Deviations from the universality of slepton masses in the MSSM

M. Gómez-Bock
Instituto de Física, Benemérita Universidad Autónoma de Puebla,
Apartado Postal J-48 Puebla, Pue. México.

Recibido el 6 de septiembre de 2007; aceptado el 23 de octubre de 2007

In this paper we propose an ansatz that applies to the slepton mass matrices. In our approach, these matrices contain a dominant sector that can be diagonalized exactly. We study the numerical results for the slepton mass eigenstates, looking for deviations from universality, which is usually assumed when one evaluates the production of sleptons at future colliders.

Keywords: S-leptons; universality; flavor; MSSM.

En este articulo proponemos un ansatz para la matriz de masa de fermiones-s aplicándolo a los leptones-s. Dentro de esta aproximación las matrices contienen un sector dominante, el cual es diagonalizable exactamente. Se hace un análisis numérico de los resultados para los eigenestados de masa de los leptones-s, buscando las desviaciones de la universalidad de las masas, dicha consideración sobre las masas es utilizada generalmente al evaluar la producción de leptones-s en los futuros colisionadores.

Descriptores: Leptones-s; universalidad; sabor; modelo estándar mínimo supersimétrico

PACS: 14.80.Ly; 11.30.Hv; 12.60.Jv

1. Introduction

Although the MSSM is the leading candidate for new physics beyond the Standard Model, and sensibly explains electroweak symmetry breaking by stabilizing the energy scale, it still leaves unanswered the open problems of the SM, among them the flavor problem [1]. Furthermore, SUSY brings a new flavor problem which is closely related to the mass generation mechanism of the superpartners, namely, that a generic sfermion mass could lead to unacceptably large FCNC, which would exclude the model [2,3]. Several conditions or scenarios have been proposed to solve this problem which reduce the number of free parameters and safely fit the experimental restrictions. The solutions handle in the literature include [4]:

i) *degeneration*, where different sfermion families have the same mass;

ii) *proportionality*, where the trilinear A-terms are proportional to the Yukawa couplings (SUGRA) [5];

iii) *decoupling*, where the superpartners are too heavy to affect the low energy physics (Split SUSY, focus point SUSY, inverted hierarchy) [6];

iv) *alignment*, in which the same physics that explains the fermion mass spectra and mixing angles would also explain the pattern of sfermion mass spectra [7].

In the MSSM, the particle mass spectra depend on the SUSY breaking mechanism. The parametrization of SUSY breaking for MSSM is called *Soft SUSY Breaking, SSB*. The scalar fields are grouped in a supermultiplet together with the fermion fields in such a way that the scalar masses are linked to the SSB energy scale[i] and the mass degeneracy could be broken by the SSB mechanism.

In this paper we are going to study the slepton mass matrices. Our goal is to determine the slepton mass eigenvalues, which are the ones that hopefully will be measured at coming (LHC) and future colliders (ILC). For this, we shall propose a hierarchy within the mass matrices which will include a sector that will have the property of being exactly diagonalizable. This sector will mostly determine the slepton masses. We also include a sector with small off-diagonal entries that will lead to lepton flavor violation (LFV), but we leave this last analysis for future work.

The organization of this paper is as follows. In the next section we present the terms that contribute to the slepton mass matrix in the MSSM. Section 3 explicitly shows the ansatz proposed for the trilinear terms that contribute to this mass matrix as two contribution orders mentioned above, obtaining the expressions for the slepton masses. We present the numerical results for the parameter space in Sec. 4. And finally, in Sec. 5, we summarize our conclusions.

2. S-lepton Mass Matrix

The SUSY invariant terms, which contribute to the diagonal elements of the mass matrix, come from the auxiliary fields, namely the F- and D-terms. However, the mass matrix also includes terms that come from the Soft SUSY Lagrangian [9,10]. Within the MSSM, this soft Lagrangian includes the following terms:

$$\mathcal{L}_{soft} = \mathcal{L}^{mass}_{sfermion} + \mathcal{L}^{mass}_{bino} + \mathcal{L}^{mass}_{gaugino} + \mathcal{L}^{mass}_{gluino} + \mathcal{L}^{mass}_{Higgsino} + \mathcal{L}_{H\tilde{f}_i\tilde{f}_j} \quad (1)$$

In order to establish the free parameters of the model coming from this Lagrangian, we write down the form of the slepton masses and the Higgs-slepton-slepton couplings, the first and last term of Eq. (1), which are given as

$$\mathcal{L}^{\tilde{l}}_{soft} = - m^2_{\tilde{E},ij} \tilde{\bar{E}}^i \tilde{\bar{E}}^{j\dagger} - m^2_{\tilde{L},ij} \tilde{L}^{i\dagger} \tilde{L}^j \\ - (A_{e,ij} \tilde{\bar{E}}^i \tilde{L}^j H_1 + h.c), \quad (2)$$

where the *trilinear terms*, or *A-terms*, are the coefficient of the scalar Higgs-sfermions couplings.

In principle, any scalar with the same quantum numbers could mix through the soft SUSY parameters [11]. This general mixing includes the parity superpartners fermionic labels, and leads us to a sfermion mass matrix given as a square 6×6 matrix, which can be written as a block matrix as

$$\tilde{M}^2_{\tilde{f}} = \begin{pmatrix} M^2_{LL} & M^2_{LR} \\ M^{2\dagger}_{LR} & M^2_{RR} \end{pmatrix} \quad (3)$$

where

$$M^2_{LL} = m^2_{\tilde{L}} + M^{(0)2}_l + \frac{1}{2}\cos 2\beta (2m^2_W - m^2_Z)\mathbf{I}_{3\times 3}, \quad (4)$$

$$M^2_{RR} = M^2_{\tilde{E}} + M^{(0)2}_l - \cos 2\beta \sin^2\theta_W m^2_Z \mathbf{I}_{3\times 3}, \quad (5)$$

$$M^2_{LR} = \frac{A_l v \cos\beta}{\sqrt{2}} - M^{(0)}_l \mu \tan\beta, \quad (6)$$

where $M^{(0)}_l$ is the lepton mass matrix.

The lepton-flavor conservation is violated by the non-vanishing off-diagonal elements of each matrix, and the size of such elements is strongly constrained from experiments. In the SUSY Standard Model based on supergravity, it is assumed that the mass matrices $m^2_{\tilde{E}}$ and $m^2_{\tilde{L}}$ are proportional to the unit matrix, while $A_{e,ij}$ is proportional to the Yukawa matrix $y_{e,ij}$. With these soft terms, the lepton-flavor number is conserved exactly [12]. However, in general soft-breaking schemes, we expect that some degree of flavor violation would be generated. A particular proposal for this pattern is presented next.

3. An ansatz for the mass matrix

The trilinear terms come directly from the Soft SUSY breaking terms, and contribute toward increasing the superparticle masses. We analyze the consequences for sfermion masses by assuming that such terms would acquire a specific flavor structure, which is represented by some *textures*. Textures represent an *a priori* assumption [13,14], in this case, for the mixtures between sfermion families. Such a structure implies that we can classify the matrix elements into three groups, those that contribute at leading order, those that could generate appreciable corrections and those that could be discarded, obtaining a hierarchal texture form.

We propose an ansatz for the trilinear A-terms in the flavor basis, and study its effects on the physical states. We work on a scheme that performs exact diagonalization. First, we parameterize off-diagonal terms, assuming a flavor asymmetry inherited from the fermionic SM sector. In general, there is no reason to expect that the sfermion mass states are exactly degenerate, and there is no solid theoretical basis to consider such patterns, although they are phenomenologically viable [8,15].

We assume, as in supergravity models, the condition of degeneracy on pure Left and pure Right contributions:

$$M^2_{LL} \simeq M^2_{RR} \simeq \tilde{m}^2_0 \mathbf{I}_{3\times 3}, \quad (7)$$

Our ansatz for the A-terms is built up using texture forms and hierarchal structure. The parametrization is obtained by assuming that the mixing between third and second families is greater than the mixing with the first family. Furthermore, it has been observed that current data mainly suppress the FCNCs associated with the first two slepton families, but allow considerable mixing between the second and third slepton families [1].

Thus, our proposal includes dominant terms that mix the second and third families, as follows:

$$A_{LO} = A'_l = \begin{pmatrix} 0 & 0 & 0 \\ 0 & w & z \\ 0 & y & 1 \end{pmatrix} A_0, \quad (8)$$

so that mixtures with the first family are treated as corrections, and are given as:

$$\delta A_l = \begin{pmatrix} e & s & r \\ s & 0 & 0 \\ r & 0 & 0 \end{pmatrix} A_0 = \begin{pmatrix} \delta A_e & \delta A_s & \delta A_r \\ \delta A_s & 0 & 0 \\ \delta A_r & 0 & 0 \end{pmatrix} \quad (9)$$

In the case of $w = 0$, we reproduce the ansatz given in Ref. 1. The dominant terms give a 4×4 decoupled block mass matrix, with respect to the basis $\tilde{e}_L, \tilde{e}_R, \tilde{\mu}_L, \tilde{\mu}_R, \tilde{\tau}_L, \tilde{\tau}_R$, as

$$\tilde{M}^2_{\tilde{l}} = \left(\begin{array}{cc|cccc} a & 0 & 0 & 0 & 0 & 0 \\ 0 & a & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & a & X_2 & 0 & A_z \\ 0 & 0 & X_2 & a & A_y & 0 \\ 0 & 0 & 0 & A_y & a & X_3 \\ 0 & 0 & A_z & 0 & X_3 & a \end{array}\right), \quad (10)$$

with

$$X_3 = (1/\sqrt{2}) A_0 v \cos\beta - \mu m_\tau \tan\beta$$

and

$$X_2 = A_w - \mu m_\mu \tan\beta.$$

Where μ is the SU(2)-invariant coupling of two different Higgs superfield doublets, A_0 is the trilinear coupling scale and $\tan\beta = v_2/v_1$ is the ratio of the two vacuum expectation values coming from the two neutral Higgs fields; these three are MSSM parameters [11,16].

The correction takes the form:

$$\delta\tilde{M}^2_{\tilde{l}} = \left(\begin{array}{cc|cccc} 0 & \delta A_e & 0 & \delta A_s & 0 & \delta A_r \\ \delta A_e & 0 & \delta A_s & 0 & \delta A_r & 0 \\ \hline 0 & \delta A_s & 0 & 0 & 0 & 0 \\ \delta A_s & 0 & 0 & 0 & 0 & 0 \\ 0 & \delta A_r & 0 & 0 & 0 & 0 \\ \delta A_r & 0 & 0 & 0 & 0 & 0 \end{array}\right). \quad (11)$$

The explicit forms of $A_{z,y,w}$ and δA are given in Table I.

TABLE I. Explicit terms of the sfermion mass matrix ansatz, assuming δA_e as a third order element.

dominant	correction
$A_z = \frac{1}{\sqrt{2}} z A_0 v \cos\beta$	$\delta A_s = \frac{1}{\sqrt{2}} s A_0 v \cos\beta$
$A_y = \frac{1}{\sqrt{2}} y A_0 v \cos\beta$	$\delta A_r = \frac{1}{\sqrt{2}} r A_0 v \cos\beta$
$A_w = \frac{1}{\sqrt{2}} w A_0 v \cos\beta$	$\delta A_e = 0$

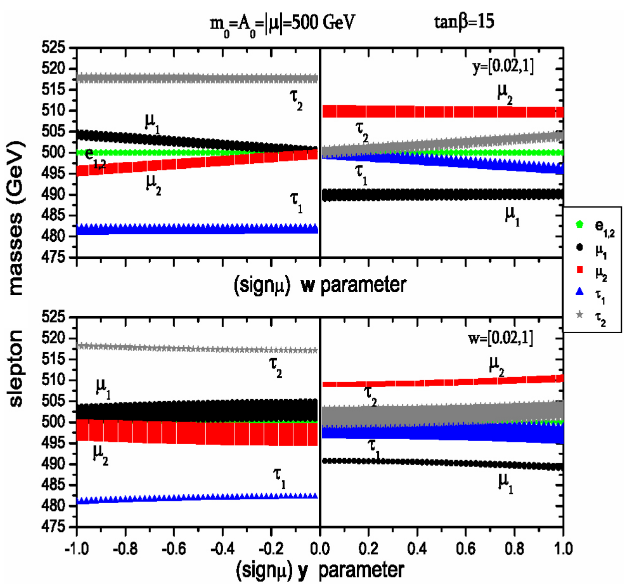

FIGURE 1. Slepton mass dependency with respect to the parameter ansatz w (up) and y (down) with $\tilde{m}_0 = A_0 = \mu_{susy} = 500$ GeV and $\tan\beta = 15$, considering $\mu_{susy} < 0$ and $\mu_{susy} > 0$.

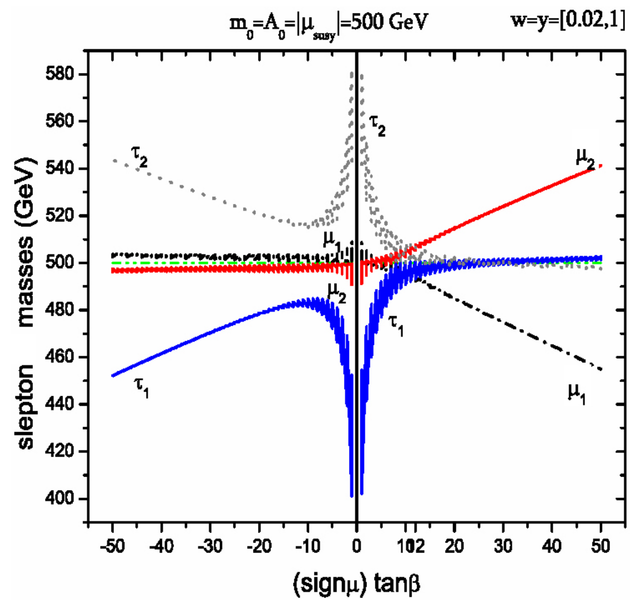

FIGURE 2. Slepton mass with respect to $\tan\beta$ for $\mu_{susy} < 0$, $\mu_{susy} > 0$ and with w=y=$[0.02, 1]$, $\tilde{m}_0$=A_0=$|\mu_{susy}|$=500 GeV.

In order to obtain the physical slepton eigenstates, we diagonalize the 4×4 mass sub-matrix given in (10). In order to achieve an analytic diagonalization, we consider that $z = y$. The rotation will be performed on this part using a hermitian matrix Z_l, such that

$$Z_l^\dagger M_{\tilde{l}}^2 Z_l = \tilde{M}_{Diag}^2, \qquad (12)$$

where

$$M_{\tilde{l}}^2 = \begin{pmatrix} \tilde{m}_0^2 & X_2 & 0 & A_y \\ X_2 & \tilde{m}_0^2 & A_y & 0 \\ 0 & A_y & \tilde{m}_0^2 & X_3 \\ A_y & 0 & X_3 & \tilde{m}_0^2 \end{pmatrix}. \qquad (13)$$

Then the rotation matrix is given by

$$\begin{pmatrix} \tilde{e}_L \\ \tilde{\mu}_L \\ \tilde{\tau}_L \\ \tilde{e}_R \\ \tilde{\mu}_R \\ \tilde{\tau}_R \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\sin\frac{\varphi}{2} & -\cos\frac{\varphi}{2} & 0 & \sin\frac{\varphi}{2} & \cos\frac{\varphi}{2} \\ 0 & \cos\frac{\varphi}{2} & -\sin\frac{\varphi}{2} & 0 & -\cos\frac{\varphi}{2} & \sin\frac{\varphi}{2} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -\sin\frac{\varphi}{2} & \cos\frac{\varphi}{2} & 0 & -\sin\frac{\varphi}{2} & \cos\frac{\varphi}{2} \\ 0 & \cos\frac{\varphi}{2} & \sin\frac{\varphi}{2} & 0 & \cos\frac{\varphi}{2} & \sin\frac{\varphi}{2} \end{pmatrix} \begin{pmatrix} \tilde{e}_1 \\ \tilde{\mu}_1 \\ \tilde{\tau}_1 \\ \tilde{e}_2 \\ \tilde{\mu}_2 \\ \tilde{\tau}_2 \end{pmatrix} = Z_B^l \tilde{l}, \qquad (14)$$

with

$$\sin\varphi = \frac{2A_y}{\sqrt{4A_y^2 + (X_2 - X_3)^2}},$$
$$\cos\varphi = \frac{(X_2 - X_3)}{\sqrt{4A_y^2 + (X_2 - X_3)^2}} \qquad (15)$$

We obtain the following hierarchy for the sleptons: $m_{\tilde{\tau}_1} < m_{\tilde{\mu}_1} < m_{\tilde{\mu}_2} < m_{\tilde{\tau}_2}$, for $\mu < 0$, having the following eigenvalues:

$$m_{\tilde{\mu}_1}^2 = \frac{1}{2}(2\tilde{m}_0^2 + X_2 + X_3 - R),$$
$$m_{\tilde{\mu}_2}^2 = \frac{1}{2}(2\tilde{m}_0^2 - X_2 - X_3 + R),$$
$$m_{\tilde{\tau}_1}^2 = \frac{1}{2}(2\tilde{m}_0^2 - X_2 - X_3 - R),$$
$$m_{\tilde{\tau}_2}^2 = \frac{1}{2}(2\tilde{m}_0^2 + X_2 + X_3 + R), \qquad (16)$$

with $R = \sqrt{4A_y^2 + (X_2 - X_3)^2}$.

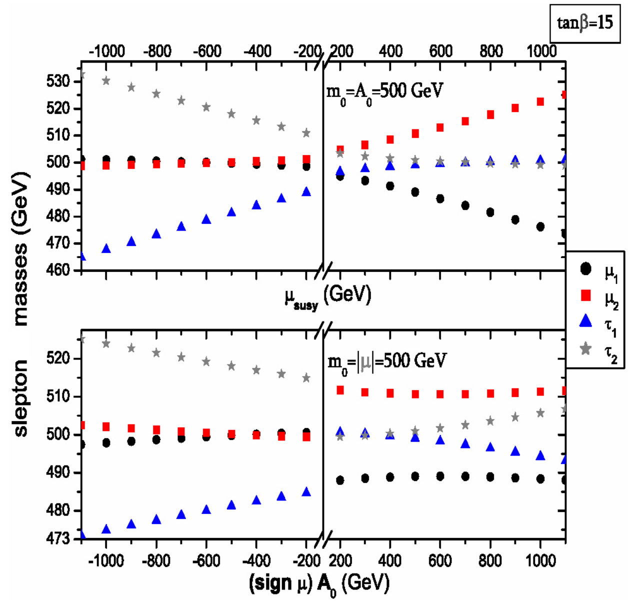

FIGURE 3. Slepton mass dependance on μ_{susy}, with $\tilde{m}_0 = A_0 = 500$ GeV (up). And slepton mass dependence on A_0 for $\mu_{susy} < 0$ and $\mu_{susy} > 0$, with $\tilde{m}_0 = |\mu_{susy}| = 500$ GeV (down). Both with $\tan\beta = 15$ and $y = w = 1$.

4. Numerical results for slepton masses

From the expressions for the slepton masses (Eq. 16), we shall analyze their parameter dependency. In Fig. 1 we show the dependence on $y(= x)$ and w. Then, in the next two figures we show the dependence of the slepton masses on the usual MSSM parameters, μ, A_0 and $\tan\beta$.

We see that X_3 and X_2 are given in terms of μ and $\tan\beta$, having a strong dependency on the sign of μ, and so we obtain a hierarchy of the slepton masses given as follows:

$\mu < 0$

$$m_{\tau_1} < m_{\mu_2} < (m_{e_1}{=}m_{e_2}) < m_{\mu_1} < m_{\tau_2} \qquad (17)$$

$\mu < 0$

$$m_{\mu_1} < m_{\tau_1} < (m_{e_1}{=}m_{e_2}) < m_{\tau_2} < m_{\mu_2} \qquad (18)$$

We observed this on the graphs of Fig. 1, where we run the values of y and w independently through a range of $[0.02, 1]$ and set the values for the soft SUSY breaking scale as $\tilde{m}_0$=500 GeV, with $\tan\beta = 15$. We have practically no dependence on parameter y; and for $w = 0$, we have degeneracy for the lightest slepton, up to 10 GeV for smuons with $w = \pm\,1$; and no dependency for the heaviest sleptons. As we mentioned above, the strongest dependency comes from the MSSM parameter, and the deviation from universality is manifested by the staus, and in the case of $\mu < 0$, giving a difference in stau masses of ~ 40 GeV.

In Fig. 2 we verify the behavior of slepton masses with $\tan\beta$, for the ansatz parameter set as $y = w = [0.02, 1]$, and $\tilde{m}_0 = 500$ GeV. We found that for $\mu < 0$ the smuons are nearly degenerate, while for $\mu > 0$ the staus are the degenerated ones. We also observed that the parameter dependency becomes diluted for $\tan\beta >\sim 12$.

Although we have considered the SSB parameters equal to the SUSY breaking mass scale $\tilde{m}_0 = |\mu| = A_0$, this is not necessarily true. We explore independently the possible values for the Higgsinos mass parameter μ from the soft mass term, as is shown at the top of Fig. 3. In the same sense we explore independently the trilinear-A coupling, the results are shown at the bottom of the same Fig. 3. In both cases we set the soft mass term as $\tilde{m}_0 = 500$ GeV. We observed again the difference in the mass hierarchy between smuons and staus depending on the μ sign. In the trilinear coupling dependency, we observe that the non-degeneration increases for $A_0 > \tilde{m}_0$.

5. Conclusions

We have studied the possible non-degeneration for the slepton masses, using an ansatz for the slepton mass matrix. Specifically, consider the mixing to occur between the second and third families, and assume that this mixing comes solely from left-right terms. We encounter the parameter space dependency of the masses, including both the MSSM parameters and the proposed model parameters. This non-degeneracy could be measured in the cases where it is about 5% of the SUSY Soft-Breaking mass scale $\tilde{m}_0$; this percentage is suggested by considering the experimental uncertainty.

We observed that the strongest dependence comes from the MSSM parameter space, while, as we expected, the parameters of the ansatz act only to accomplish this for some non-zero terms.

A dependence on the μ sign is strongly manifested. The mass hierarchy changes whether μ is positive or negative, which leads us to the conclusion that if the hierarchy mass spectrum is the most expected one, *i.e.* $m_{\tilde{\tau}_1}{<}m_{\tilde{\mu}_1}{<}m_{\tilde{\mu}_2}{<}m_{\tilde{\tau}_2}$ then μ must be negative. Also we observed that, for each case,

- For $\mu > 0$, we obtain non-degeneration on smuons, and the difference between $\tilde{\mu}_1$ and $\tilde{\mu}_2$ could be larger than 10% for $\tan\beta \sim> 30$ and $|\mu|/\tilde{m}_0 \sim> 2$, while we obtain approximately, stau degeneration, where only for $A_0/\tilde{m}_0 > 2$, we reach a difference of $> 3\%$ of the $\tilde{m}_0$.

  Analyzing the ansatz parameters, we obtain an increased mass difference for $y = w = 1$ up to 2%, with the strongest dependency being on the w parameter.

- For $\mu < 0$, the non-degeneration is obtained for the staus, with a difference between them of 10% or more, for $\tan\beta \sim> 30$ and $|\mu|/\tilde{m}_0 \sim> 1.6$. And we have, practically, smuon degeneration. In this case, considering $A_0/\tilde{m}_0 > 2$ generates a difference in stau masses of $\sim$ 10% of $\tilde{m}_0$, with $\tan\beta = 15$, while for the smuons we reach only 1%. For the ansatz parameters we also have an increase in mass difference up to 2% for $y = w = 1$.

For $\tan\beta$ we conclude that, if degenerate masses are measured, then the $\tan\beta$ value should be around 10, while in the other case, non-degeneration is manifested either for small $\tan\beta$ (less than ~ 5) or for a large value.

The mass difference found here could be tasted possibly at LHC, with some difficulties, but certainly at ILC.

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i See for instance Ref. 8.

1. J.L. Diaz-Cruz, H.J. He and C.P. Yuan, *Phys. Lett. B* **530** (2002) 179 [arXiv:hep-ph/0103178].
2. W.J. Marciano, *FCNC: a Perspective.* Proceedings of Symposium of FCNC: Present and Future Studies, Editor Davd B. Cline (Word Scientific, 1997).
3. J.L. Diaz-Cruz, R. Noriega-Papaqui, and A. Rosado, *Phys. Rev. D* **71** (2005) 015014 [arXiv:hep-ph/0410391].
4. J.L. Diaz-Cruz, M. Gomez-Bock, R. Noriega-Papaqui, and A. Rosado, arXiv:hep-ph/0512168.
5. L.J. Hall, V.A. Kostelecky, and S. Raby, *Nucl. Phys. B* **267** (1986) 415.
6. J.L. Feng and F. Wilczek, *Phys. Lett. B* **631** (2005) 170 [arXiv:hep-ph/0507032].
7. Y. Nir and N. Seiberg, *Phys. Lett. B* **309** (1993) 337 [arXiv:hep-ph/9304307].
8. S. Weinberg, *The Quantum Theory of Fields*, Vol. III. Supersymmetry (Cambridge Univesity Press, 1996).
9. For a review see: See for example: S.P. Martin, arXiv:hep-ph/9709356.
10. M. Kuroda, arXiv:hep-ph/9902340.
11. For a review see: Ian. J.R. Aitchison, arXiv:hep-ph/0505105.
12. J. Hisano, T. Moroi, K. Tobe, M. Yamaguchi, and T. Yanagida, *Phys. Lett. B* **357** (1995) 579 [arXiv:hep-ph/9501407].
13. Z.z. Xing and H. Zhang, *J. Phys. G* **30** (2004) 129 [arXiv:hep-ph/0309112].
14. H. Fritzsch and Z.Z. Xing, *Phys. Lett. B* **555** (2003) 63 [arXiv:hep-ph/0212195].
15. H.E. Haber, *Nucl. Phys. Proc. Suppl.* **101** (2001) 217 [arXiv:hep-ph/0103095].
16. used as phenomenological parameters as in E. Accomando *et al.* [ECFA/DESY LC Physics Working Group Collaboration], *Phys. Rept.* **299** (1998) 1 [arXiv:hep-ph/9705442].