

# Nonlinear element of a chaotic generator

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A mathematical model of a nonlinear electronic circuit, which is the core of the electronic chaotic oscillator, is presented. This mathematical model or nonlinear function has a direct relationship with the values of the components used to build an experimental electronic system. In order to get a good approximation to the characteristic response curve of the nonlinear circuit, the mathematic model considers the current-voltage curve of nonlinear elements.

*Keywords:* Chaotic oscillator; bifurcation parameter; nonlinear converter.

Se presenta un modelo matemático de un circuito electrónico no lineal, el cual es el corazón del oscilador caótico. Este modelo matemático o función no lineal tiene una relación directa con los valores de los componentes usados para construir el sistema electrónico experimental, de tal forma que, para obtener una buena aproximación a la curva de respuesta característica del circuito no lineal, el modelo matemático considera la curva de corriente-voltaje de los elementos no lineales.

*Descriptores:* Oscilador caótico; parámetro de bifurcación; convertidor no lineal.

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## 1. Introduction

The employment of electronic circuits that display various forms of nonlinear behavior are useful in many physical, engineering, biological and other systems. In general, electronic circuits with chaotic behavior use nonlinear elements; for instance, the Chua’s electronic circuit employs a nonlinear negative resistance [1], and the Rössler electronic circuit proposed in Ref. 2 also includes a nonlinear element in order to generate chaos. In these cases, the diode produces the nonlinear behavior. At present, the most positive application seems to be in improving the performance of existing nonlinear circuits: frequency multipliers, mixers, chaotic communication systems, etc.

In order to study chaos synchronization in experiments, some articles [1–4] have employed chaotic circuits, where a nonlinear element is the core of the chaotic behavior. In Refs. 3 and 4, a chaotic oscillator is employed which consists of a nonlinear circuit, called nonlinear converter (NC for short), and linear feedback. The main functionality of the NC is to transform an input voltage into output voltage. The output voltage bears a resemblance to an odd function with a nonlinear dependence. Rulkov *et al.* [4] used an empirical nonlinear function in order to model the NC, where the parameters of this function do not have any relationship with the values of the electronic components used to build the NC. In a similar way, in Ref. 5, a different model of the output voltage is considered, but again without any relationship with values of electronic components. These models have been employed in several numerical simulations.

The goal of this paper is to present a mathematical model for the NC having a direct relationship with the electronic components used to build it. This paper is organized in the following way. Section 2 describes the model of the NC, whereas in Sec. 3, we describe the chaotic generator using the NC. Finally, Sec. 4 presents the conclusions.

## 2. Nonlinear converter

In this section, we shall discuss how to obtain a direct relationship between the electronic components of the NC and the mathematical model. The schematic diagram of the NC is shown in Fig. 1.

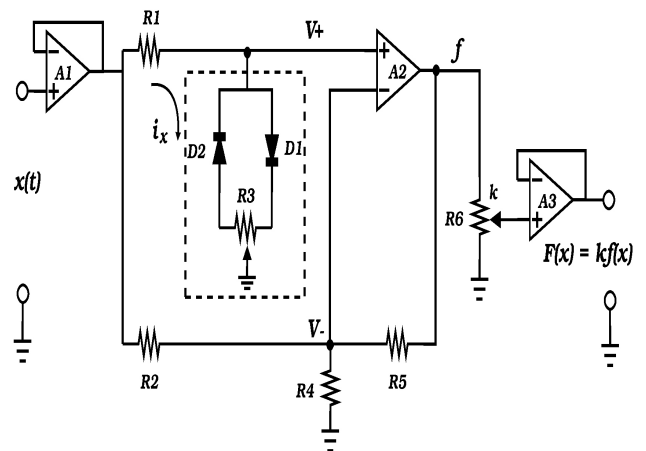


FIGURE 1. Schematic diagram of the nonlinear converter NC.

TABLE I. The component values employed for the NC shown in Fig. 1.

Nonlinear converter	
Components	Values
$R_1$	2.7 K $\Omega$
$R_2, R_4$	7.5 K $\Omega$
$R_3$	50 $\Omega$
$R_5$	177 K $\Omega$
$R_6$	2 K $\Omega$
A1, A2	Op. Amp. TL082
A3	Op. Amp. LF356N
D1, D2	Silicon diode 1N4148

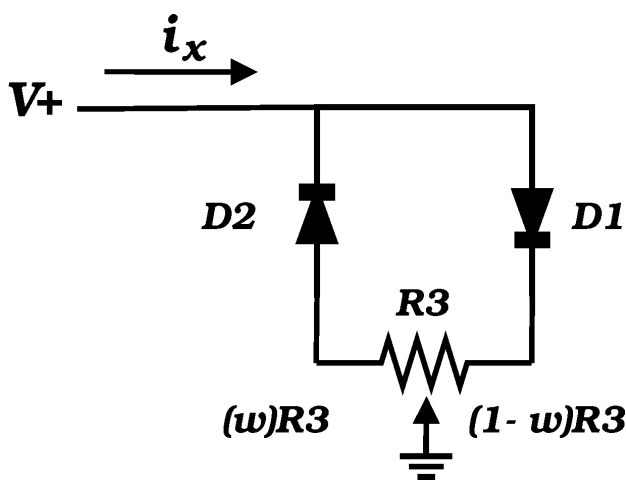


FIGURE 2. The circuit diagram of the pair of diodes of the NC.

This converter transforms the input voltage  $x(t)$  into the output voltage, which is expressed by the nonlinear function  $F(x) = kf(x)$ , where the parameter  $k$  corresponds to the gain of the converter at  $x = 0$ . The values of electronic components for the NC are shown in Table I.

The nonlinear behavior of the NC circuit is due to the functionality of turn on-off of the pair of diodes. In Fig. 1, we can see that the  $k$  parameter belongs to interval  $[0, 1]$  (potentiometer  $R_6$ ).

Now, we shall focus on finding an expression of  $f$  in terms of the input voltage  $x$ . The procedure for computing  $f(x)$  is the following:

1. Let  $i_x(V)$  be the current-voltage relation of the nonlinear module shown in Fig. 2.
2. Applying a KVL(Kirchhoff's voltage law) in the  $A_1 - R_1 - D_1/D_2 - R_3$  network elements (see Fig. 1) we can obtain the voltage  $V_+$ , which is expressed as

$$V_+ = x - i_x(V_+)R_1, \tag{1}$$

where  $i_x(V_+)$  is supposed to be a known function for the moment.

3. By voltage superposition,  $V_-$  is given as

$$V_- = af + bx, \tag{2}$$

where

$$a = \frac{R_2 || R_4}{R_5 + R_2 || R_4}$$

and

$$b = \frac{R_5 || R_4}{R_2 + R_5 || R_4}.$$

In standard conditions of operation, the Op-Amp adjusts its input voltage in order to get

$$V(x) := V_+ = V_-. \tag{3}$$

From Eqs. (2) and (3), we have that

$$f(x) = \frac{V(x) - bx}{a}. \tag{4}$$

According to Eq. (1),  $V(x)$  is solution of the following equation:

$$i_x(V) = \frac{x - V}{R_1}. \tag{5}$$

Now we need to compute  $V$  in terms of  $x$ ; this is accomplished considering two cases: (a)Sudden Commutation Model. In this case, we consider the on-off commutation of the diodes and, (b)Smooth Commutation Model. This case consider the current-voltage relation using Shockley's diode law.

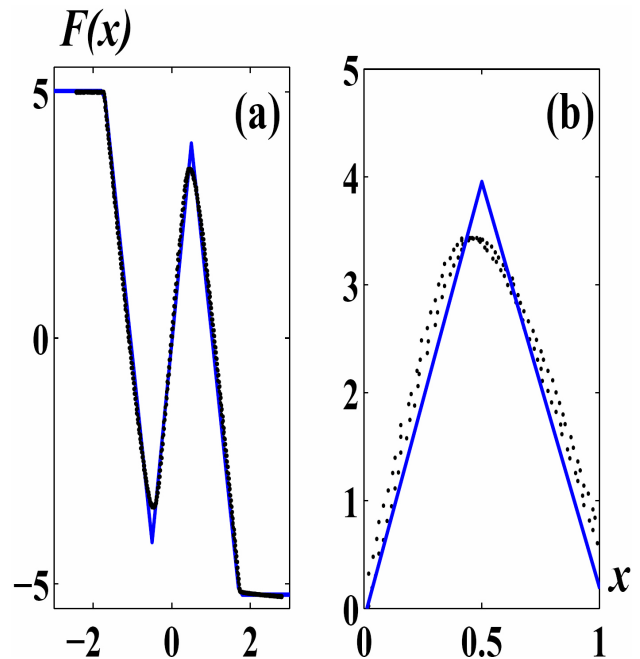


FIGURE 3. (a) Characteristic response curve of the nonlinear converter (circuit NC). (b) The solid line is obtained with the mathematical model (7) with  $k = 1/3$ , and dotted line is obtained from the experimental data.

**2.1. Sudden Commutation Model**

If we suppose that the set of diodes, along with passive linear components, of Fig. 2 has a sudden commutation, and that it occurs in a voltage  $V_D$ , then the current-voltage relation is given by

$$i_x(V) = \begin{cases} \frac{V - V_D}{(1 - w)R_3}, & \text{if } V > V_D \\ 0 & \text{if } |V| \leq V_D \\ \frac{V + V_D}{wR_3}, & \text{if } V < -V_D \end{cases}, \quad (6)$$

where  $w$  is the balance parameter given by potentiometer  $R_3$ . The sudden commutation model for  $f(x)$  is obtained by solving (5) for  $V$ , and considering expression (6) for  $i_x(V)$ . For this situation there exist the following three cases.

**I:** If  $|V| \leq V_D$ , then  $i_x(V) = 0$ . As a consequence the voltage in  $R_1$  is zero and therefore  $V(x) = x$ . From (4) we have that

$$f(x) = \left(\frac{1 - b}{a}\right)x, \quad \text{with } |x| \leq V_D.$$

**II:** If  $V > V_D$ , then the current flows through  $(1 - w)R_3$ . From Eqs. (5)-(6), we find a solution for  $V(x)$  and, solving for  $f$  in (4),

$$f(x) = \frac{[(1 - b)(1 - w)R_3 - bR_1]x + R_1V_D}{[R_1 + (1 - w)R_3]a},$$

with  $x > V_D$ .

**III:** If  $V < -V_D$ , then the current flows through  $wR_3$ . In a similar way to the above, we find

$$f(x) = \frac{[(1 - b)wR_3 - bR_1]x - R_1V_D}{(R_1 + wR_3)a},$$

with  $x < -V_D$ .

Finally, we have the output voltage function  $F$  in terms of the input voltage  $x$  as the following piecewise function:

$$F(x) = kf(x), \quad (7)$$

where

$$f(x) = \begin{cases} \frac{[(1 - b)(1 - w)R_3 - bR_1]x + R_1V_D}{(R_1 + (1 - w)R_3)a}, & \text{if } x > V_D \\ \left(\frac{1 - b}{a}\right)x, & \text{if } |x| \leq V_D \\ \frac{[(1 - b)wR_3 - bR_1]x - R_1V_D}{(R_1 + wR_3)a}, & \text{if } x < -V_D \end{cases}.$$

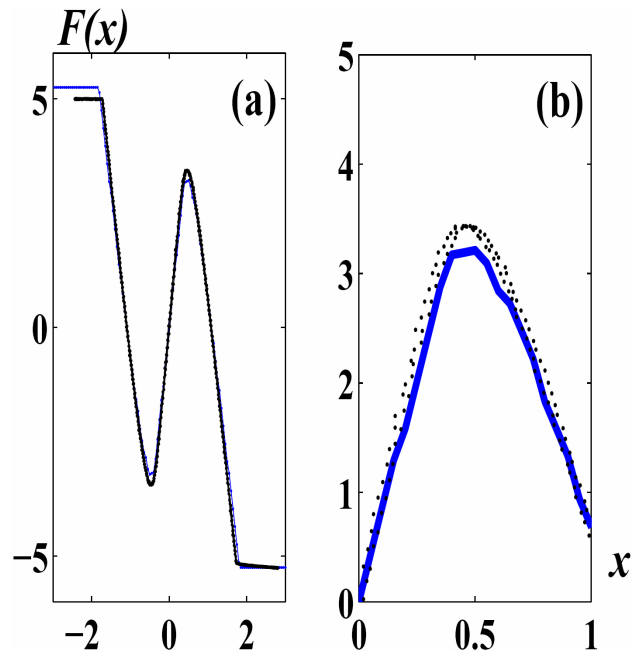


FIGURE 4. (a) The characteristic response curve of the nonlinear converter NC. (b) The solid line is obtained from smooth commutation given by Eqs. (11) and (9). The dotted line shows the data acquired from the experimental circuit.

Figure 3a shows the characteristic response curve of the NC. Parameters  $V_D$  and  $w$  were fixed to  $1/2 V$  and  $w = 1/2$ , respectively. Figure 3b shows a zoom of this response curve, where the dotted line corresponds to data obtained from the experimental measurements, and the solid line corresponds to the mathematical model of Eq. (7). This mathematical model of the transfer function is directly related to the components employed to build the NC. This transfer function  $F(x)$  was obtained using the on-off commutation in diodes. In the next subsection, Shockley's law is used.

**2.2. Smooth Commutation Model**

On the other hand, if we consider the following current-voltage relation (Shockley's law):

$$i_x(V) = i_{sat} \left( e^{[(V - V_D)/V_T]} - 1 \right), \quad (8)$$

from Eq. (8) and the nonlinear block of the schematic diagram shown in Fig. 2, we have that the current-voltage relation can be written as follows:

$$V(i_x) = i_x R_3 + \text{sign}(i_x) \left( V_D + V_T \ln \left( 1 + \frac{i_x}{i_{sat}} \right) \right), \quad (9)$$

where  $V_D = 1/3 V$ ,  $V_T = 1/10 V$ , and  $i_{sat} = 0.04242A$ . In order to compute the transference function  $F(x)$ , we have to solve (5), which can be written as follows:

$$V(i_x) = x - i_x R_1. \quad (10)$$

TABLE II. Component values employed to build the experimental chaotic oscillator shown in Fig. 5.

Chaotic Oscillator	
Components	Values
$C'$	200 nF
$C$	99.8 nF
$L$	26.1 mH
$r$	70 $\Omega$
$R$	1139 $\Omega$

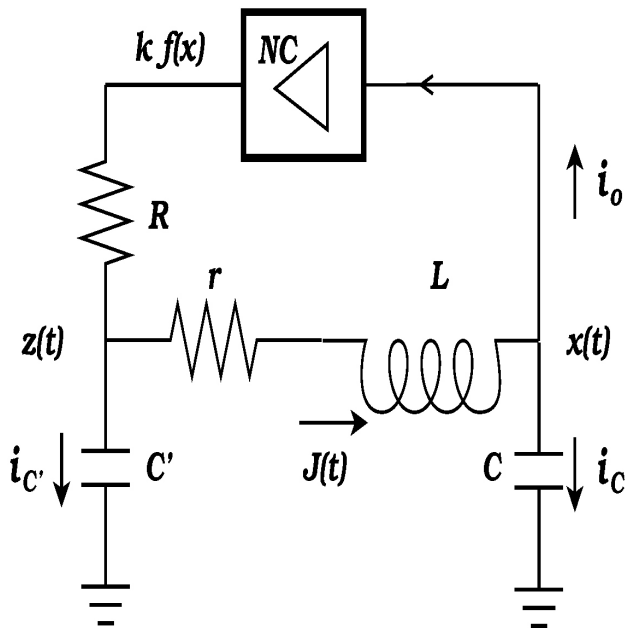


FIGURE 5. The circuit diagram of a nonlinear and chaotic oscillator.

The only difference from (5) is that here the current-voltage relation of the nonlinear block shown in Fig. 2 is expressed as a function  $V(i_x)$ .

Once solution  $i_x(x)$  has been obtained from (10) and (9), it is possible to compute  $f(x)$  with (4). We have that

$$F(x) = kf(x), \tag{11}$$

where

$$f(x) = \frac{V(i_x(x)) - bx}{a}.$$

Figure 4a shows the response of the smooth commutation model given by Eq. (11) and experimental data of the NC. Figure 4b is a zoom in the rupture voltage of the diodes, the solid line is given by Eq. (11) and the dotted line is the curve generated experimentally by the NC. The  $k$  parameter were fixed to  $1/3$ , and parameters  $a = 0.17$  and  $b = 0.489$  according to the values of the Table I. With this model it is possible to find the response of the nonlinear converter, if the values of the components change. Unfortunately, the mathematical model given by Eq. (11) needs more computation than the

model given by Eq. (7). The chaotic generator that uses the NC is discussed in the next section.

### 3. Chaotic Generator

The electronic circuit of Fig. 5 is called Chaotic Generator(CG). This CG is cheap and easily implemented experimentally way. Some values of the electronic components, which have been employed to build the CG experimentally, are given in Table II.

The linear feedback in the schematic diagram of the CG consists of a low-pass filter  $RC'$ , and a resonator circuit  $rLC$ . Depending on parameters of the linear feedback and the parameter  $k$ , the behavior of the CG can be in regimes of periodic or chaotic oscillations.

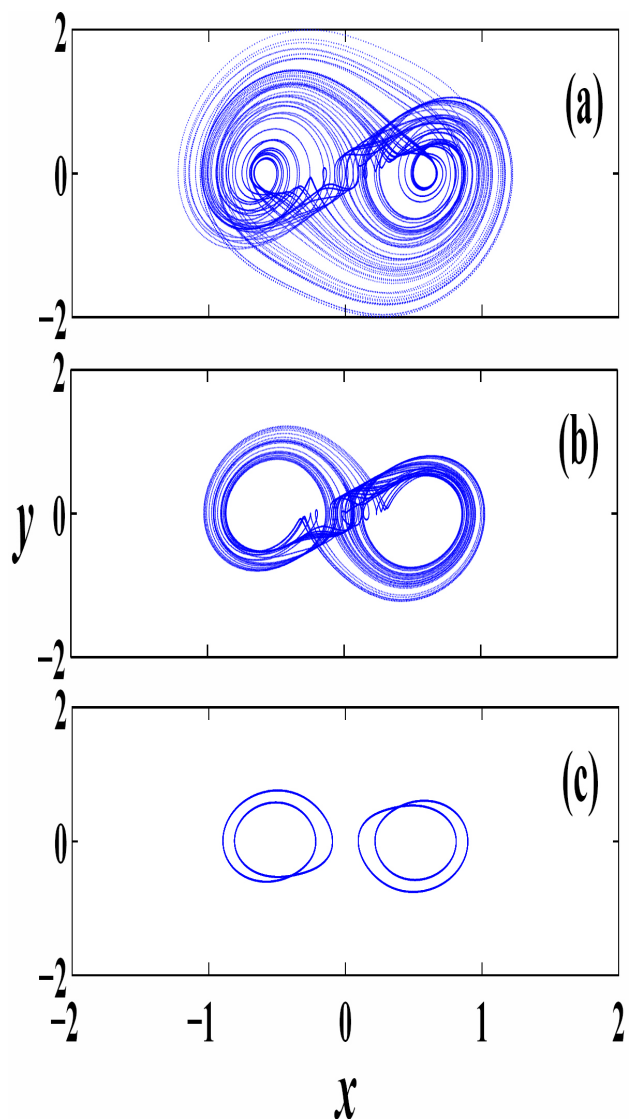


FIGURE 6. The chaotic attractors of the CG projected on the  $(x, y)$  plane for different values of  $k$ :(a) 0.4167, (b) 0.3125 and (c) 0.2083.

The dynamic of the CG shown in Fig. 5 is very well modelled by the following set of differential equations:

$$\begin{aligned}\dot{x} &= y, \\ \dot{y} &= z - x - \delta y, \\ \dot{z} &= \gamma [kf(x) - z] - \sigma y,\end{aligned}\quad (12)$$

where  $x(t)$  and  $z(t)$  are voltages across the capacitors  $C$  and  $C'$ , respectively, and  $y(t) = J(t)(L/C)^{1/2}$  is the current through the coil. The parameter  $k$  is the gain of the nonlinear converter at  $x = 0$ . The unit time is given by  $\tau = \sqrt{LC}$ . The other parameters of the model depend on the physical values of the circuit elements:  $\gamma = \sqrt{LC}/RC'$ ,  $\delta = r\sqrt{C/L}$  and  $\sigma = C/C'$ .

Figure 6 shows different projections of the attractors on the plane  $(x, y)$ , generated by system (12) and the nonlinear function given by Eq. (7). We used a fourth order Runge-Kutta and a step equal to 0.01. These attractors can be obtained just by changing the  $k$  gain parameter, which is the bifurcation parameter. We choose the value of the gain to be equal to  $k = 0.4167$  and  $0.3125$  for the attractors of Figs. 6a-6b, respectively. These attractors take the shape of a family of double scroll oscillations. For the attractors of Fig. 6c, we

select the value of  $k = 0.2083$ , but with different initial conditions. These attractors take the shape of a family of Rössler systems. The initial conditions are  $(-0.1, -0.1, -0.1)$  for the left attractor of Fig. 6c and  $(0.1, 0.1, 0.1)$  for the right attractor.

## 4. Conclusions

We presented a mathematical model for the nonlinear function  $F$  in terms of the input voltage  $x$ . This mathematical model  $F(x)$  has a direct relationship with the components used to build the nonlinear electronic circuit experimentally. Also, different projections of the chaotic generator onto the  $(x, y)$  plane have been presented for different values of  $k$  parameter, where is a bifurcation parameter  $k$  inside the nonlinear function. The mathematical model is in a good agreement with the experimental measured curves.

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1. L.O. Chua, L. kocarév, K. Eckert, and M. Itoh, *Int. J. Bif. and Chaos* **2** (1992) 705.
  2. T.L. Carroll, *Am. J. Phys.* **63** (1995) 377.
  3. N.F. Rulkov, *Chaos* **6** (1996) 262.
  4. N.F. Rulkov and M.M. Sushchik, *Int. J. Bif. and Chaos* **7** (1996) 625.
  5. H.D.I. Abarbanel, T.W. Frison, and L.S. Tsimring, *IEEE Signal Processing Magazine* **3** (1998) 49.