On the Application of the numerical Laplace transform for accurate electromagnetic transient analysis

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This work presents an overview of a methodology based on the Numerical Laplace Transform (NLT) and applied to the analysis of electromagnetic transient phenomena in power systems. The basic development of the method is described, with its main qualitative advantages as compared to conventional time domain methods, such as the method of characteristics and professional programs for transient simulation such as EMTDC and ATP/EMTP. Current practices for reducing errors derived from the truncation and discretization of the analytical equations are also discussed. Finally, some important results obtained recently with this tool are shown. Comparisons with time domain methods reveal a high accuracy of the Numerical Laplace Transform in several studies.

Keywords: Electromagnetic transients, frequency domain analysis, Numerical Laplace Transform.

En este artículo se revisa una metodología basada en la Transformada Numérica de Laplace (TNL) y aplicada al análisis de fenómenos transitorios electromagnéticos en sistemas eléctricos de potencia. Se expone el desarrollo básico del método con sus principales ventajas cualitativas con respecto a métodos convencionales basados en técnicas del dominio del tiempo como son el método de las características y los programas profesionales de simulación de transitorios EMTDC y ATP/EMTP. Se discuten también las prácticas actuales para reducir las fuentes de error derivadas del truncamiento y la discretización de las ecuaciones analíticas. Finalmente, se muestran algunos resultados importantes obtenidos recientemente con esta herramienta. Comparaciones con métodos en el dominio del tiempo demuestran una alta eficiencia de la Transformada Numérica de Laplace en estudios diversos.

Descriptores: Transitorios electromagnéticos; análisis en el dominio de la frecuencia; transformada numérica de Laplace.

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1. Introduction

Electromagnetic transients, mainly due to switching operations, faults and lightning, cause overvoltages that are dangerous to the power system. Therefore, an accurate analysis of these disturbances is very important for the insulation coordination design and testing stages of power system equipment, such as transmission lines and cables, rotatory plant, transformers, grounding systems, and so on. This analysis can be performed either with time or frequency domain methods. However, the latter are preferred, mainly for the following reasons:

- short computer processing time is required,
- nonlinear and time varying elements can be directly accounted for, and
- it is suitable for real time simulation.

Besides, the time domain program EMTP (Electromagnetic Transients Program) is nowadays the most widely known and used tool for analyzing electromagnetic transients in power systems [1].

The inclusion of frequency dependent elements, such as transmission lines, has always been an inherent difficulty in time domain methods. Several approaches have been applied to overcome this problem the since early 70s [2-7], but even the most advanced line models consider approximations that are prone to error in systems with high frequency dependence [8]. In contrast, when using frequency domain methods, such as those based on the Fourier or Laplace transforms, frequency dependent elements can be included in a straightforward manner. Thus, a frequency domain method offers the most theoretically exact transient solution.

In this work, the methodology basis of a frequency domain method, namely the Numerical Laplace Transform, is reviewed, discussing its accuracy as compared with time domain methods and presenting some of its most recent applications.

2. Historical review of the NLT

The Fourier and Laplace transforms are very powerful analysis tools for the solution of differential and integral equations. However, their application to practical problems is limited, given that the transformation from time to frequency domain and vice versa can be very difficult or even impossible. Besides, the time domain function may not be defined analytically, but rather through graphics, experimental measurements, sections or in discrete form. In particular, the analytical solution of systems with nonlinear frequency dependence, such as transmission lines, is practically impossible. To overcome these situations, numerical transformations have been used instead of the analytical expressions.

The numerical inversion of the Laplace transform was introduced in the 60s by Bellman *et al.*, approximating the Laplace integral by a Gauss-Legendre polynomial [9]. From 1965 to 1973, a group lead by Mullineux applied discrete Fourier transforms in analyzing transients in power systems [10-13], naming their technique "Modified Fourier Transform" (MFT), since the algorithm was adapted to reducing truncation and discretization errors. In 1969, the MFT was successfully applied by Medepohl *et al*. to the computation of transients in multiconductor transmission lines [14].

In 1973, Ametani introduced the use of the Fast Fourier Transform algorithm (FFT) to obtain computer time savings, and the MFT became a much more attractive analysis method [15]. The term "Numerical Laplace Transform" was introduced in 1978 by Wilcox, who formulated the MFT in terms of the Laplace transform theory [16].

In 1988, Nagaoka *et al*. developed an electromagnetic transient program in the frequency domain based on the MFT, which included lumped and distributed parameters, as well as switches and nonlinear elements [17].

The Numerical Laplace Transform has been successfully applied in analyzing transients in particular elements such as uniform transmission lines, as well as nonuniform and field excited transmission lines, underground cables, transformer and machine windings, etc. [8], [18-23]. Besides, the NLT has been widely used in testing new time domain model developments.

3. Basic Development of the NLT

Let $f(t)$ be a causal time domain function and $F(s)$ its image in the frequency domain. Direct and inverse Laplace transforms are given by

$$F(s) = \int_0^\infty f(t) e^{-st} dt, \tag{1}$$

$$f(t) = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} F(s) e^{st} ds \tag{2}$$

Defining the Laplace variable as s=c+$j\omega$, (1) and (2) can be rewritten as

$$F(c + j\omega) = \int_0^\infty \left[f(t) e^{-ct} \right] e^{-j\omega t} dt, \tag{3}$$

$$f(t) = \frac{e^{ct}}{2\pi} \int_{-\infty}^{\infty} F(c + j\omega) e^{j\omega t} d\omega, \tag{4}$$

where ω is the angular frequency and c is a stability constant. It can be noticed that when c=0, (3) and (4) correspond to the Fourier transforms:

$$F(j\omega) = \int_0^\infty f(t) e^{-j\omega t} dt, \tag{5}$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(j\omega) e^{j\omega t} d\omega. \tag{6}$$

A comparison of (3) and (5) shows that the Laplace transform can be obtained by applying the Fourier integral to $f(t)$exp(-*ct*), *i.e.* a damped version of $f(t)$. Hence, c is also known as a damping constant and, as will be seen, its correct definition is fundamental in order to reduce aliasing errors.

As previously mentioned, the application of (3) and (4) [or (5) and (6)] for real practical systems can be very difficult or even impossible. In consequence, these expressions need to be evaluated numerically, giving rise to truncation and discretization errors. Practical techniques for reducing numerical errors when inverting from Laplace to time domain are addressed in the following subsections.

3.1. Truncation errors

It will be assumed in this section that c=0; application of c as a damping factor will be introduced in 3.2. For the numerical evaluation of (6), the finite range [-Ω, Ω] is considered:

$$f(t) = \frac{1}{2\pi} \int_{-\Omega}^{\Omega} F(j\omega) e^{j\omega t} d\omega. \tag{7}$$

Equation (7) can be rewritten as

$$f'(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(j\omega) H(\omega) e^{j\omega t} d\omega, \tag{8}$$

where

$$H(\omega) = \begin{cases} 1, & -\Omega < \omega < \Omega \\ 0, & \Omega < \omega < -\Omega \end{cases}. \tag{9}$$

From (6) and (8):

$$F'(j\omega) = F(j\omega) H(\omega), \tag{10}$$

and from the convolution theorem:

$$f'(t) = f(t) * h(t), \tag{11}$$

where $h(t)$ is the inverse Laplace transform of $H(\omega)$, computed as follows:

$$h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\omega) e^{j\omega t} d\omega = \frac{\Omega}{\pi} \frac{\sin(\Omega t)}{\Omega t}. \tag{12}$$

According to (11) and (12), truncation of the frequency spectrum is equivalent to the convolution of $f(t)$ and a *sinc* function in time domain. As an example, let $f(t)$ be a unit step function. The waveform obtained from its convolution with $h(t)$ shows high frequency oscillations near the discontinuities (Fig. 1), known as Gibbs oscillations, which lead to amplitude errors that are unacceptable for transient analysis purposes. This magnitude can be reduced to an acceptable value by the introduction of some suitable data window $\sigma(\omega)$, *e.g.* by multiplying $F(j\omega)$ by $\sigma(\omega)$. Among a variety of existing data windows for digital signal processing, Day *et al.* introduced the use of the Lanczos window for transient analysis in 1965 [10], while Wedepohl proposed in 1983 the use of the Hamming window [24]. More recently, the Hanning (Von Hann) and Blackman windows have also been tested, yielding satisfactory results [8]. Figure 2 shows these data windows, while Table I lists their respective equations.

TABLE I. Defining equations of data windows.

Window	Equation
Blackman	$\sigma(\omega) = 0.45 + 0.5\cos\left(\pi\frac{\omega}{\Omega}\right) + 0.08\cos\left(2\pi\frac{\omega}{\Omega}\right)$
Hanning	$\sigma(\omega) = \frac{1+\cos(\pi\omega/\Omega)}{2}$
Lanczos	$\sigma(\omega) = \frac{1+\cos(\pi\omega/\Omega)}{\pi\omega/\Omega}$
Riez	$\sigma(\omega) = 1.0 - \left\lvert\frac{\omega}{\Omega}\right\rvert^2$

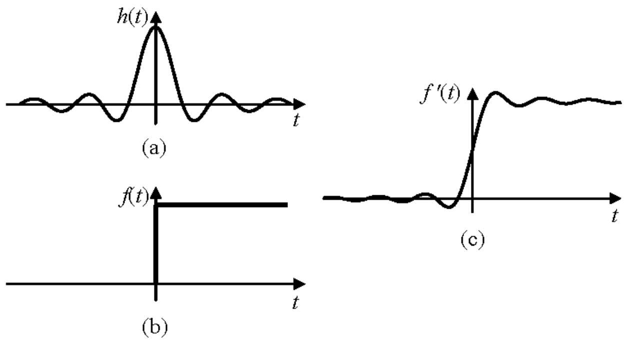

FIGURE 1. Convolution of $f(t)$ and $h(t)$.

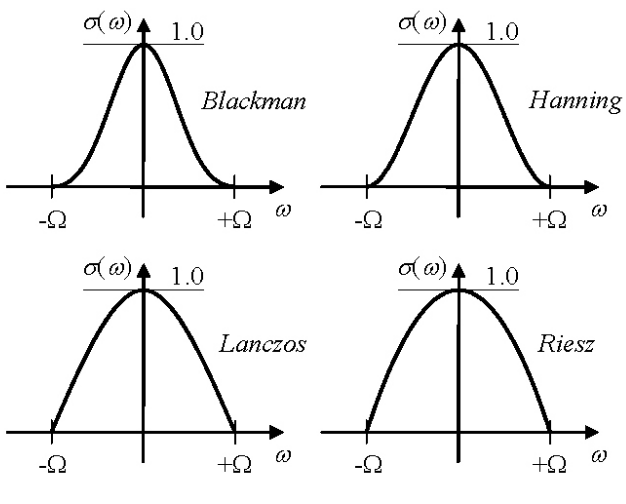

FIGURE 2. Data windows.

3.2. Discretization errors

Equation (6) can be expressed in discrete form as

$$f_1(t) = \frac{\Delta\omega}{2\pi}\sum_{n=-\infty}^{\infty} F(jn\Delta\omega)e^{jn\Delta\omega t}, \tag{13}$$

where $\Delta\omega$ is the spectrum integration step. From the sampling property of a Dirac function, the term inside the summation can be expressed as follows:

$$f_1(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)G(\omega)e^{j\omega t}d\omega, \tag{14}$$

where $G(\omega)$ is a Dirac comb in the frequency domain:

$$G(\omega) = \Delta\omega\sum_{n=-\infty}^{\infty}\delta(\omega - n\Delta\omega), \tag{15}$$

or in the time domain

$$g(t) = \sum_{n=-\infty}^{\infty}\delta(t - nT), \tag{16}$$

with $T = \Delta\omega/2\pi$. From (14) and the definition of the inverse Fourier transform:

$$F_1(j\omega) = F(j\omega)G(\omega). \tag{17}$$

Using the convolution theorem, the discrete approximation $f_1(t)$ is given by the convolution of the original function $f(t)$ and the Dirac comb $g(t)$:

$$f_1(t) = f(t) * g(t) = \sum_{n=-\infty}^{\infty} f(t - nT). \tag{18}$$

Equation (18) shows that $f_1(t)$ is obtained from a superposition of $f(t)$ and its time-displaced versions $f(t+T)$, $f(t+2T)$, etc., as shown in Fig. 3. This causes *aliasing* errors, which can be reduced by multiplying $f(t)$ by the damping factor exp(-*ct*), as in (3), so that $f(t)$ tends to zero for $t > T$. Function $f_1(t)$ will accurately approximate $f(t)$ in the interval $0 < t < T$ if the damping constant c is appropriately chosen. Given that exp(-*ct*) is used to damp $f(t)$, it could be supposed that a high value for c is required. Unfortunately, other errors arise if c is chosen too high, since the expression exp(*ct*) in the inverse Laplace transform acts as an amplifier which, when multiplied by $f(t)$, magnifies the remaining errors related to truncation and quantification.

Determination of c is still mostly based on empirical rules. Wilcox [16] proposed the following criterion:

$$c = 2\Delta\omega. \tag{19}$$

Wedepohl [24] found a relationship between the number of samples N and the choice of c, given by

$$c = \frac{\ln\left(N^2\right)}{T}. \tag{20}$$

Using (20), aliasing errors can be directly reduced by increasing the number of samples.

3.3. Direct numerical Laplace transform

Considering $f(t)$ to be not only causal but also real and measurable, and taking a finite integration range [0,T], (3) can be written in discrete form according to

$$F(c+jm\Delta\omega) = \sum_{n=0}^{N-1} f(n\Delta t)e^{-cn\Delta t}e^{-jm\Delta\omega n\Delta t}\Delta t, \tag{21}$$

where n= 0, 1, 2,..., N-1 and Δt is the time step. Moreover, (21) can be expressed in terms of the well known Discrete Fourier Transform (DFT):

$$F_m = \sum_{n=0}^{N-1} f_n D_n \exp\left(-\frac{j2\pi mn}{N}\right), \tag{22}$$

where

$$D_n = \Delta t \exp\left(-cn\Delta t - \frac{j\pi n}{N}\right). \tag{23}$$

3.4. Inverse numerical Laplace transform

Taking a finite integration range [0,Ω] and including the data window $\sigma(\omega)$, (4) can be expressed as

$$f(t) \cong \frac{e^{ct}}{\pi} Re\left\{\int_0^{\Omega} F(c+j\omega)\sigma(\omega)e^{j\omega t}d\omega\right\}. \tag{24}$$

For the numerical evaluation of (24), an odd sampling of ω is considered in order to avoid singularities of $F(j\omega)$ at ω=0. Bearing this in mind, the discrete form of (24) is as follows:

$$f(n\Delta t) = \frac{e^{cn\Delta t}}{\pi} Re\left\{\sum_{m=1,3,5,\ldots}^{2N} F\left(c+jm\Delta\omega\right) \times \sigma\left(m\Delta\omega\right)e^{jm\Delta\omega\, n\Delta t}2\Delta\omega\right\}. \tag{25}$$

Equation (25) can be expressed in terms of the Inverse Discrete Fourier Transform (IDFT):

$$f_n = Re\left\{C_n\left[\frac{1}{N}\sum_{m=1,3,5,\ldots}^{2N} F_m\sigma_m \exp\left(\frac{j2\pi mn}{N}\right)\right]\right\}, \tag{26}$$

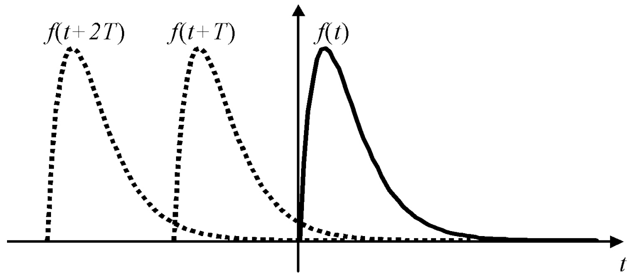

FIGURE 3. Superposition of $f(t)$ and its time-displaced versions.

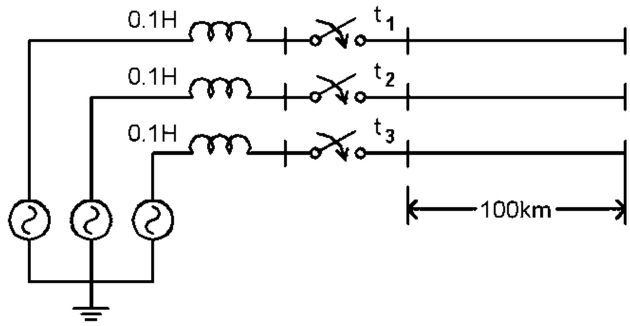

FIGURE 4. Circuit for example 4.1.

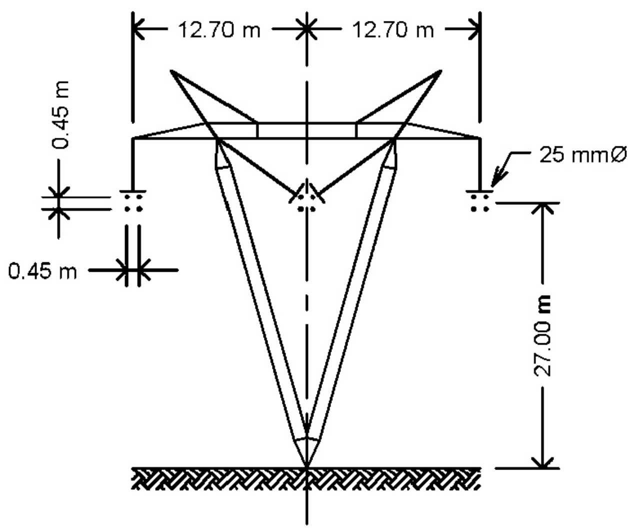

FIGURE 5. Arrangement of the conductors for example 4.1.

where

$$F_m = F(c+jm\Delta\omega), \tag{27}$$

$$f_n = f(n\Delta t), \tag{28}$$

$$C_n = \frac{2}{\Delta t}\exp\left(cn\Delta t + \frac{j\pi n}{N}\right), \tag{29}$$

$$\sigma_m = \sigma(m\Delta\omega). \tag{30}$$

Equations (22) and (26) can be solved using the FFT (Fast Fourier Transform) and inverse FFT, respectively, to get computer time savings.

TABLE II. Effect of increasing N in EMTDC.

N	max. absolute difference (p.u.)	max. relative error (%)
1×1024	0.1249	7.1213
2×1024	0.0796	4.5408
4×1024	0.0621	3.5451
6×1024	0.0529	3.0156
8×1024	0.0522	2.9768
10×1024	0.0551	3.1433

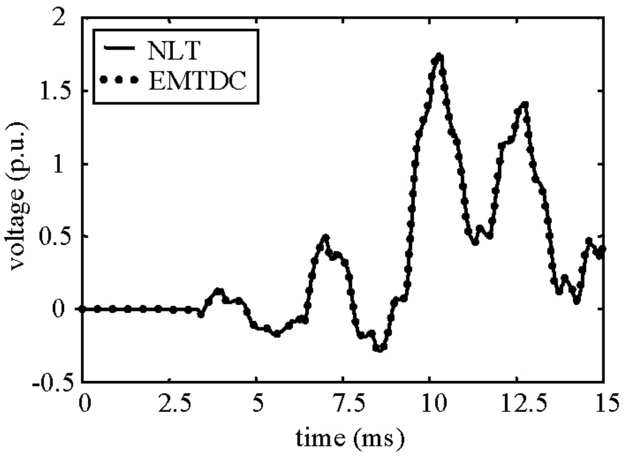

FIGURE 6. Voltage at phase C of the receiving end of the line.

4. Application examples

In order to illustrate the use of the NLT, three application examples related to electromagnetic transient phenomena are presented in this section: sequential energization of a transmission line, fast transient overvoltage in machine winding, and switching transients related to the restoration process of a power network. Comparisons with EMTDC, ATP and the Method of Characteristics are provided to show the accuracy of the NLT.

4.1. Sequential energization of a transmission line

A 400 kV 3-phase transmission line shown in Fig. 4, with the conductors arrangement depicted in Fig. 5 is considered. Sequential energization with closing times of 3, 6 and 9 ms for phases A, B and C, respectively, is analyzed using the NLT and the commercial time domain program EMTDC, with a total observation time of 15 ms. The Phase Domain Line Model, which takes into account the frequency dependence of the line electrical parameters, was used for the EMTDC simulation [7]. The number of samples for the NLT algorithm was fixed to 1024 (2^{10}), while in EMTDC it was increased from 1024 to 10×1024, for a total of 10 simulations.

TABLE III. Winding data.

Slot width	0.75m
Slot material	Iron
Turn area	3×9 mm
Turn length	3.8 m
Slot length	0.75 m
Overhang length	1.15 m
Conductor material	Copper

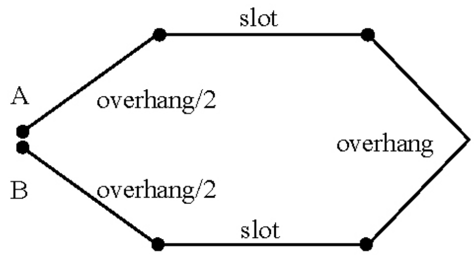

FIGURE 7. Winding representation using line segments.

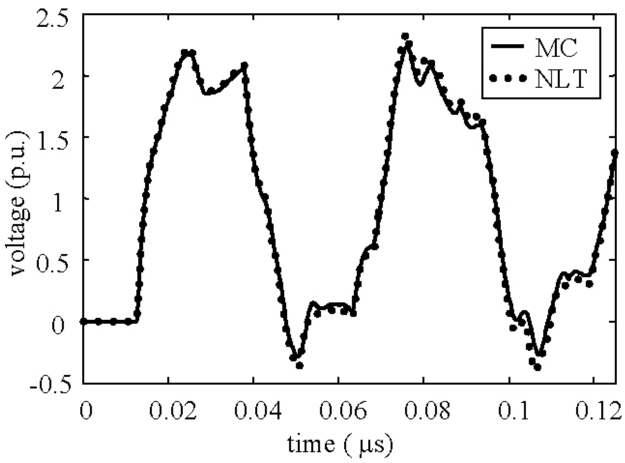

FIGURE 8. Voltage at node B of the winding.

In Fig. 6, the transient overvoltage at phase C of the receiving node is shown, with N=10×1024 in EMTDC.

Table II shows a comparison between NLT and EMTDC results for different number of samples of the latter. It can be noticed that from 6×1024 onwards, the relative error between the two methods remains at approximately 3%. It is clear that the number of samples required in EMTDC for this example is on the order of 6 times those of NLT to assure similar results.

4.2. Fast transient overvoltage in machine winding

A machine winding is modeled using 6 distributed parameter segments, as shown in Fig. 7. The coil electrical parameters are computed according to [25] and the data listed in Table III. A unit step voltage source is connected to node A,

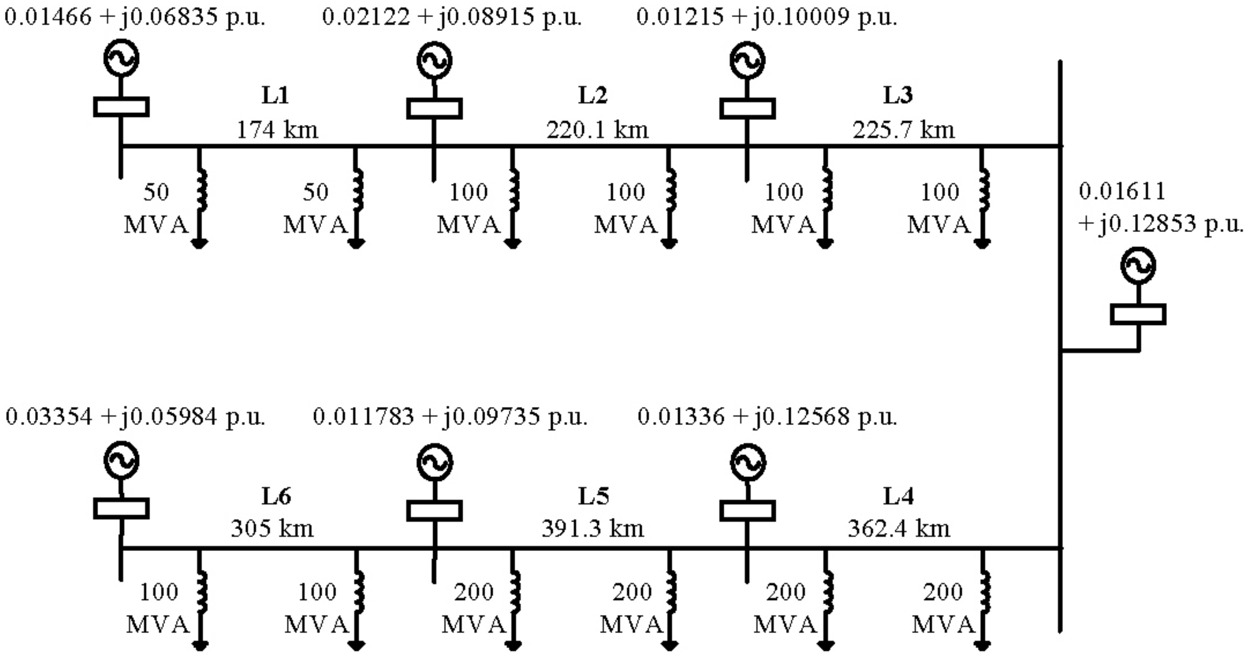

FIGURE 9. One-line diagram for example 4.3.

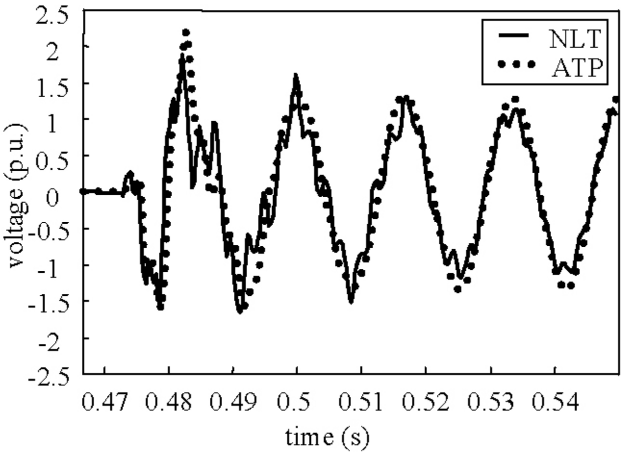

FIGURE 10. Voltage at phase A of the open end of the line L2. Same number of samples for NLT and ATP.

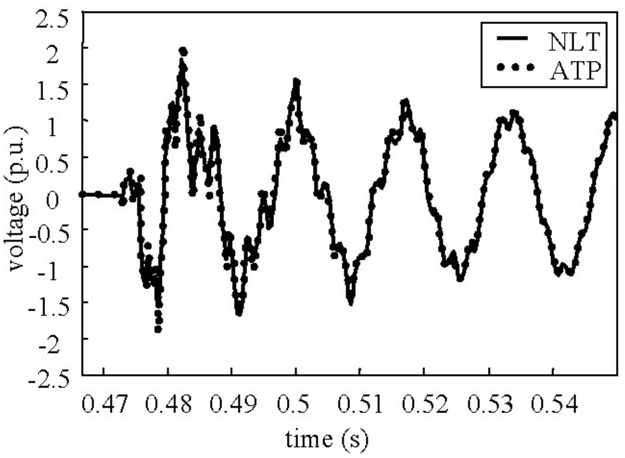

FIGURE 11. Voltage at phase A of the open end of the line L2. Number of samples for ATP 15 times greater than with NLT.

while node B is left open. Figure 8 shows the voltage waveform at node B, comparing the results with those obtained with the Method of Characteristics (MC) [26]. A better approximation between the waveforms could not be achieved, since the MC requires a rational approximation in order to consider the frequency dependence of the coil parameters, which can be difficult for non-smooth frequency spectra.

4.3. Switching transients related to the restoration process of a power network

In order to analyze transient overvoltages related to transmission line energization during a restoration process, the NLT and the superposition principles [8], [18] were applied to the test system shown in Fig. 9. Each switch operation is performed in separated simulation processes to obtain more accurate results. Waveforms are compared with those obtained in the time domain using the ATP. To analyze the most severe overvoltages, each sequential energization is considered to be critical, *i.e.* each switch pole closes at the maximum voltage value present, and neither pre-insertion resistors nor arresters are included. The ATP simulation was performed using the J. Marti Line Model, which considers the frequency dependence of the line electrical parameters [6].

As an example, Fig. 10 shows transient overvoltage at phase A of the open end of line L2 when energized from its left end, with line L1 previously connected. For both ATP and NLT simulations, N=2048 samples were used. An important difference in waveforms obtained with the frequency and time domain methods can be noticed. Closing times of switch poles were 0.475, 0.47222 and 0.47777 s for phases A, B and C, respectively, considering a damping time of 0.4666 s (28 cycles) for the transient produced by previous connection of line L1.

Figure 11 shows transient overvoltage at phase A of the same line, when the frequency domain analysis is performed with N=2048 but 15N points are considered in ATP. This gives very similar results, showing that in this case the frequency domain method is much more accurate than ATP.

5. Conclusions

In this article, the basic development of the Numerical Laplace Transform has been presented. This technique has proven to be efficient for the analysis of electromagnetic transients in power systems. The main advantages of the NLT are summarized below:

1. The modeling of components with distributed and frequency dependent parameters can be done in a straightforward manner.

2. Since its basic principles are different from those of time domain methods, the NLT is very useful to verifying time domain methods, as well as in the development of new time domain models and techniques.

3. The application of the NLT can be very important when a high accuracy of results is mandatory. The examples given show that time domain methods may require a much smaller discretization step.

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