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Electroweak-Higgs unification in the two Higgs doublet model: masses and couplings of the neutral and charged Higgs bosons

J.L. Díaz-Cruz
Facultad de Ciencias Físico-Matemáticas, BUAP,
Apartado Postal 1364, 72000 Puebla, Pue., México.

A. Rosado
Instituto de Física, BUAP,
Apartado Postal J-48, 72570 Puebla, Pue., México.

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We obtain the mass spectrum and the Higgs self-couplings of the two Higgs doublet model (THDM) in an alternative unification scenario where the parameters of the Higgs potential λ_i ($i = 1, 2, 3, 4, 5$) are determined by imposing their unification on the electroweak gauge couplings. An attractive feature of this scenario is the possibility of determining the Higgs boson masses by evolving the λ_i from the electroweak-Higgs unification scale M_{GH} down to the electroweak scale. The unification condition for the gauge (g_1, g_2) and Higgs couplings is written as $g_1 = g_2 = f(\lambda_i)$, where $g_1 = k_Y^{1/2} g_Y$, and k_Y isthe normalization constant. Two variants for the unification condition are discussed: Scenario I is defined through the linear relation: $g_1 = g_2 = k_H(i)\lambda_i(M_{GH})$, while Scenario II assumes a quadratic relation $g_1^2 = g_2^2 = k_H(i)\lambda_i(M_{GH})$. In Scenario I, by setting *ad hoc* $-k_H(5) = \frac{1}{2}k_H(4) = \frac{3}{2}k_H(3) = k_H(2) = k_H(1) = 1$, taking $\tan\beta = 1$ and using the standard normalization ($k_Y = 5/3$), we obtain the following spectrum for the Higgs boson masses: $m_{h^0} = 109.1$ GeV, $m_{H^0} = 123.2$ GeV, $m_{A^0} = 115.5$ GeV, and $m_{H^\pm} = 80.3$ GeV, with similar results for other normalizations such as $k_Y = 3/2$ and $k_Y = 7/4$.

Keywords: Electroweak-Higgs unification; Higgs boson masses and couplings; Two Higgs doublet model.

Se obtienen el espectro de masas y los autoacoplamientos de los bosones de Higgs en el modelo de dos dobletes de Higgs, en un escenario de unificación alternativo donde los parámetros del potencial de Higgs λ_i ($i = 1, 2, 3, 4, 5$) son determinados imponiendo su unificación con los acoplamientos de norma electrodébiles. Una característica atractiva de este escenario es la posibilidad de determinar las masas de los bosones de Higgs mediante la evolución de las λ_i,s de la escala de unificación electrodébil-Higgs M_{GH} a la escala electrodébil. La condición de unificación para los acoplamientos de norma (g_1, g_2) y de Higgs es $g_1 = g_2 = f(\lambda_i)$, donde $g_1 = k_Y^{1/2} g_Y$, y k_Y es la constante de normalización. Dos variantes para la condición de unificación son discutidas. Escenario I definido a través de la relación lineal: $g_1 = g_2 = k_H(i)\lambda_i(M_{GH})$, mientras en el Escenario II se supone una relación cuadrática: $g_1^2 = g_2^2 = k_H(i)\lambda_i(M_{GH})$. Trabajando en el Escenario I, fijando *ad hoc* $-k_H(5) = \frac{1}{2}k_H(4) = \frac{3}{2}k_H(3) = k_H(2) = k_H(1) = 1$, tomando $\tan\beta = 1$ y usando la normalización estándar ($k_Y = 5/3$), se obtiene el siguiente espectro de masas para los bosones de Higgs $m_{h^0} = 109.1$ GeV, $m_{H^0} = 123.2$ GeV, $m_{A^0} = 115.5$ GeV, y $m_{H^\pm} = 80.3$ GeV, con resultados similares para otras normalizaciones, tales como $k_Y = 3/2$ and $k_Y = 7/4$.

Descriptores: Unificación electrodébil-Higgs; masas y acoplamientos de bosones de Higgs; Modelo de dos dobletes de Higgs.

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1. Introduction

The Standard Model (SM) of the strong and electroweak (EW) interactions has met with extraordinary success; it has been already tested at the level of quantum corrections [1, 2]. These corrections give some hints about the nature of the Higgs sector, pointing towards the existence of a relatively light Higgs boson, with a mass of the order of the EW scale, $m_{\phi_{SM}} \simeq v$ [3]. However, it is widely believed that the SM cannot be the final theory of particle physics, in particular because the Higgs sector suffers from naturalness problems, and we do not really have a clear understanding of electroweak symmetry breaking (EWSB).

These problems in the Higgs sector can be stated as our present inability to find a satisfactory answer to some questions regarding its structure, which can be stated as follows:

1. What determines the size (and sign) of the *dimensionful parameter* μ_0^2 that appears in the Higgs potential? This parameter determines the scale of EWSB in the SM; in principle it could be as high as the Planck mass; however, it needs to be fixed to much lower values.

2. What is the nature of the quartic Higgs coupling λ? This parameter is not associated with a known symmetry, and we expect all interactions in nature to be associated somehow with gauge forces, as these are the ones we understand best [4].

An improvement on our understanding of EWSB is provided by the supersymmetric (SUSY) extensions of the SM [5], where loop corrections to the tree-level parameter μ_0^2 are under control, thus making the Higgs sector more natural. The quartic Higgs couplings are nicely related to gauge couplings through relations of the form $\lambda = (g_2^2 + g_Y^2)/8$. In the SUSY alternative, it is even possible to (indirectly) explain the sign of μ_0^2 as a result of loop effects and the breaking of the symmetry between bosons and fermions. Further progress to understand the SM structure is achieved in Grand Unified Theories (GUT), where the strong and electroweak gauge interactions are unified at a high-energy scale (M_{GUT}) [6]. However, certain consequences of the GUT idea seem to indicate that this unification, by itself, may be too drastic (within the minimal SU(5) GUT model one actually gets inexact unification, too large proton decay, doublet-triplet problem, incorrect fermion mass relations, etc.), and some additional theoretical tool is needed to overcome these difficulties. Again, SUSY offers an improvement in these problems. When SUSY is combined with the GUT program, one gets a more precise gauge coupling unification and some aspects of proton decay and fermion masses are under better control [7, 8].

In order to verify the realization of SUSY-GUT in nature, it will be necessary to observe plenty of new phenomena such as superpartners, proton decay or rare decay modes.

As attractive as these ideas may appear, it seems worthwhile to consider other approaches for physics beyond the SM. For instance, it has been shown that additional progress towards understanding the SM origin can be achieved by postulating the existence of extra dimensions. These theories have received a great deal of attention, mainly because of the possibility they offer in addressing the problems of the SM in a new geometrical perspective. These range from a new approach to the hierarchy problem [9–13] to a possible explanation of flavor hierarchies in terms of field localization along the extra dimensions [14]. Models with extra dimensions have been applied to neutrino physics [15–19] and Higgs phenomenology [20, 21], among many others. In the particular GUT context, it has been shown that it is possible to find viable solutions to the doublet-triplet problem [22,23]. More recently, new methods in strong interactions have also been used in an attempt to revive the old models (TC, ETC, topcolor, etc) [24]. Other ideas have motivated new types of models as well (little Higgs [25], AdS/CFT composite Higgs models [26], etc).

In this paper we are interested in exploring further an alternative unification scenario, of a weakly-interacting type, that could also offer a direct understanding of the Higgs sector too and that was first discussed in Ref. 27. Namely, we shall explore the consequences of a scenario in which the electroweak $SU(2)_L \times U(1)_Y$ gauge interactions are unified with the Higgs self-interactions on an intermediate scale M_{GH}. Here, we further explore this idea within the context of the THDM, which allows us to predict the Higgs spectrum of this model. The dependence of our results on the choice for the normalization for the hypercharge is also discussed, as well as a possible test of this EW-Higgs unification idea at future colliders, such as ILC. Besides predicting the Higgs spectrum, namely the masses for the neutral CP-even states (h^0, H^0), the neutral CP-odd state (A^0) and the charged Higgs ($H^{\pm}$), we also discuss the Higgs couplings in gauging bosons and fermions. As we mentioned in our previous paper [27], it is relevant to compare our approach with the so-called Gauge-Higgs unification program, as they share some similarities. We think that our approach is more model independent, as we first explore the consequences of a parametric unification, without really choosing a definite model at higher energies. In fact, at higher energies both the SUSY models and the framework of extra dimensions could work as an ultraviolet completion to our approach. The SUSY models could work because they allow us to relate the scalar quartic couplings to the gauge couplings, thanks to the D-terms [4]. On the other hand, within the extra dimensions it is also possible to obtain similar relations, when the Higgs fields are identified as the extra-dimensional components of gauge fields [28–40]. Actually, we feel that the work of Ref. 41 and 42 has a similar spirit to ours; in their case they look for gauge unification of the Higgs self-couplings that appear in the superpotential of the NMSSM, and then they justify their work with a concrete model in 7D. However, in the present work, we do not discuss further the unification of the EW-Higgs couplings with the strong constant, which can be done within the context of extra-dimensional Gauge-Higgs unified theories.

2. Gauge-Higgs unification in the SM: review

In the EW-Higgs unified scenario, one assumes that there exists a scale where the gauge coupling constants g_1, g_2, associated with the gauge symmetry $SU(2)_L \times U(1)_Y$, are unified, and that on this scale they are also unified with the SM Higgs self coupling λ, *i.e.* $g_1 = g_2 = f(\lambda)$ at M_{GH}. The precise relation between g_1 and g_Y (the SM hypercharge coupling) involves a normalization factor k_Y, *i.e.* $g_1 = k_Y^{1/2} g_Y$, which depends on the unification model. The standard normalization gives $k_Y = 5/3$, which is associated with minimal models such as $SU(5), SO(10), E_6$. However, in the context of string theory, it is possible to have such standard normalization without even having a unification group. For other unification groups that involve additional $U(1)$ factors, one would also have exotic normalizations, and similarly for the case of GUT models in extra dimensions. In what follows we shall present results for the cases: $k_Y = 5/3, 3/2$ and $7/4$, which indeed arise in string-inspired models [43]. Note that these values fall in the range $3/2 < 5/3 < 7/4$ and so they can illustrate what happens when one chooses a value below or above the standard normalization. The form of the unification condition will depend on the particular realization of this scenario is carried out, which should be as generic as possible. However, in order to be able to make predictions for the Higgs boson mass, we shall consider two specific cases. Scenario I will be based on the linear relation: $g_2 = g_1 = k_H \lambda(M_{GH})$, where the factor k_H is included in order to maintain some generality, for instance to take into account possible unknown group theoretical or normalization factors. Motivated by specific models, such as SUSY itself, as well as an argument based on the power counting of the beta coefficients in the RGE for scalar couplings, *i.e.*, the fact that β_λ varies with $O(g^4)$, we shall also define Scenario II, through the quadratic unification condition: $g_1^2 = g_2^2 = k_H \lambda(M_{GH})$. The expressions for the SM renormalization group equations at the two-loop level can be found in Ref. 44.

In practice, one first determines the scale M_{GH} at which g_2 and g_1 are unified, then one fixes the quartic Higgs coupling λ by imposing the unification condition and finally, by evolving the quartic Higgs coupling down to the EW scale, we are able to predict the Higgs boson mass. For the numerical calculations, discussed in Ref.27, we employed the full two-loop SM renormalization group equations involving the gauge coupling constants $g_{1,2,3}$, the Higgs self-coupling λ, the top-quark Yukawa coupling g_t, and the parameter k_Y [44,45]. We also take the values for the coupling constants as reported in the Review of Particle Properties [46], while for the top quark mass we take the value recently reported in [47,48].

Now, let us summarize our previous results with a full numerical analysis. For $k_Y = 5/3$, we find that $M_{GH} \cong 1.0 \times 10^{13}$ GeV and by taking $\tan\beta = 1$, results for the Higgs boson mass are given as a function of the parameter k_H over a range $10^{-1} < k_H < 10^2$, which covers three orders of magnitude (We stress here that the expected natural value for k_H is 1). For this range of k_H, the Higgs boson mass takes on the values: $176 < m_H < 275$ GeV for Scenario I, while for $k_H = 1$ we obtain a prediction for the Higgs boson mass: $m_H = 229,\ 234,\ 241$ GeV, for a top quark mass of $m_{top} = 165,\ 170,\ 175$ GeV [47, 48], respectively. On the other hand, for Scenario II, we find that the Higgs boson mass can take on the values: $175 < m_H < 269$ GeV, while for $k_H = 1$ we obtain: $m_H = 214,\ 222,\ 230$ GeV.

Then, when we compare our results with the Higgs boson mass obtained from EW precision measurements, which imply $m_H \lesssim 190$ GeV, we notice that in order to have compatibility with this value, our model seems to prefer high values of k_H. For instance, by taking the lowest value that we consider here for the top mass, $m_t = 165$ GeV, and getting $k_H = 10^2$, we obtain the minimum value for the Higgs boson mass equal to $m_H = 176$ GeV in Scenario I, while Scenario II implies a minimal value that is slightly lower, $m_H = 175$ GeV.

For k_Y=$3/2$, we find that M_{GH}=4.9×10^{14} GeV, higher than in the previous case, but for which one still gets a mass gap between M_{GH} and a possible M_{GUT}. In this case, and by taking $\tan\beta = 1$, we find values that are a little higher for the Higgs boson mass; for instance, for $k_H = 1$, one gets $m_H = 225,\ 232,\ 238$ ($m_H = 212,\ 220,\ 218$) GeV for Scenario I (II).

On the other hand, for $k_Y = 7/4$, we find that $M_{GH} = 1.8 \times 10^{12}$ GeV, which is lower than in the previous cases, and has an even larger mass gap between M_{GH} and a possible M_{GUT}. In this case, and by taking $\tan\beta = 1$, we also find values slightly higher for the Higgs boson mass; for instance, for $k_H = 1$, one finds that $m_H = 230,\ 236,\ 243$ ($m_H = 215,\ 223,\ 231$) GeV for Scenario I (II).

At this point, rather than continuing discussions on the precise Higgs boson mass, we would like to emphasize that our approach based on the EW-Higgs unification idea is very successful in giving a Higgs boson mass that has indeed the correct order of magnitude, and that once measured at the LHC, we shall be able to fix the parameter k_H and find connections with other approaches for physics beyond the SM, such as the one to be discussed next.

In fact, for the Higgs boson mass range that is predicted in our approach, it turns out that the Higgs boson will decay predominantly into the mode $h \to ZZ$, which may provide us with good chances to measure the Higgs boson mass within a precision of 5% [49,50], thus making it possible to bound k_H to within a few percent. Further tests of our EW-Higgs unification hypothesis would involve testing more implications of the quartic Higgs coupling. For instance, one could use the production of Higgs boson pairs ($e^+e^- \to \nu\bar{\nu}hh$) at a future linear collider, such as the ILC. This is just another example of the complementarity of future studies at LHC and ILC.

3. EW-Higgs unification in the Two-Higgs doublet model

Let us now discuss the implications of EW-Higgs unification for the two Higgs doublet model (THDM). This model includes two scalar doublets (Φ_1, Φ_2), and the Higgs potential can be written as follows [51]:

$$\begin{aligned} V(\Phi_1,\Phi_2) &= \mu_1^2\Phi_1^\dagger\Phi_1 + \mu_2^2\Phi_2^\dagger\Phi_2 + \lambda_1(\Phi_1^\dagger\Phi_1)^2 \\ &+ \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ &+ \frac{1}{2}\lambda_5[(\Phi_1^\dagger\Phi_2)^2 + (\Phi_2^\dagger\Phi_1)^2]. \end{aligned} \tag{1}$$

It is clear that, by absorbing a phase in the definition of Φ_2, one can make λ_5 real and negative, which pushes all potential CP violating effects into the Yukawa sector:

$$\lambda_5 \le 0. \tag{2}$$

In order to prevent the spontaneous breakdown of the electromagnetic $U(1)$ [52], the vacuum expectation values must have the following form:

$$\langle\Phi_1\rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix}\ ,\ \langle\Phi_2\rangle = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}\ , \tag{3}$$

$v_1^2 + v_2^2 \equiv v^2 = (246GeV)^2$. This configuration is indeed a minimum of the tree level potential if the following conditions are satisfied:

$$\begin{gathered} \lambda_1 \ge 0\ , \quad \lambda_2 \ge 0\ , \quad \lambda_4 + \lambda_5 \le 0\ , \\ 4\lambda_1\lambda_2 \ge (\lambda_3 + \lambda_4 + \lambda_5)^2\ . \end{gathered} \tag{4}$$

The scalar spectrum in this model includes two CP-even states (h^0, H^0), one CP-odd (A^0) and two charged Higgs bosons ($H^{\pm}$). The tree level expressions for the masses and mixing angles are given as follows:

$$\tan\beta = \frac{v_2}{v_1}\,, \tag{5}$$

$$\sin\alpha = -(\mathrm{sgn}C)\left[\frac{1}{2}\frac{\sqrt{(A-B)^2+4C^2}-(B-A)}{\sqrt{(A-B)^2+4C^2}}\right]^{\frac{1}{2}}, \tag{6}$$

$$\cos\alpha = \left[\frac{1}{2}\frac{\sqrt{(A-B)^2+4C^2}+(B-A)}{\sqrt{(A-B)^2+4C^2}}\right]^{1/2}, \tag{7}$$

$$M^2_{H^\pm} = -\frac{1}{2}(\lambda_4+\lambda_5)v^2\,, \tag{8}$$

$$M^2_{A^0} = -\lambda_5 v^2\,, \tag{9}$$

$$M^2_{H^0,h^0} = \frac{1}{2}\left[A+B\pm\sqrt{(A-B)^2+4C^2}\right]\,, \tag{10}$$

where $A = 2\lambda_1 v_1^2$, $B = 2\lambda_2 v_2^2$, $C = (\lambda_3+\lambda_4+\lambda_5)v_1v_2$.

The two Higgs doublet models are described by 7 independent parameters which can be taken to be α, β, $m_{H^\pm}$, m_{H^0}, m_{h^0}, m_{A^0}, while the top quark mass is given as:

$$m_t = g_t v \sin\beta\,. \tag{11}$$

Now, we write the THDM renormalization group equations at the one-loop level involving the gauge coupling constants $g_{1,2,3}$, the Higgs self-couplings $\lambda_{1,2,3,4,5}$, the top-quark Yukawa coupling g_t, and the parameter k_Y, as follows [44, 51]:

$$\frac{dg_i}{dt} = \frac{b_i^{thdm}}{16\pi^2}g_i^3, \tag{12}$$

$$\frac{dg_t}{dt} = \frac{g_t}{16\pi^2}\left[\frac{9}{2}g_t^2 - \left(\frac{17}{12k_Y}g_1^2+\frac{9}{4}g_2^2+8g_3^2\right)\right], \tag{13}$$

$$\frac{d\lambda_1}{dt} = \frac{1}{16\pi^2}\Big[24\lambda_1^2+2\lambda_3^2+2\lambda_3\lambda_4+\lambda_5^2+\lambda_4^2 -3\lambda_1(3g_2^2+\frac{1}{k_Y}g_1^2)+12\lambda_1 g_t^2+\frac{9}{8}g_2^4 +\frac{3}{4k_Y}g_1^2g_2^2+\frac{3}{8k_Y^2}g_1^4-6g_t^4\Big], \tag{14}$$

$$\frac{d\lambda_2}{dt} = \frac{1}{16\pi^2}\Big[24\lambda_2^2+2\lambda_3^2+2\lambda_3\lambda_4+\lambda_5^2+\lambda_4^2-3\lambda_2 \times(3g_2^2+\frac{1}{k_Y}g_1^2)+\frac{9}{8}g_2^4+\frac{3}{4k_Y}g_1^2g_2^2+\frac{3}{8k_Y^2}g_1^4\Big], \tag{15}$$

$$\frac{d\lambda_3}{dt} = \frac{1}{16\pi^2}\Big[4(\lambda_1+\lambda_2)(3\lambda_3+\lambda_4)+4\lambda_3^2+2\lambda_4^2 +2\lambda_5^2-3\lambda_3(3g_2^2+\frac{1}{k_Y}g_1^2)+6\lambda_3 g_t^2+\frac{9}{4}g_2^4 -\frac{3}{2k_Y}g_1^2g_2^2+\frac{3}{4k_Y^2}g_1^4\Big], \tag{16}$$

$$\frac{d\lambda_4}{dt} = \frac{1}{16\pi^2}\Big[4\lambda_4(\lambda_1+\lambda_2+2\lambda_3+\lambda_4)+8\lambda_5^2 -3\lambda_4(3g_2^2+\frac{1}{k_Y}g_1^2)+6\lambda_4 g_t^2+\frac{3}{k_Y}g_1^2g_2^2\Big], \tag{17}$$

$$\frac{d\lambda_5}{dt} = \frac{1}{16\pi^2}\Big[\lambda_5\Big((4\lambda_1+4\lambda_2+8\lambda_3+12\lambda_4 -3(3g_2^2+\frac{1}{k_Y}g_1^2)+6g_t^2\Big)\Big], \tag{18}$$

where $(b_1^{thdm}, b_2^{thdm}, b_3^{thdm}) = (7/k_Y, -3, -7)$; μ denotes the scale at which the coupling constants are defined, and $t = \log(\mu/\mu_0)$.

The form of the unification condition will depend on the particular way this scenario is carried out, which should be as generic as possible. However, in order to make predictions for the Higgs mass, we shall again consider two specific cases. Scenario I will be based on the linear relation:

$$g_1 = g_2 = k_H(i)\,\lambda_i(M_{GH}) \quad (i = 1,2,3,4,5)\,, \tag{19}$$

where the factors $k_H(i)$ are included in order to take into account possible unknown group theoretical or normalization factors. We shall also define Scenario II, which uses quadratic unification conditions, as follows:

$$g_1^2 = g_2^2 = k_H(i)\,\lambda_i(M_{GH}) \quad (i = 1,2,3,4,5)\,. \tag{20}$$

Now, we present first the results of the numerical analysis for the Higgs boson masses in the context of the two Higgs doublet model for $\tan\beta = 1$ and taking $m_{top} = 170.0$ GeV. In order to get an idea of the behavior of the masses of the Higgs bosons (h^0, H^0, A^0, $H^\pm$, we make the following *ad hoc* choice:

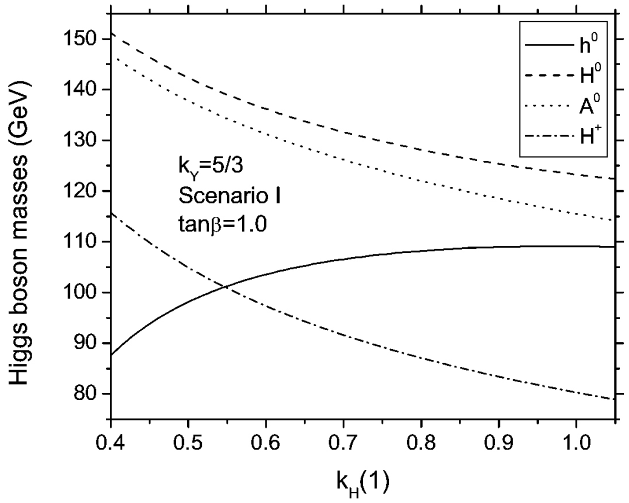

FIGURE 1. Prediction for the Higgs boson masses as a function of $k_H(1)$ in the context of the THDM with $k_Y = 5/3$, within the framework of Scenario I, taking $\tan\beta = 1$ and $m_{top} = 170.0$ GeV.

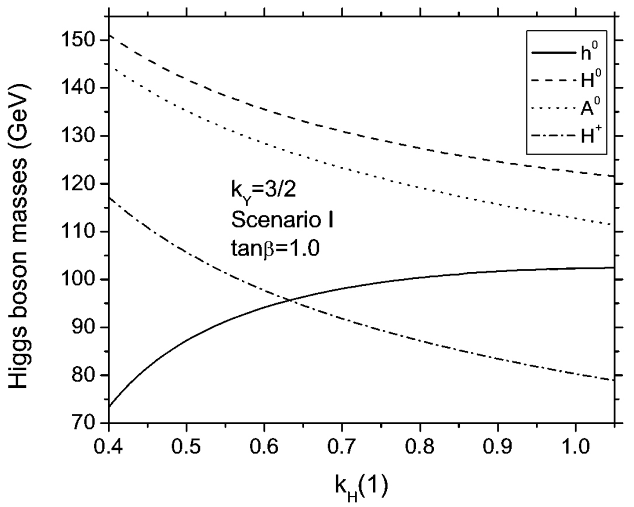

FIGURE 2. Prediction for the Higgs boson masses as a function of $k_H(1)$ in the context of the THDM with $k_Y = 3/2$, within the framework of Scenario I, taking $\tan\beta = 1$ and $m_{top} = 170.0$ GeV.

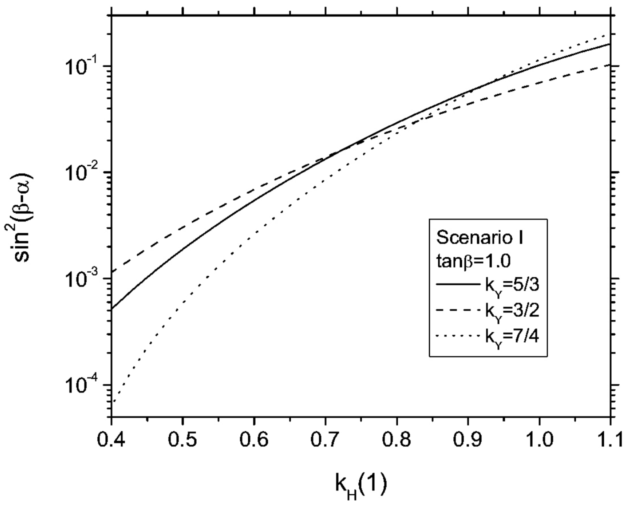

FIGURE 4. Prediction for the $sin^2(\beta - \alpha)$ as a function of $k_H(1)$ in the context of the THDM for $k_Y = 5/3, 3/2, 7/4$, within the framework of Scenario I, taking $\tan\beta = 1$ and $m_{top} = 170$ GeV.

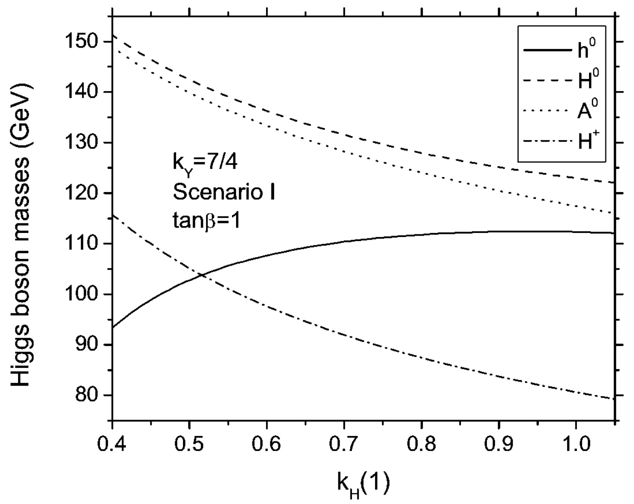

FIGURE 3. Prediction for the Higgs boson masses as a function of $k_H(1)$ in the context of the THDM with $k_Y = 7/4$, within the framework of Scenario I, taking $\tan\beta = 1$ and $m_{top} = 170.0$ GeV.

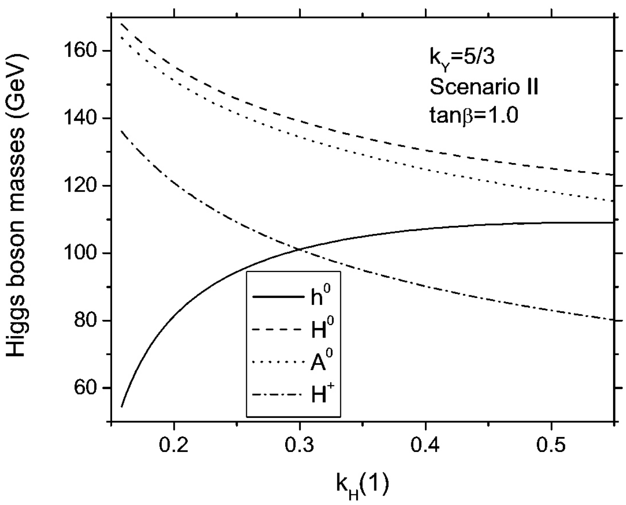

FIGURE 5. Prediction for the Higgs boson masses as a function of $k_H(1)$ in the context of the THDM with $k_Y = 5/3$, within the framework of Scenario II, taking $\tan\beta = 1$ and $m_{top} = 170.0$ GeV.

$$-k_H(5) = \frac{1}{2}k_H(4) = \frac{3}{2}k_H(3) = k_H(2) = k_H(1)\,, \quad (21)$$

for both Scenarios I and II.

Also presented in this section will be a complete discussion of the resulting couplings of the neutral CP even Higgs bosons with gauge vector boson pairs in the THDM, which are related to the corresponding SM couplings as follows [53]:

$$\frac{g_{h^0VV}}{g_{h^0_{sm}VV}} = \sin(\beta-\alpha)\,, \quad \frac{g_{H^0VV}}{g_{h^0_{sm}VV}} = \cos(\beta-\alpha)\,, \quad (22)$$

where $V = W$ or Z. For the moment suffice it to stress that the factor $\sin^2(\beta - \alpha)$ fixes the coupling of the lightest CP even Higgs boson with ZZ pairs, relative to the SM value, and therefore scales the result for the cross-section of the reaction $e^+e^- \to h^0 + Z$, which in turn allows us to determine the Higgs masses within LEP bounds. Hence, results for the Higgs boson masses and $\sin^2(\beta - \alpha)$ are given as a function of the parameter $k_H(1)$, looking for regions which are acceptable according to the available experimental data. In fact, first we shall make use of the experimental results reported in Table 14 of Ref.54 which, assuming SM decay rates, allow for a simple and direct check of our results for m_{h^0} and $\sin^2(\beta-\alpha)$. We would like to emphasize the following: even though the analysis of the EW-Higgs Unification within the THDM implies that the lightest neutral CP-even Higgs boson has a mass (~ 100 GeV) that is somewhat below the LEP bounds, 114.4 GeV [54, 55], it should be mentioned that this bound refers to the SM Higgs boson. The bound on the lightest Higgs boson of the THDM depends on the factor $\sin^2(\beta-\alpha)$, which could be less than 1, thus resulting in weaker Higgs boson mass bounds. Secondly, we shall use the experimental bound reported for $m_{H^\pm}$ in the literature [56]:

$$m_{H^\pm} > 79.3\,\text{GeV} \qquad \text{(95\% C.L.)}. \qquad (23)$$

Even though these two comparisons lead to a parameter space that is drastically reduced, from Figs.1-8 we observe that there are still two region allowed for Scenarios I and II, *viz*, $0.4 \lesssim k_H(1) \lesssim 1.1$ for Scenario I and $0.15 \lesssim k_H(1) \lesssim 0.55$ for Scenario II.

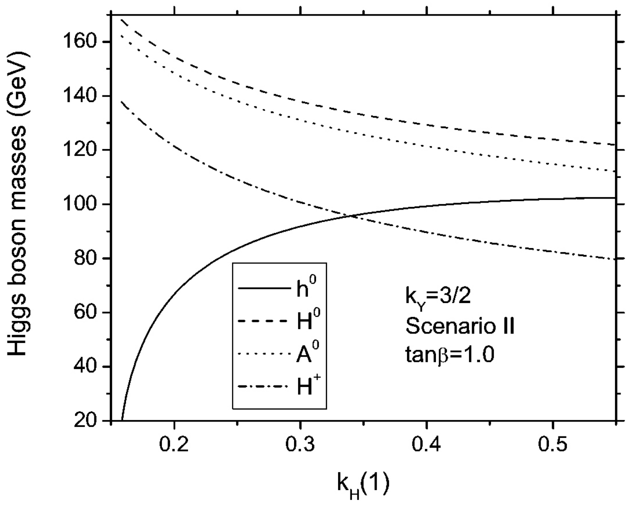

FIGURE 6. Prediction for the Higgs boson masses as a function of $k_H(1)$ in the context of the THDM with $k_Y = 3/2$, within the framework of Scenario II, taking $\tan\beta = 1$ and $m_{top} = 170.0$ GeV.

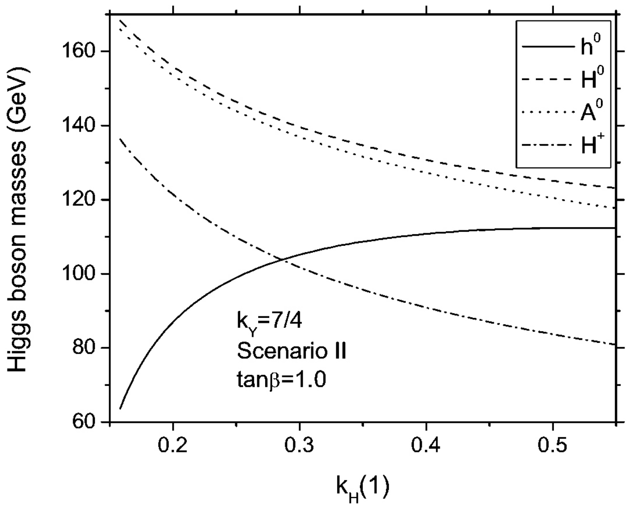

FIGURE 7. Prediction for the Higgs boson masses as a function of $k_H(1)$ in the context of the THDM with $k_Y = 7/4$, within the framework of Scenario II, taking $\tan\beta = 1$ and $m_{top} = 170.0$ GeV.

TABLE I. Prediction for the lightest neutral Higgs boson h^0 mass, $\sin^2(\beta-\alpha)$ and the charged Higgs boson $H^\pm$ mass as a function of $\tan\beta$ in the context of the THDM with $k_Y = 5/3$, within the framework of Scenario I with $k_H(1) = 1$, taking $m_{top} = 170.0$ GeV.

$\tan\beta$	m_{h^0} (GeV)	$\sin^2(\beta-\alpha)$	$m_{H^\pm}$ (GeV)
0.900	105.3	0.3674	77.26
0.925	106.5	0.3155	78.13
0.950	107.6	0.2540	78.92
0.975	108.5	0.1816	79.63
1.000	109.1	0.1020	80.28
1.025	109.3	0.0315	80.87
1.050	109.0	0.0001	81.41
1.075	108.2	0.0238	81.90
1.100	107.0	0.0828	82.36
1.125	105.6	0.1497	82.78
1.150	104.0	0.2107	83.17
1.175	102.4	0.2629	83.53
1.200	100.8	0.3068	83.86
1.225	99.24	0.3442	84.17
1.250	97.70	0.3763	84.46
1.275	96.21	0.4044	84.73
1.300	94.75	0.4293	84.99

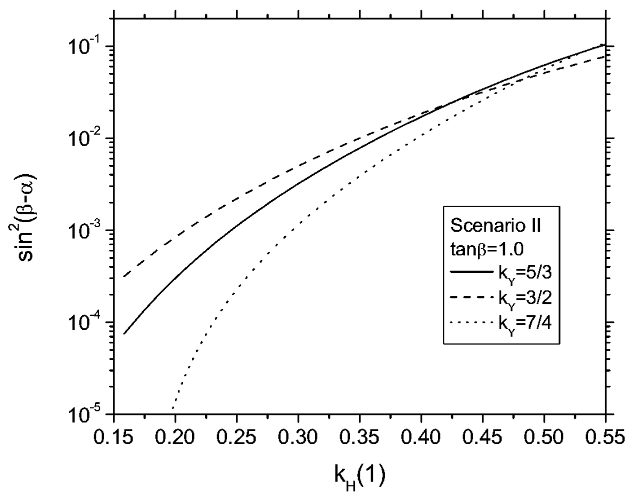

FIGURE 8. Prediction for the $\sin^2(\beta-\alpha)$ as a function of $k_H(1)$ in the context of the THDM for $k_Y = 5/3, 3/2, 7/4$, within the framework of Scenario II, taking $\tan\beta = 1$ and $m_{top} = 170$ GeV.

From now on, we shall restrict ourselves to continuing our numerical analysis only in Scenario I, assuming $k_H(1)$=1.

Now we present results in terms of the parameter $\tan\beta$ for the lightest neutral and charged Higgs boson masses (m_{h^0} and $m_{H^\pm}$) and the coupling of the lightest neutral CP-even Higgs boson with ZZ pairs, with respect to the correspond-

TABLE II. Prediction for the lightest neutral Higgs boson h^0 mass, $\sin^2(\beta-\alpha)$ and the charged Higgs boson $H^\pm$ mass as a function of $\tan\beta$ in the context of the THDM with $k_Y = 3/2$, within the framework of Scenario I with $k_H(1) = 1$, taking $m_{top} = 170.0$ GeV.

$\tan\beta$	m_{h^0} (GeV)	$\sin^2(\beta-\alpha)$	$m_{H^\pm}$ (GeV)
0.900	99.93	0.3063	76.91
0.925	100.8	0.2503	77.89
0.950	101.6	0.1898	78.78
0.975	102.1	0.1277	79.57
1.000	102.3	0.0697	80.30
1.025	102.3	0.0249	80.95
1.050	101.9	0.0020	81.55
1.075	101.2	0.0044	82.10
1.100	100.2	0.0279	82.61
1.125	99.06	0.0643	83.07
1.150	97.78	0.1060	83.50
1.175	96.43	0.1483	83.90
1.200	95.04	0.1886	84.27
1.225	93.65	0.2259	84.61
1.250	92.27	0.2601	84.93
1.275	90.90	0.2913	85.23
1.300	89.56	0.3198	85.51

TABLE III. Prediction for the lightest neutral Higgs boson h^0 mass, $\sin^2(\beta-\alpha)$ and the charged Higgs boson $H^\pm$ mass as a function of $\tan\beta$ in the context of the THDM with $k_Y = 7/4$, within the framework of Scenario I with $k_H(1) = 1$, taking $m_{top} = 170.0$ GeV.

$\tan\beta$	m_{h^0} (GeV)	$\sin^2(\beta-\alpha)$	$m_{H^\pm}$ (GeV)
0.900	107.9	0.4004	77.79
0.925	109.3	0.3518	78.61
0.950	110.5	0.2908	79.35
0.975	111.6	0.2121	80.02
1.000	112.3	0.1144	80.63
1.025	112.6	0.0229	81.18
1.050	112.2	0.0064	81.69
1.075	111.1	0.0777	82.16
1.100	109.5	0.1693	82.58
1.125	107.9	0.2460	82.98
1.150	106.1	0.3054	83.34
1.175	104.4	0.3519	83.68
1.200	102.7	0.3893	84.00
1.225	101.1	0.4204	84.29
1.250	99.50	0.4470	84.56
1.275	97.97	0.4701	84.82
1.300	96.48	0.4907	85.06

TABLE IV. Predicted Higgs mass spectrum as a function of $\tan\beta$ in the context of the THDM with $k_Y = 5/3$, within the framework of Scenario I with $k_H(1) = 1$, taking $m_{top} = 170.0$ GeV.

$\tan\beta$	m_{h^0} (GeV)	m_{H^0} (GeV)	m_{A^0} (GeV)	$m_{H^\pm}$ (GeV)
0.975	108.5	125.4	114.7	79.63
1.000	109.1	123.2	115.5	80.28
1.025	109.3	121.6	116.2	80.87
1.050	109.0	120.6	116.9	81.41
1.075	108.2	120.2	117.6	81.90
1.100	107.0	120.3	118.2	82.36
1.125	105.6	120.7	118.7	82.78
1.150	104.0	121.3	119.2	83.17

TABLE V. Predicted Higgs mass spectrum as a function of $\tan\beta$ in the context of the THDM with $k_Y = 3/2$, within the framework of Scenario I with $k_H(1) = 1$, taking $m_{top} = 170.0$ GeV.

$\tan\beta$	m_{h^0} (GeV)	m_{H^0} (GeV)	m_{A^0} (GeV)	$m_{H^\pm}$ (GeV)
0.975	102.1	124.4	111.8	79.57
1.000	102.3	122.4	112.8	80.30
1.025	102.3	120.9	113.6	80.95
1.050	101.9	119.9	114.3	81.55
1.075	101.2	119.3	115.0	82.10
1.100	100.2	119.1	115.6	82.61
1.125	99.06	119.1	116.2	83.07
1.150	97.78	119.4	116.8	83.50
1.175	96.43	119.7	117.3	83.90
1.200	95.04	120.2	117.7	84.27

TABLE VI. Predicted Higgs mass spectrum as a function of $\tan\beta$ in the context of the THDM with $k_Y = 7/4$, within the framework of Scenario I with $k_H(1) = 1$, taking $m_{top} = 170.0$ GeV.

$\tan\beta$	m_{h^0} (GeV)	m_{H^0} (GeV)	m_{A^0} (GeV)	$m_{H^\pm}$ (GeV)
0.950	110.5	127.9	115.7	79.35
0.975	111.6	125.2	116.6	80.02
1.000	112.3	123.0	117.4	80.63
1.025	112.6	121.3	118.1	81.18
1.050	112.2	120.6	118.8	81.69
1.075	111.1	120.6	119.4	82.16
1.100	109.5	121.1	119.9	82.58
1.125	107.9	121.8	120.5	82.98

ing SM value ($|g_{h^0ZZ}/g_{h^0_{sm}ZZ}|^2 = \sin^2(\beta-\alpha)$), looking again for regions which are acceptable according to the experimental data currently available, for $k_Y = 5/3$ (Table I), $k_Y = 3/2$ (Table II) and $k_Y = 7/4$ (Table III). As can be seen from Tables I-III, there are values of $\tan\beta$ for which the ratio $|g_{h^0ZZ}/g_{h^0_{sm}ZZ}|^2$ is substantially reduced, which therefore will make it possible to overcome the constraints imposed by the LEP search for neutral Higgs bosons. Lastly, taking into account the bound on $m_{H^\pm}$ given in expression (23), we conclude that the following regions for $\tan\beta$ are experimentally permitted:

$$0.975 \leq \tan\beta \leq 1.15 \qquad \text{for } k_Y = 5/3\,, \tag{24}$$

$$0.975 \leq \tan\beta \leq 1.20 \qquad \text{for } k_Y = 3/2\,, \tag{25}$$

$$0.95 \leq \tan\beta \leq 1.125 \qquad \text{for } k_Y = 7/4\,. \tag{26}$$

Now, using the ranges given in (24),(25), and (26), we plot in Figs. 9, 10, and 11, the results for the Higgs boson masses as a function of $\tan\beta$ for $k_Y = 5/3$, $k_Y = 3/2$ and $k_Y = 7/4$, respectively. We also present the same results in Tables IV, V and VI.

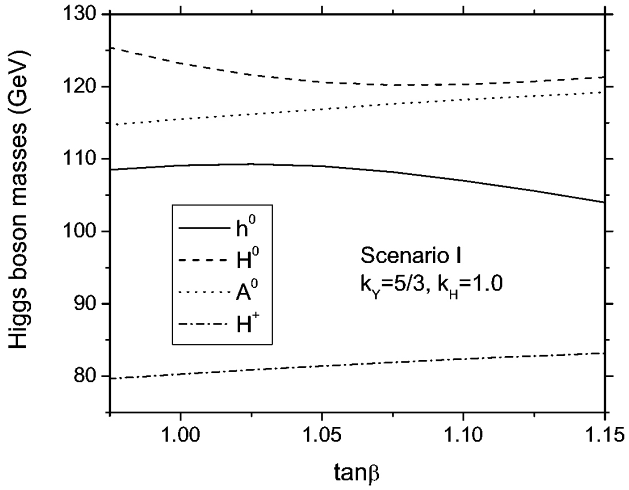

FIGURE 9. Prediction for the Higgs boson masses as a function of $\tan\beta$ in the context of the THDM with $k_Y = 5/3$, within the framework of Scenario I, taking $k_H(1) = 1$ and $m_{top} = 170.0$ GeV.

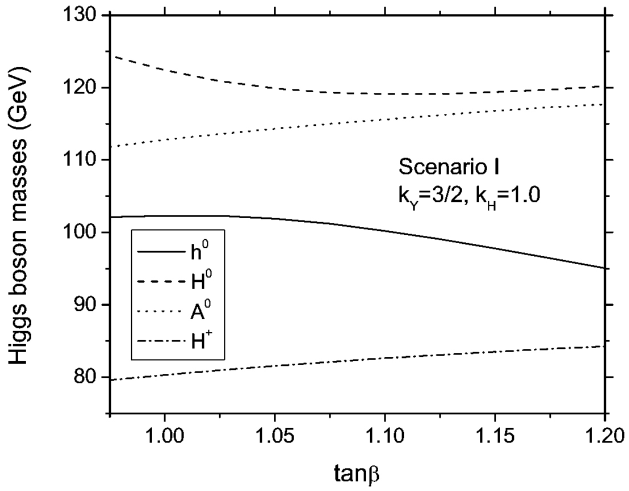

FIGURE 10. Prediction for the Higgs boson masses as a function of $\tan\beta$ in the context of the THDM with $k_Y = 3/2$, within the framework of Scenario I, taking $k_H(1) = 1$ and $m_{top} = 170.0$ GeV.

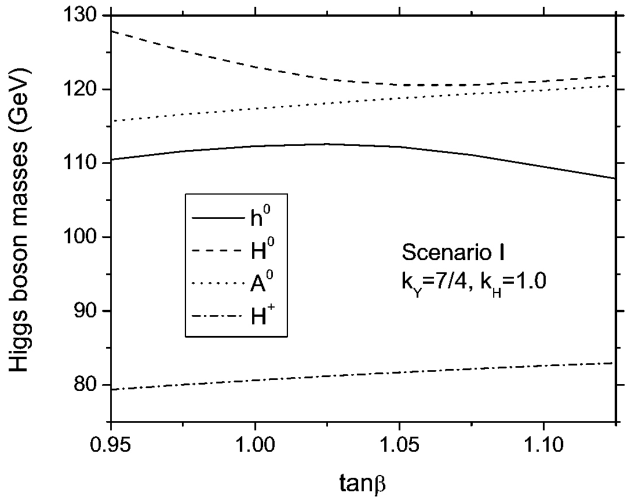

FIGURE 11. Prediction for the Higgs boson masses as a function of $\tan\beta$ in the context of the THDM with $k_Y = 7/4$, within the framework of Scenario I, taking $k_H(1) = 1$ and $m_{top} = 170.0$ GeV.

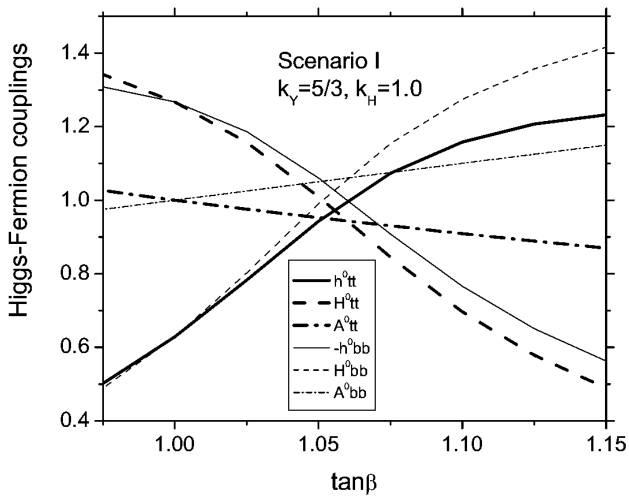

FIGURE 12. Prediction for the Higgs-fermion couplings as a function of $\tan\beta$ in the context of the THDM with $k_Y = 5/3$, within the framework of Scenario I, taking $k_H(1) = 1$ and $m_{top} = 170.0$ GeV. The curves correspond to: 1) $h^0 t\bar{t}$, 2) $H^0 t\bar{t}$, 3) $A^0 t\bar{t}$, 4) $-h^0 b\bar{b}$, 5) $H^0 b\bar{b}$ and 6) $A^0 b\bar{b}$.

Let us briefly discuss the results of the numerical analysis of the Higgs mass spectrum. For $k_Y = 5/3$, we find that $M_{GH} = 1.3 \times 10^{13}$ GeV, and by taking $\tan\beta = 1$, we obtain the following Higgs mass spectrum: $m_{h^0} = 109.1$ GeV, m_{H^0}=123.2 GeV, m_{A^0}=115.5 GeV, and $m_{H^\pm}$=80.3 GeV. In turn, for $k_Y = 3/2$, we find that $M_{GH} = 5.9 \times 10^{14}$ GeV, somewhat higher than in the previous case, but for which one still obtains a mass gap between M_{GH} and a possible M_{GUT}. One finds similar values for m_{H^0}, m_{A^0}, and $m_{H^\pm}$ and slightly lower values for m_{h^0}; for instance, for $\tan\beta = 1$, we get $m_{h^0} = 102.3$ GeV, $m_{H^0} = 122.4$ GeV, $m_{A^0} = 112.8$ GeV, and $m_{H^\pm} = 80.3$ GeV. On the other hand, for $k_Y = 7/4$ we find that $M_{GH} = 2.2 \times 10^{12}$ GeV, which is lower than in the previous cases, and has an even larger mass gap between M_{GH} and a possible M_{GUT}. We obtain similar values for m_{H^0}, m_{A^0}, and $m_{H^\pm}$ and slightly higher values for m_{h^0}. For instance, for $\tan\beta = 1$, one gets $m_{h^0} = 112.3$ GeV, $m_{H^0} = 123.0$ GeV, $m_{A^0} = 117.4$ GeV, and $m_{H^\pm} = 80.6$ GeV.

The numerical results presented in Figs. 9 to 11 (Tables IV-VI) lead us to conclude that the Higgs mass spectrum is almost independent of the value of k_Y. However, the unification scale M_{GH} depends strongly on the value of k_Y, going from 2.2×10^{12} GeV up to 5.9×10^{14} GeV for $7/4 > k_Y > 3/2$ (for $k_Y = 5/3$, we obtain $M_{GH} = 1.3 \times 10^{13}$GeV).

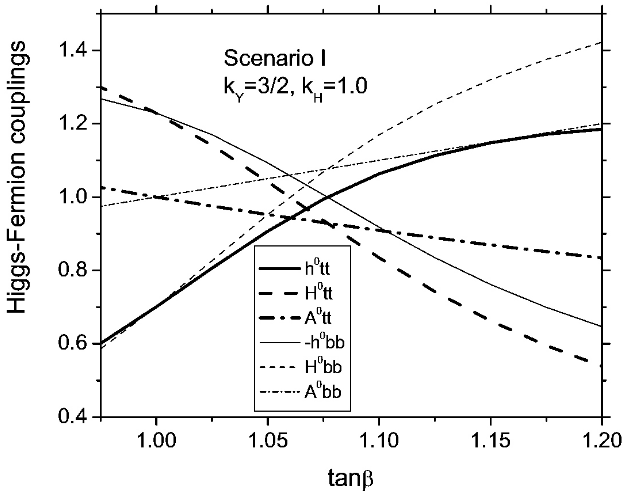

FIGURE 13. Prediction for the Higgs-fermion couplings as a function of $\tan\beta$ in the context of the THDM with $k_Y = 3/2$, within the framework of Scenario I, taking $k_H(1) = 1$ and $m_{top} = 170.0$ GeV. The curves correspond to: 1) $h^0 t\bar{t}$, 2) $H^0 t\bar{t}$, 3) $A^0 t\bar{t}$, 4) $-h^0 b\bar{b}$, 5) $H^0 b\bar{b}$ and 6) $A^0 b\bar{b}$.

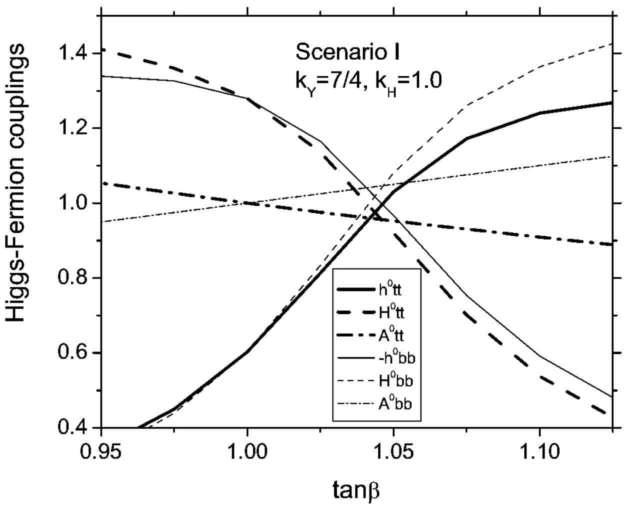

FIGURE 14. Prediction for the Higgs-fermion couplings as a function of $\tan\beta$ in the context of the THDM with $k_Y = 7/4$, within the framework of Scenario I, taking $k_H(1) = 1$ and $m_{top} = 170.0$ GeV. The curves correspond to: 1) $h^0 t\bar{t}$, 2) $H^0 t\bar{t}$, 3) $A^0 t\bar{t}$, 4) $-h^0 b\bar{b}$, 5) $H^0 b\bar{b}$ and 6) $A^0 b\bar{b}$.

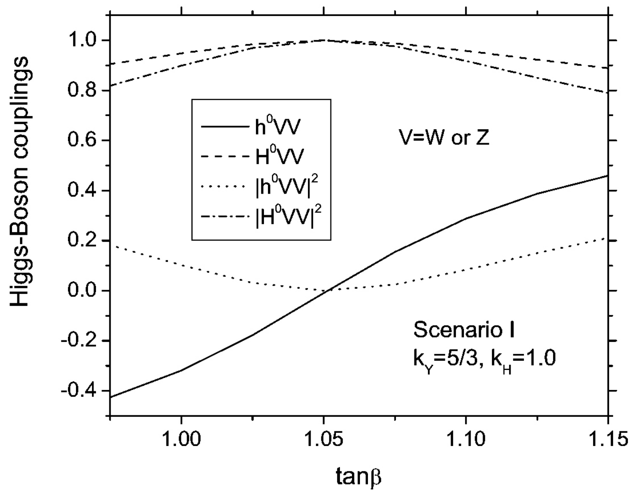

FIGURE 15. Prediction for the Higgs-boson couplings as a function of $\tan\beta$ in the context of the THDM with $k_Y = 5/3$, within the framework of Scenario I, taking $k_H(1)$=1 and m_{top}=170.0 GeV. The curves correspond to: 1) $h^0VV = g_{h^0VV}/g_{h^0_{sm}VV}$, 2) $H^0VV = g_{H^0VV}/g_{h^0_{sm}VV}$, 3) $|h^0VV|^2 = |g_{h^0VV}/g_{h^0_{sm}VV}|^2$ and 4) $|H^0VV|^2 = |g_{H^0VV}/g_{h^0_{sm}VV}|^2$, where $V = W$ or Z.

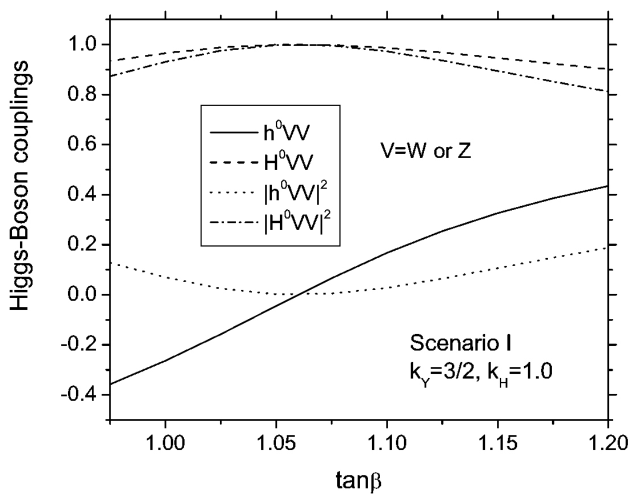

FIGURE 16. Prediction for the Higgs-boson couplings as a function of $\tan\beta$ in the context of the THDM with k_Y=3/2, within the framework of Scenario I, taking $k_H(1)$=1 and m_{top}=170.0 GeV. The curves correspond to: 1) $h^0VV = g_{h^0VV}/g_{h^0_{sm}VV}$, 2) $H^0VV = g_{H^0VV}/g_{h^0_{sm}VV}$, 3) $|h^0VV|^2 = |g_{h^0VV}/g_{h^0_{sm}VV}|^2$ and 4) $|H^0VV|^2 = |g_{H^0VV}/g_{h^0_{sm}VV}|^2$, where $V = W$ or Z.

At this point, we want to recall the relation between the Higgs-fermion couplings, which can be expressed relative to the SM value and is given by [53]:

$$
\begin{aligned}
&H^0 t\bar{t} : \frac{\sin\alpha}{\sin\beta}\,, && H^0 b\bar{b} : \frac{\cos\alpha}{\cos\beta}\,,\\
&h^0 t\bar{t} : \frac{\cos\alpha}{\sin\beta}\,, && h^0 b\bar{b} : \frac{-\sin\alpha}{\cos\beta}\,,\\
&A^0 t\bar{t} : \cot\beta\,, && A^0 b\bar{b} : \tan\beta\,.
\end{aligned}
\tag{27}
$$

Now, we use the ranges given in (24),(25), and (26), and in Figs. 12, 13, and 14 we present the results for the fermion

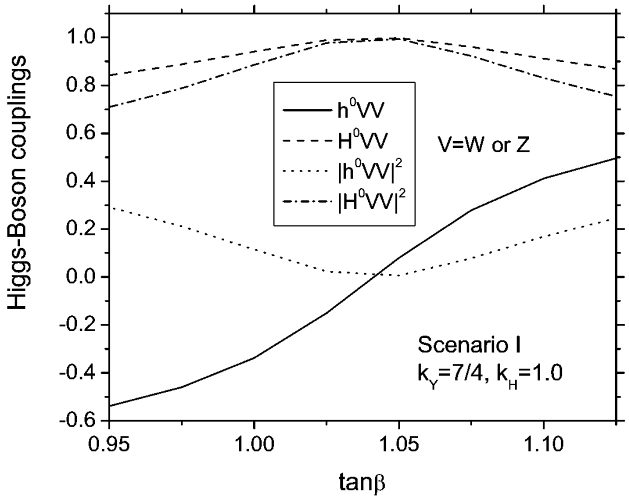

FIGURE 17. Prediction for the Higgs-boson couplings as a function of $\tan\beta$ in the context of the THDM with $k_Y = 7/4$, within the framework of Scenario I, taking $k_H(1)$=1 and m_{top}=170.0 GeV. The curves correspond to: 1) $h^0VV = g_{h^0VV}/g_{h^0_{sm}VV}$, 2) $H^0VV = g_{H^0VV}/g_{h^0_{sm}VV}$, 3) $|h^0VV|^2 = |g_{h^0VV}/g_{h^0_{sm}VV}|^2$ and 4) $|H^0VV|^2 = |g_{H^0VV}/g_{h^0_{sm}VV}|^2$, where $V = W$ or Z.

couplings as a function of $\tan\beta$ for $k_Y = 5/3$, $k_Y = 3/2$ and $k_Y = 7/4$, respectively.

Finally, making use of the ranges given in (24),(25), and (26), in Figs. 15, 16, and 17 we present the results for the Higgs-boson couplings as a function of $\tan\beta$ for $k_Y = 5/3$, $k_Y = 3/2$ and $k_Y = 7/4$, respectively.

From our results shown in Figs. 9-17 and Tables IV-VI, we also conclude that the Higgs mass spectrum does not depend strongly on the value of $\tan\beta$. On the other hand, the fermion couplings and the boson couplings depend strongly on the value of $\tan\beta$.

We find that for $\tan\beta = 1$, the coupling of h^0 to up-type (d-type) quarks is suppressed (enhanced), which will have important phenomenological consequences [57, 58]: for instance, it will suppress the production of h^0 at hadron colliders through gluon fusion, while the associated production with $b\bar{b}$ quarks will be enhanced. The couplings of H^0 show the opposite behavior, namely the couplings with d-type (up-type) quarks are suppressed (enhanced). This behavior changes as $\tan\beta$ takes on higher values, and is already reversed for $\tan\beta = 1.1$. Similar results are obtained for other normalizations. We end this section by saying that similar results are obtained in the regions that are experimentally allowed for Scenario II.

4. Comments and conclusions

In this paper we have obtained the Higgs mass spectrum, the Higgs-fermion couplings and the Higgs-boson couplings of the THDM in a framework in which it is possible to unify the Higgs self-coupling with the gauge interactions.

The hypercharge normalization plays an important role in identifying the EW-Higgs unification scale. For the canonical value $k_Y = 5/3$, we get $M_{GH} = 1.3 \times 10^{13}$ GeV. For lower values such as $k_Y = 3/2$, the scale is $M_{GH} = 5.9 \times 10^{14}$ GeV, which is closer to the GUT scale ($\approx 10^{16}$ GeV); but for higher values, such as $k_Y = 7/4$, which gives $M_{GH} = 2.2 \times 10^{12}$ GeV, the EW-Higgs unification becomes clearly distinctive.

The present approach still lacks a solution to the hierarchy problem; at the moment we must subscribe to the argument that fundamental physics could accept some fine-tuning [59]. Another option would be to consider one of the simplest early attempts to solve the problem of quadratic divergences in the SM, namely through an accidental cancellation [60]. In fact, this kind of cancellation implies a relationship between the quartic Higgs coupling and the Yukawa and gauge constants that has the form: $\lambda = y_t^2 - (1/8)[3g^2 + g'^2]$. Unfortunately, this relation implies a Higgs mass $m_\phi = 316$ GeV, and that seems to be already excluded. Nevertheless, this relation could work if one takes into account the running of the coupling and Yukawa constants. This particular point will be the subject of future studies.

Acknowledgments

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