An alternative deduction of relativistic transformations in thermodynamics

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In this article, we propose an alternative deduction to the Ares de Parga *et al.* proposal for the relativistic transformation laws in thermodynamics [*J. of Phys. A: Math and Gen.* **38** (2005) 2821]. A generalization of the so-called thermal work is proposed. The covariance of the theory is discussed.

Keywords: Relativistic transformations; thermodynamics; covariance.

En este artículo, se propone una deducción alterna de la propuesta de Ares de Parga *et al.* sobre las leyes de las transformaciones relativistas en termodinámica [*J. of Phys. A: Math and Gen.* **38** (2005) 2821]. Una generalización del llamado trabajo térmico es propuesto. La covariancia de la teoría se discute.

Descriptores: Transformaciones relativistas; termodinámica; covariancia.

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1. Introduction

Recently, Ares de Parga *et al.* [1] [AA] proposed a new set of relativistic transformation laws for thermodynamics quantities. They arrived at the new proposal by using what they have called the "renormalization of thermodynamics" which consists in subtracting a bulk energy from the Einstein-Planck energy. Indeed, we can summarize that for different proposals of thermodynamic relativistic transformations, work transforms as $dW = \gamma^a dW_o + \varphi d\zeta$ where a represents the Balescu parameter, which is different for each proposal ($a = -1$ for the Planck-Einstein proposal [2–4], $a = 1$ for the Ott proposal [4, 5] and $a = 0$ for one of the different proposals made by Landsberg [4, 6]), $\gamma = (1 - [u^2/c^2])^{-\frac{1}{2}}$ where u is the velocity between both the rest and the moving reference frames, the subscript $_o$ denotes the quantities in the rest frame, and $\varphi d\zeta$ differs for each proposal. Indeed, nowadays the impossibility of a relativistic transformation law is still supported by some authors [7–9]. However, by using the "g" function of finite time thermodynamics [10], AA proved that the efficiency is an invariant no matter what the value of the Balescu parameter may be. Consequently, they arrived at the conclusion that the differential $\varphi d\zeta$ needs to be an exact differential [1]. On the other hand, they concluded that only the value $a = -1$ is consistent with the free adiabatic expansion of an ideal gas. Nevertheless, they found that the Planck-Einstein proposal does not maintain the invariant form of thermodynamics. By using the exact differential of the Planck-Einstein transformation the work, they defined a bulk energy which made it possible to obtain an invariant relativistic thermodynamics with changes of reference frames. In the present article, we find a different way to deduce the so-called renormalized thermodynamics [1] without using any concept from finite time thermodynamics, such as the "g" function, and therefore independently of the second law; that is, we shall not use the "g" function, which is defined based on the second law. The second section will be devoted to showing the non-invariance of the Planck-Einstein proposal and just by using the first law we shall obtain the energy transformation law. In Sec. 3, the connection between Planck-Einstein energy and renormalized or thermal energy will be described. Section 4 will generalize renormalized or thermal work. The covariance of the theory will be explain in Sec. 5. In the concluding remarks, Sec. 6, a set of related work to be done will be proposed.

Before continuing, let us present a summary of renormalized thermodynamics. The new proposal of relativistic transformations of thermodynamic quantities, made by Ares de Parga *et al.* [1], is the Table I. Where V, P, S, T, Q and F represent volume, pressure, entropy, heat and Helmholtz free energy respectively. ξ is the so-called thermal energy of the system and is defined as:

$$\xi = E - \gamma(E_o + PV_o)\frac{u^2}{c^2}, \tag{1}$$

where E represents Einstein-Planck energy. We can notice that the transformations of the different thermodynamic quantities coincide with the Einstein-Planck theory except for energy. The purpose of the next section consists in explaining this difference and its consequences.

2. Transformation for Helmholtz free energy

Starting from the definition of Helmholtz free energy in the rest frame,

TABLE I.

V	P	S	T	dQ	ξ	F
$\gamma^{-1}V_o$	P_o	S_o	$\gamma^{-1}T_o$	$\gamma^{-1}dQ_o$	$\gamma^{-1}\xi_o = E_o\gamma^{-1}$	$\gamma^{-1}F_o$

$$F_o = E_o - T_o S_o \tag{2}$$

and supposing that the invariance form of thermodynamics is preserved in a moving frame, we must have:

$$F = E - TS. \tag{3}$$

By differentiating both sides of the last equation, we obtain

$$dF = dE - d(TS). \tag{4}$$

By supposing that the invariant form of thermodynamics is preserved, at first glance the first law and work may be written as

$$dE = dQ - dW \text{ and } dW = PdV. \tag{5}$$

Then, Eq. (5) can be written as

$$dE = dQ - PdV. \tag{6}$$

Therefore, Eq. (4) is now expressed as:

$$dF = dQ - PdV - d(TS). \tag{7}$$

Let us consider that the Einstein-Planck transformation laws of volume, temperature and heat are valid (see Table I); then we arrive at:

$$dF = \gamma^{-1}\left[dQ_o - PdV_o - d(T_o S_o)\right], \tag{8}$$

where $d(\gamma^{-1}f) = \gamma^{-1}df$, since the velocity of the system is constant. Eq. (8) can be expressed as:

$$dF = \gamma^{-1}\left[dQ_o - dW_o - d(T_o S_o)\right]. \tag{9}$$

By using the first law of thermodynamics in the rest frame, we arrive at:

$$dF = \gamma^{-1}\left[d(E_o - T_o S)\right]. \tag{10}$$

The term inside brackets represents the Helmholtz free energy in the rest frame. Then

$$dF = \gamma^{-1} dF_o, \tag{11}$$

or equivalently,

$$F = \gamma^{-1} F_o. \tag{12}$$

This represents the same transformation law for the Helmholtz free energy of the Einstein-Planck theory [2,3,11] (see Table I). Now, let us analyze the energy transformation law by using Eqs. (3) and (12). We know that if the invariance of form in thermodynamics is satisfied, we must have

$$E = F + TS. \tag{13}$$

By using Table I, except for the energy transformation, it is easy to see that

$$E = \gamma^{-1} F_o + \gamma^{-1} T_o S_o\,, \tag{14}$$

that is,

$$E = \gamma^{-1}(F_o + T_o S_o). \tag{15}$$

The quantity in brackets represents the energy of the system in the proper frame.

$$E = \gamma^{-1} E_o. \tag{16}$$

We conclude that in order to conserve the invariance of form in thermodynamics, the energy must transform as Eq. (16). Since E does not coincide with the Einstein-Planck energy transformation [2, 3, 11], we can conclude that the regular concept of energy E and the transformed work $dW = PdV$ must be redefined in order to keep the invariance of the form in thermodynamics. We can also note that to obtain Eq. (16), we only use the first law of thermodynamics and we have shown that it is not necessary to use the second law of thermodynamics to obtain the transformation law of the redefined energy. Returning to the new definition of energy, we must notice that Einstein-Planck [2, 3, 11] calculated the transformation energy based on a regular energy which is connected with the momentum of the system $\overrightarrow{G} = \gamma\left(E_o + PV_o\right)\frac{\overrightarrow{u}}{c^2}$ and is a component of the stress tensor. As a consequence of this, they obtained the following transformation law for energy and work:

$$E = \gamma(E_o + P_o V_o \frac{u^2}{c^2}) \text{ and } W = PdV - \gamma\frac{u^2}{c^2}(E_o + PV_o)$$
$$= PdV - \overrightarrow{u}.d\overrightarrow{G} \tag{17}$$

This last result is not compatible with invariance form thermodynamics. Therefore, as we note above, the energy and work concepts need to be changed in relativistic thermodynamics.

3. Renormalized or thermal energy

As we noted above, Ares de Parga *et al.* [1] defined a thermal energy that has been deduced by subtracting the so-called bulk energy from the regular energy and they obtained a quantity that transforms as Eq. (16). The new expression was called the thermal energy and based on it they obtained the invariant form in thermodynamics. The question now is to deduce this expression for energy without using the "g" function, that is, just by using Eqs. (16) and (17). We must obtain a thermodynamic quantity, which we shall call "thermal energy", such that:

$$\xi = E - A = \gamma^{-1} E_o. \tag{18}$$

A represents the bulk energy used by Ares de Parga *et al.* [1] but in this new deduction, we do not yet know what its value is. Therefore,

$$\gamma(E_o + P_o V_o \frac{u^2}{c^2}) - A = \gamma^{-1} E_o. \tag{19}$$

Simple algebra will give:

$$A = \gamma(E_o + PV_o)\frac{u^2}{c^2} = \overrightarrow{u} \cdot \overrightarrow{G}. \tag{20}$$

This quantity A coincides with the bulk energy of the proposal made by Ares de Parga *et al.* [1]. By using this result we can construct any of the thermal quantities obtained by Ares de Parga *et al.* [1]. On the other hand, in order to maintain the invariance of form in relativistic thermodynamics, it is necessary to add the bulk energy to the regular work; that is,

$$d\Omega = dW + \overrightarrow{u}.d\overrightarrow{G}, \tag{21}$$

$d\Omega$ being the thermal energy. Therefore, the first law may be written as

$$d\xi = dQ - d\Omega. \tag{22}$$

For the particular case where $dW_o{=}PdV_o$, we have $d\Omega = PdV$.

We can mention that thermodynamical quantities are divided into two groups: the first is made up of temperature and some extensive qualitiesas heat, volume, Helmholtz free energy and enthalpy, which transform as:

$$\Gamma = \gamma^{-1}\Gamma_o; \tag{23}$$

and the second comprises entropy and specific variables such as thermal energy, density and pressure, which are invariant. Other examples of invariant thermodynamical quantities are represented by the Massieu functions,

$$-\frac{F}{T} = -\frac{F_o}{T_o} \text{ and } -\frac{G}{T} = -\frac{G_o}{T_o} \tag{24}$$

where F and G represent the free Helmholtz and Gibbs energies.

4. Generalized thermal quantities

As we mentioned above, there are two kinds of thermodynamic quantities which transform as Einstein-Planck temperature or as invariants. Nevertheless, we should note that to obtain thermal work, Ares de Parga *et al.* [1] consider that it is necessary to add bulk energy to work. Indeed, the objective is to express the first law in the moving frame in the same way as in the rest frame; that is:

$$dW_o = dQ_o - dE_o. \tag{25}$$

In the moving frame, if we wish to keep the regular definition of work,

$$dW = dQ - dE = \gamma^{-1}dQ_o - \gamma(E_o + P_oV_o\frac{u^2}{c^2}). \tag{26}$$

We will not have a regular transformation of work unless we define a thermal work where we add bulk energy to work, as Ares de Parga *et al.* [1] did. That is:

$$d\Omega = dW + \overrightarrow{u}\cdot d\overrightarrow{G} = dQ - (dE - \overrightarrow{u}\cdot d\overrightarrow{G}) = dQ - d\xi \tag{27}$$

which correspond to subtracting bulk energy from the regular energy; that is, by using the thermal energy to define thermal work. For the particular case of $dW_o = PdV_o$, since $dW = \gamma^{-1}dW_o - \overrightarrow{u} \cdot d\overrightarrow{G}$ [1], we know that $dW \neq PdV$ and $d\Omega = PdV = \gamma^{-1}PdV_o = \gamma^{-1}dW_o$. The reason is that the energy required to bring the system in a moving frame from zero energy to an energy E_o is already considered in the thermal energy $\xi = E - \overrightarrow{u} \cdot \overrightarrow{G}$. This reason will be analyzed in the next section, when we discuss the covariance of the theory. All the different representations of work must follow this criterion even if they are not described by the typical PdV. So let us consider the first law for any kind of work:

$$dE_o = dQ_o - \sum_i X_{oi}dY_{oi}. \tag{28}$$

In order to preserve the invariance of form, work must be expressed as $d\Omega = \sum_i X_i dY_i$. We can construct generalized thermal work just by using the first law.

$$\sum_i X_i dY_i = d\Omega = dQ - d\xi = \gamma^{-1}(dQ_o - dE_o)$$
$$= \gamma^{-1}\sum_i X_{oi}dY_{oi}. \tag{29}$$

As an example we can consider chemical work, with $X = \mu$ representing the chemical potential and $Y = N$ being the particle number. Therefore, since the number of particles is invariant,

$$d\Omega{=}\gamma^{-1}dW_o \Rightarrow \mu dN{=}\gamma^{-1}\mu_o dN_o{\Rightarrow}\mu{=}\gamma^{-1}\mu_o. \tag{30}$$

In general, in order to satisfy the first law of thermodynamics, work must satisfy the following property:

$$d\Omega = \sum_i X_i dY_i = \sum_i \gamma^{-1}X_{oi}dY_{oi}$$
$$\Rightarrow (X_i = \gamma^{-1}X_{oi} \quad \text{and} \quad Y_i = Y_{oi})$$
$$\text{or}(X_i = X_{oi} \quad \text{and} \quad Y_i = \gamma^{-1}Y_{oi}) \tag{31}$$

For the other quantities as enthalpy, the definition must be given in a similar way. That is, in order to keep the invariance of form, we need to define thermal enthalpy as:

$$d\Psi{=}dH{-}\overrightarrow{u}\cdot\overrightarrow{dG}{=}dE{-}\overrightarrow{u}\cdot\overrightarrow{dG}{+}PdV{=}d\xi{+}PdV. \tag{32}$$

Therefore, it is natural that

$$\Psi = \gamma^{-1}H_o. \tag{33}$$

5. Covariance

With each relativistic theory, a covariant model must be included. For the Einstein-Planck proposal, Staruszkiewicz [12] has developed a covariant theory which consists in defining some 4-vectors with the aid of the velocity of the system ($H^{\mu} = (\gamma H_o, uH)$). Recently, Nakamura [13] developed a covariant theory by using the van Kampen covariance [14]. However, they assumed that the Einstein-Planck proposal conserves the form invariance of thermodynamics, which has been proven to fail, as we saw in Sec. 2 in the present paper and has already been shown by Ares de Parga *et al.* [1]. It should be noted that the interesting covariant theory developed by van Kampen [14] was based on the third law of thermodynamics, leading to a scalar theory of entropy, temperature and heat, which is just the Rohrlich Light proposal [15]. Nevertheless, in this proposal it is assumed that $V = \gamma V_o = \gamma V_o$ which is not physically acceptable.

In our case, following regular covariance, we can define the 4-thermodynamic vector from thermodynamic quantities; that is,

$$\Gamma^{\mu} = \begin{pmatrix} \gamma^2 \Gamma \\ \gamma^2 \overrightarrow{\beta} \Gamma \end{pmatrix} = \begin{pmatrix} \gamma \Gamma_o \\ \gamma \overrightarrow{\beta} \Gamma_o \end{pmatrix} = \Gamma_o v^{\mu}$$

$$\text{with } g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\Rightarrow \Gamma^{\mu}\Gamma_{\mu} = \Gamma^{\mu} g_{\mu\nu} \Gamma^{\nu} = \left(\gamma^2 - \gamma^2\beta^2\right)\Gamma_o^2 = \Gamma_o^2. \quad (34)$$

Then the first law of thermodynamics may be expressed as

$$dQ^{\mu} = d\xi^{\mu} + d\Omega^{\mu}. \quad (35)$$

It should be noted that some confusion can arise from the classical explanation. Indeed, Tolman considers the stress tensor in the same way as Laue [16] does. In this scheme, the pressure P is an invariant, but since it just represents a component of the tensor in a rest frame, it can be thought not to be an invariant. However, it is easy to show that the pressure is an invariant by just showing that the force over the area is an invariant, as Tolman [11] demostrated. Nevertheless, the correct way to deal with thermodynamical quantities consists in using the regular stress tensor described by Weinberg [17] or Landau and Lifshitz [18] (The tensor used in § 85 Eq. 85.7 p 217 in Tolman's book corresponds to the Landau and Lifshitz definition of stress tensor). That is (the difference with Weinberg's book is due to the signature):

$$T^{\mu\nu} = (P + e)u^{\mu}u^{v} - Pg^{\mu\nu}. \quad (36)$$

the flux of energy is represented by

$$T^{0i} = (P + e)\gamma^2 v^i = \frac{(P + e)v^i}{1 - \beta^2} = s^i. \quad (37)$$

Therefore, the total momentum can be expressed as

$$\overrightarrow{G} = \overrightarrow{s}\frac{V}{c^2} = \frac{P + e}{1 - \beta^2}\frac{V}{c^2}\overrightarrow{u} = \gamma\left(E_o + PV_o\right)\frac{\overrightarrow{u}}{c^2}. \quad (38)$$

The energy dw required to bring the system from an energy level E_o to an energy $E_o + dE_o$ in the rest frame, corresponds to the flux of energy times the $\overrightarrow{u}$ velocity:

$$dw = \gamma\left(dE_o + dPV_o\right)\frac{u^2}{c^2}. \quad (39)$$

Then, the energy that will play an important role will be represented by

$$d\xi = dE - dw. \quad (40)$$

This represents the realm of existence of thermal energy.

6. Concluding remarks

Nowadays, even if there are some opinions against the existence of a relativistic transformation in thermodynamics [9] based on the second law of thermodynamics, this last result supports the idea that renormalized thermodynamics, that is, the Ares de Parga *et al.* proposal, represents the compatible form with the first law of thermodynamics for describing the relativistic transformation laws of thermodynamics. But there are stil some problems to solve. One of them consists in analyzing if there exists a reference frame where the 2.7 K blackbody radiation background is at rest. This has already been used by Henry [19] to explain part of the anisotropies of the 2.7 K blackbody radiation background. But the existence or not of such a reference frame has been an open problem since Dirac and Wigner discussed the theme at Carbondale in the seventies [20]. Indeed the existence of such a frame has been described by many authors [21], [22] and it corresponds to being at Alfa Centauro with a velocity of 627 kms^{-1} in the direction l = 276 deg, and b = 30 deg.

However, when gravitational effects are included, the theory becomes more complicated that it would at first appear. Nevertheless, Tolman-Landau-Lifshitz temperature transformation [23] when a gravitational field is present is compatible with renormalized thermodynamics. So renormalized thermodynamics may be a good point of departure for analyzing gravitational effects.

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