range and the dispersion of the resulting fiber is calculated.

The investigation purpose was to find a fast and easy way to adjust the position (*i.e.* wavelength and the value) of the PCF dispersion maximum. Therefore, we have only taken the value and the wavelength of the local dispersion maximum in case the PCF possesses one.

3.1. Global sensitivity analysis of the PCF dispersion

In general, a sensitivity analysis refers to a study of how the uncertainty in the output of a mathematical model or system can be apportioned to different sources of uncertainty in its inputs [14]. A related practice is uncertainty analysis which has a greater focus on uncertainty quantification and propagation of uncertainty. Ideally, uncertainty and sensitivity analysis should be run in tandem.

In this work, there has been investigated sensitivity of the wavelength and the value of the PCF dispersion maximum according to variation of its geometric parameters. Namely, we varied the PhC pitch defining the scale of the fiber, the relative radii of the holes that are closest to the fiber core and the radius of the rest of holes.

4. Sensitivity analysis for center-symmetric fiber core with 5 parameters

The PCF refractive index profile has been defined as a system of holes organized into a hexagonal 2D structure in fused silica. The core is created by missing central hole. The core is, therefore, surrounded by several rings of holes.

First, there has been studied the sensitivity of the dispersion maximum value and its wavelength to variation of the holes radii of three rings closest to the core, the radii of the rest of the holes and the pitch. The first parameter named P1 is the pitch while other 4 parameters correspond to radii of the holes circles as demonstrated in Fig. 2.

All 5 parameters were randomly generated within defined limits. Namely, the pitch varied within 1 μm - 2 μm. Empirically it has been found that the PCF with pitch outside these limits can barely possess all-normal dispersion with a local maximum. As for the holes radii, their natural limits are $0 - 0.5 \cdot a$. However, basing on empirical data, we varied the radii within $0.1 \cdot a - 0.5 \cdot a$, where a stands for PhC pitch.

During the simulation, not all dispersion curves have been taken into account. We only took into account those characteristics where the local maximum is found. Moreover, due to the PWE method peculiarities, the field distribution should be analyzed to avoid fake results after the fundamental mode cut-off. Analyzing the mode spatial distribution, we can detect the fundamental mode cut-off when the field intensity far from the PCF core exceeds 1% of the intensity in the center of the mode.

Each characteristics that satisfy the above mentioned conditions are analyzed and the maximum wavelength as well as corresponding dispersion value is extracted. These characteristics are plotted against each parameter. In Figs. 3 and 4

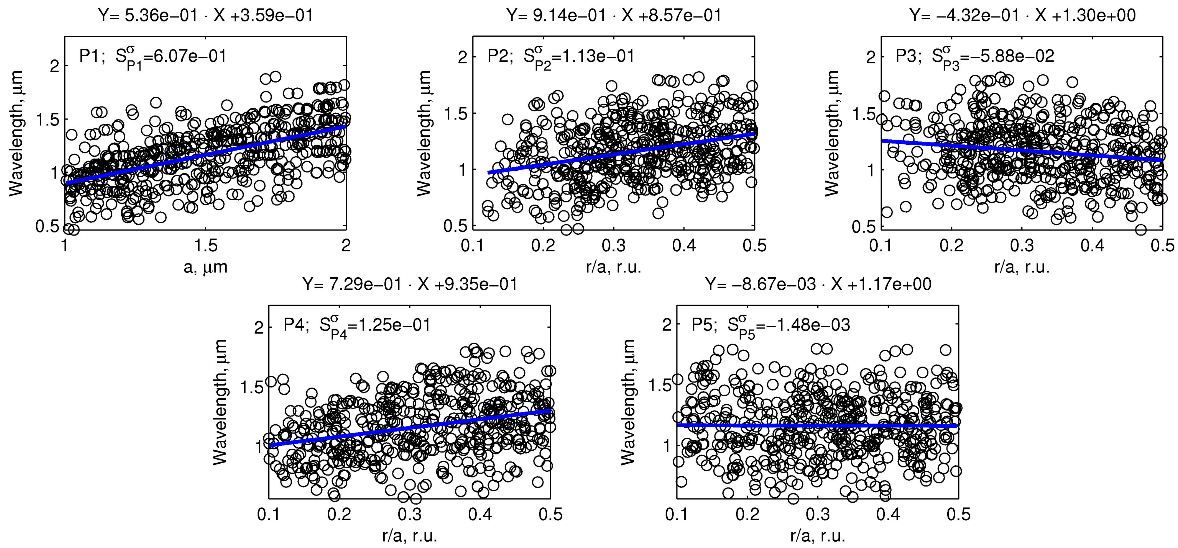


FIGURE 3. Global sensitivity of the dispersion maximum wavelength.

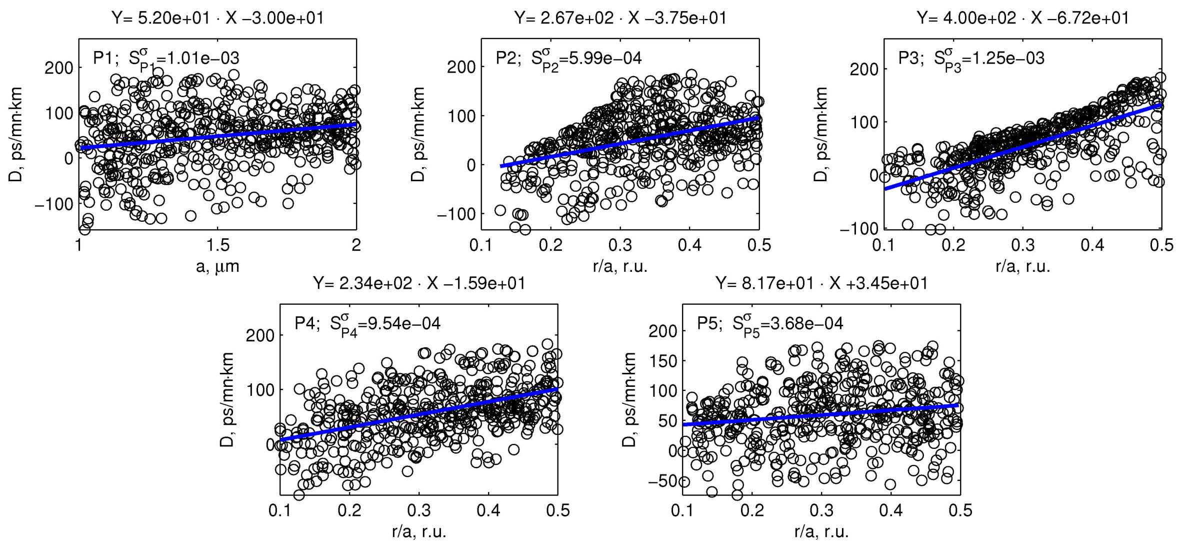


FIGURE 4. Global sensitivity of the dispersion maximum value.

the name of the parameter as well as the approximating equation is shown above each plot. The sensitivity as respect to each parameter is found by calculating the derivatives of approximating linear function as follows:

$$S^{\sigma}_{P_i} = \frac{\sigma_{P_i} \partial Y}{\sigma_Y \partial P_i}, \qquad (3)$$

where σ_{P_i} and σ_Y are standard deviations of the parameter and the computed value.

Therefore, the less scattered the plot is, the more correlated is the output to the parameter. For the PCF this means that if the absolute value of $S^{\sigma}_{P_i}$ is high, the contribution of corresponding parameter is high as well and varying this parameter one can tune the PCF characteristic efficiently. For instance, in Fig. 3 the derivative (*i.e.* inclination) for the P2 is slightly larger than the one for the P4. However, due to larger points scattering its sensitivity is lower. Hence the maximum dispersion wavelength tuning will be more efficient if one vary the radii of the external PhC holes (P2) rather than the radii of the second ring (P4).

Analyzing the sensitivity of the dispersion maximum wavelength presented in Fig. 3 we can mention almost linear wavelength shift with increasing PhC period. This is quite obvious and follows from the scalability of the PhC characteristics. Even though the refractive index dispersion is taken into account, it does not contribute essentially into a characteristic nonlinearity.

The wavelength sensitivity to the parameter P2 that is the radius of all the holes outside the third ring, possesses almost the same inclination as for the P4. On the other hand, the wavelength sensitivity to the radii of the first (parameter P3) and the third (parameter P5) rings are negative. The deriva-

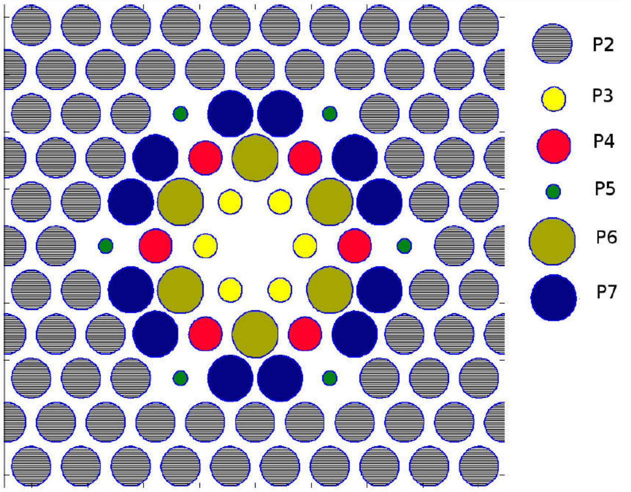


FIGURE 5. Variable parameters for center-symmetric PCF.

tive as respect to the parameter P5 is about 200 times lower than the one of P3 and the scattering is obviously much higher. Therefore we see quite low wavelength sensitivity to the parameter P5.

As for the sensitivity of the dispersion local maximum value, the results are quite different. First, all the derivatives are positive. The dispersion value possesses quite large inclination as respect to varying pitch, however, the scattering here is large as well which gives quite low sensitivity. The most influent parameter in this case is P3 which corresponds to the holes radius of the first ring. The dispersion possesses much lower sensitivities as respect to P2, P4 and P5 parameters.

There should be mentioned several important details when selecting the PCF parameters. Namely, in the Fig. 4 in case of the parameters P2 and P3 the dispersion maximum values are concentrated below certain linear boundary. Particularly, the dispersion maximum when P2 parameter is less than 0.3 is limited with the line $Y = 833 \cdot X - 100$. Moreover, no dispersion maximum is found at low P2 parameter values (*i.e.* radii of the external holes). As for the variation of the P3 parameter, the dispersion maximum here is limited from above with the line $Y = 333 \cdot X - 8.3$.

Therefore, we can see that the sensitivities of the dispersion maximum wavelength and value are different for the same parameters. This fact provides high flexibility in tuning the PCF dispersion characteristic.

5. Sensitivity analysis for hexagonal-symmetric fiber core with 7 parameters

To improve the accuracy of the PCF dispersion tuning, in the next step there have been studied the sensitivities of the dispersion maximum wavelength and value as respect to variation of 7 parameters. The parameters are taken to form hexagonal symmetric PCF since the modes in this kind of fibers usually possess hexagonal symmetry as well. Again, for the simplicity we consider circular holes only.

As in previous study, the P1 parameter stands for PhC pitch. The rest of parameters are shown in Fig. 5. As is seen

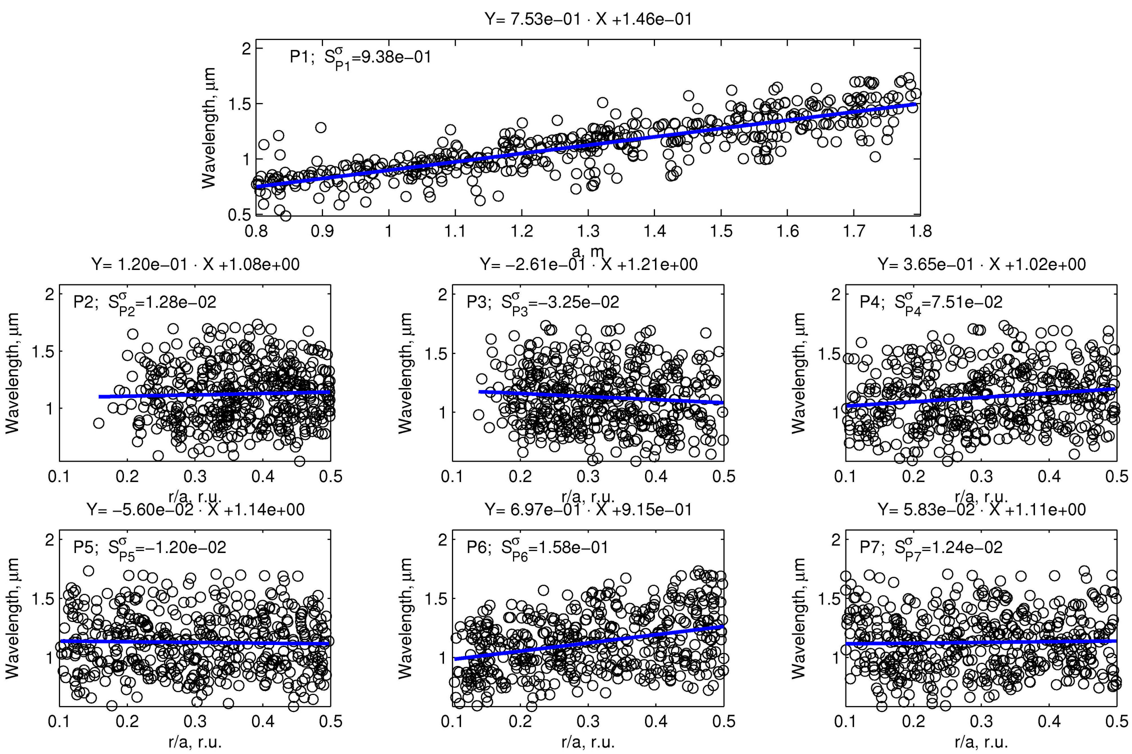


FIGURE 6. Global sensitivity of the dispersion maximum wavelength.

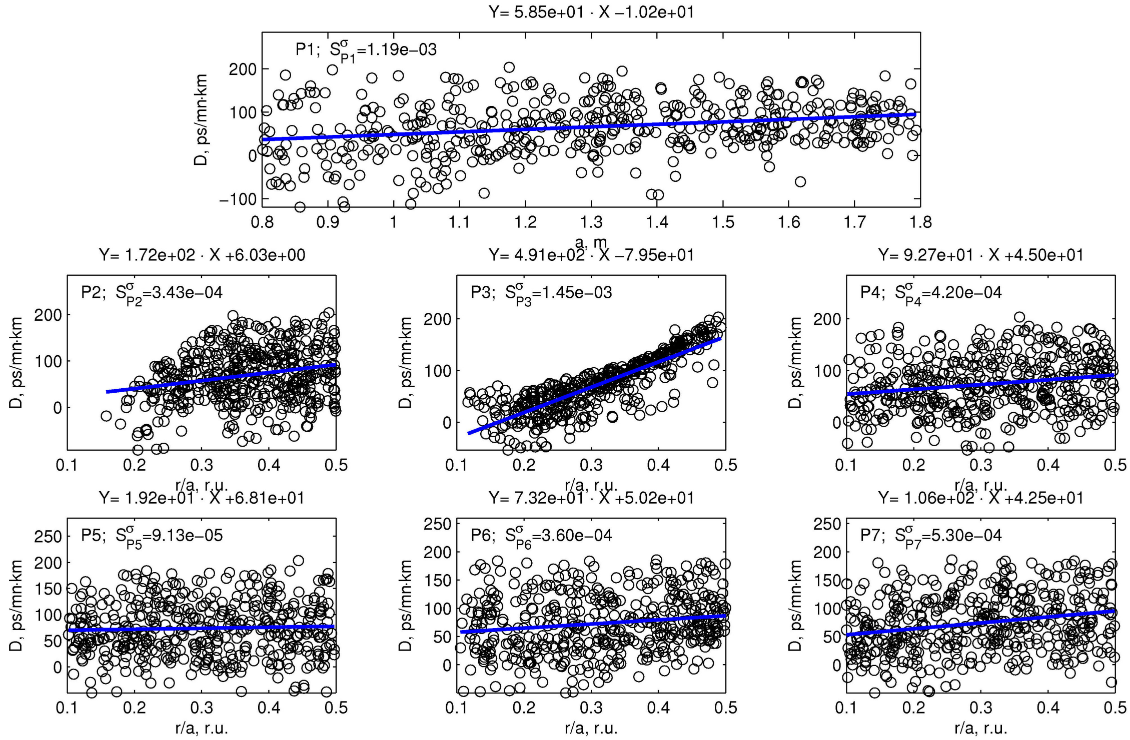


FIGURE 7. Global sensitivity of the dispersion maximum value.

from the figure, the parameters P2 and P3 are the same as in previous study. Again, the P2 holes confine the radiation within several hole-rings around the PCF core. As for the P3 parameters, it is impossible to divide it into smaller parts without breaking the hexagonal symmetry.

The analysis of the results discovers similarity of the sensitivity as respect to the variation of the P1, P2 and P3 parameters which is quite obvious since the parameters are the same as in previous case.

On the other hand, the sensitivity accordint to the rest of parameters in this case is lower than that of the central-symmetric fiber. This allows two-stage parameters selection. During the first stage, the approximate values of the dispersion maximum position are set by selecting P1–P3 parameters. Then the fine-tuning is made by varying the rest of the holes radii.

In general, extra parameters do not introduce significant tuning possibilities into PCF dispersion maximum tuning. However, they may be important when tuning high-order dispersion of the PCF.

6. PCF dispersion tuning

As an example of computed characteristics application, there has been found the structure of the PCF which possesses all-normal dispersion and the dispersion maximum wavelength is 0.9 μm. For simplicity, we use 5-parameters model. However, fine-tuning the dispersion characteristic may require 7-parameters model.

Since the sensitivity analysis included the dispersions with local maximum only, we have first found approximate PCF parameters which give the dispersion characteristic local maximum. This PCF parameters as well as further tuning process details are depicted in the Table I.

The dispersion of the fiber with initial set of parameters corresponds to the line 1 in the Fig. 8. In this case, the dispersion maximum wavelength is 0.87μm and the dispersion curve in its maximum is positive.

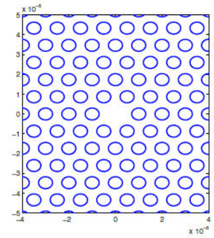
First, we modify the PCF parameters to move all the dispersion curve into negative values. According to the sensitivity analysis results, the value of the dispersion maximum is the most sensitive to the variation of the parameter P3, which in this case should be lowered.

Resulting PCF (line 2 in Fig. 8) possesses all-normal dispersion. However, the wavelength of the maximum is now shifted into the long-wavelength region. To lower the wavelength, there are two options. The most obvious way is to lower the PhC period (parameter P1).

The PCF dispersion (line 3 in Fig. 8) is now shifted to 0.9 as desired. The only disadvantage of scalability is that the fundamental mode cut-off decreases as well, which may be critical in case of wide-spectrum applications.

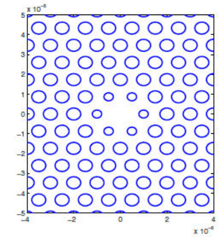
The other way is to tune the parameter P4. Lowering the P4 results in short-wavelength shift of the dispersion maximum while decreasing the dispersion value which allows maintain the characteristic below zero dispersion. On the other hand, the PhC period and, therefore, the scale of the

P1	$1\mu m$
P2	0.3
P3	0.3
P4	0.3
P5	0.3



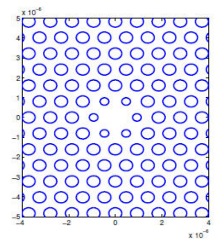
Step 1

P1	$1\mu m$
P2	0.3
P3	0.2
P4	0.3
P5	0.3



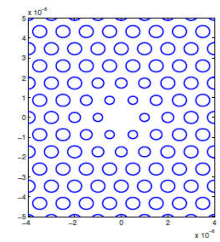
Step 2

P1	$0.93\mu m$
P2	0.3
P3	0.2
P4	0.3
P5	0.3



Step 3

P1	$1\mu m$
P2	0.3
P3	0.2
P4	0.25
P5	0.3



Step 4

TABLE I. The PCF dispersion at different tuning stages.

fiber does not change which does not vary the fundamental mode cut-off.

The resulting fiber (with dispersion shown with the line 4 in Fig. 8) possesses all-normal dispersion at the desired wavelength and the fundamental mode cut-off is not changed as respect to the initial fiber.

7. Conclusions

In the paper, there has been made an attempt to generalize PCF dispersion tuning. For this reason, global sensitivity analysis of the dispersion maximum position has been performed and analyzed.

Although present study still does not demonstrate direct correlation between the PCF dispersion curve and its parameters, the sensitivity analysis of the fiber dispersion maximum behavior can be effectively used for wide-range dispersion tuning as well as for fine-tuning the final dispersion characteristic.

In memory of professor Igor Sukhoivanov.

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