Diffraction of hermite-gaussian beams by Ronchi and aperiodic rulings

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In this paper, the diffraction of beams by Ronchi rulings is considered. The theory of diffraction is based on the Rayleigh-Sommerfeld integral equation with Dirichlet conditions. The diffraction of Gaussian and Hermite-Gaussian beams is studied numerically. The transmitted power and the normally diffracted energy are analyzed as a function of the spot size. The diffraction patterns obtained at maximum and minimum transmitted power are also deal with. We show that the two methods to determine the Gaussian spot size with Ronchi rulings which were experimentally compared in a previous paper [*Rev. Mex. Fís.* **53** (2007) 133] cannot be extended to the case of Hermite-Gaussian beams. Finally, for aperiodic rulings we propose a method for obtaining the Gaussian spot size by means of the normally diffracted energy.

Keywords: Diffraction; gratings.

En este artículo se estudia la difracción de haces por redes de difracción de Ronchi. La teoría de la difracción utilizada esta basada en la ecuación integral de Rayleigh-Sommerfeld con condiciones de Dirichlet. La difracción de haces tipo Gauss y Hermite-Gauss es estudiada numéricamente. La potencia transmitida y la energía difractada normalmente a la pantalla son estudiadas. Mostramos que los dos métodos para determinar el ancho de un haz tipo Gauss, los cuales fueron considerados en un artículo previo [*Rev. Mex. Fís.* **53** (2007) 133], no pueden extenderse al caso de haces Hermite-Gauss. Finalmente, utilizando una red de difracción no periódica se propone un método para obtener el ancho de un haz de Gauss por medio de la energía difractada normalmente.

Descriptores: Difracción; redes de difracción.

PACS: 42.25.Fx; 42.10.H.C

1. Introduction

The methods for calculation of diameters of Gaussian beams at the present time have a wide range of applications[1]. This paper can be considered to be the continuation of a previously published article where two methods for Gaussian spot size measurement by means of Ronchi rulings were studied[1]. The reader is referred to Ref. 1 for a more complete list of references.

Some methods for determining the size of the Gaussian beams have been proposed which are based on the properties of the transmitted power by rulings. Thus, Ronchi rulings[2] (grating with alternate clear and dark fringes of square profile per period), sinusoidal rulings[2-3], triangular rulings[2-3], periodic exponential rulings[4-5], and aperiodic gratings[6] have been considered. In all the mentioned papers[2-6] the beam diameters have been determined by means of the maximum and the minimum transmitted power. However, two exceptions are given in Refs. 7 and 8 where the normally diffracted energy to the gratings was considered.

It is important to notice that, to our knowledge, only the measurement of Gaussian beam diameters has been considered in the literature and no attention has been paid to the determination of Hermite-Gaussian beam diameters. So, in this paper we examine whether the techniques mentioned can be extended to the case of Hermite-Gaussian beams. These kinds of beams are described by the product of Hermite polynomials and Gaussian functions. At present, the two-dimensional Hermite-Gaussian beams can easily be excited, for instance, with end-pumped solid-state lasers[9] or by inserting a cross wire into the laser cavity with the wires aligned with the nodes of the desired mode[10]. In Ref. 9 it was demonstrated that it is possible to generate two-dimensional Hermite-Gaussian modes up to the $\mathrm{TEM}_{0,80}$ mode. In passing, we mention that these beams have been considered in relation to some other diffraction problems[11-14].

In this paper two diffractional methods to determine the beam diameters of Gaussian and Hermite-Gaussian beams by means of Ronchi rulings are considered. These two methods are based on the maximum and the minimum transmitted power, and in the normally diffracted energy. Also, the diffraction patterns obtained at maximum and minimum transmitted power are studied. Finally, the aperiodic ruling is considered, and it is shown that the Gaussian spot size can be determined from the normally diffracted energy. The results obtained for the aperiodic ruling can be useful in that they use only the diffracted energy close to the normal direction instead of the total transmitted power as is usually done with conventional[2] or non-conventional rulings[3-6].

2. Formulation

We have a ruling made of alternate opaque (width ℓ) and transparent zones (width ℓ), whose period is $d = 2\ell$. We fixed a cartesian coordinate system with the O_z-axis parallel to the ruling as shown in Fig. 1. The ruling is illuminated by a beam independent of the z coordinate (cylindrical incident wave). The time dependence $\exp(-iwt)$ is used in what follows.

The theory of diffraction is only outlined here and the reader is referred to Ref. 1 for most details. Let $E(x)$, $E_i(x)$, and $t(x)$ be the transmitted field, the input or incident field, and the grating transmittance function, related as follows:

$$E(x) = t(x)E_i(x), \tag{1}$$

where the function $t(x)$ is null in the opaque zones, and has the unit value in the transparent zones. From this expression, the field just below the ruling can be obtained. From the knowledge of the field $E(x)$ it is possible, by means of the two-dimensional Rayleigh-Sommerfeld theory with Dirichlet conditions, to get the field $E(x_o, y_o)$ at any point below the ruling

$$\begin{aligned} E(x_0, y_0) &= \frac{i}{2}\int_{-\infty}^{\infty} E(x)\frac{\partial}{\partial y_0}H_0^1(kr)dx \\ &= \frac{i}{2}\int_{-\infty}^{\infty} t(x)E_i(x)\frac{\partial}{\partial y_0}H_0^1(kr)dx, \end{aligned} \tag{2}$$

where $k = 2\pi/\lambda$, with the wavelength λ of the incident radiation, $r^2 = (x - x_0)^2 + y_0^2$ with $P(x_o, y_o)$ the observation point as illustrated in Fig. 1, and H_o^1 is the Hankel function of the first kind and order zero. From Eq. (2), the far field can be obtained by looking at the asymptotic behavior of the field E when $kr \gg 1$. In this approximation we have

$$E(x_0, y_0) = f(\theta)\exp(ikr_0)/\sqrt{r_0}, \tag{3}$$

where $\sin\theta = x_0/r_0$, and $\cos\theta = -y_o/r_o$ (see Fig. 1). This is the expression of a cylindrical wave with the oblique factor $t(\theta)$ given by:

$$f(\theta)\sqrt{k}\exp(-i\pi/4)\cos\theta\hat{E}(k\sin\theta) \tag{4}$$

with

$$\begin{aligned} \hat{E}(\alpha) &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} E(x)\exp(-i\alpha\, x)dx \\ &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} t(x)E_i(x)\exp(-i\alpha\, x)dx. \end{aligned} \tag{5}$$

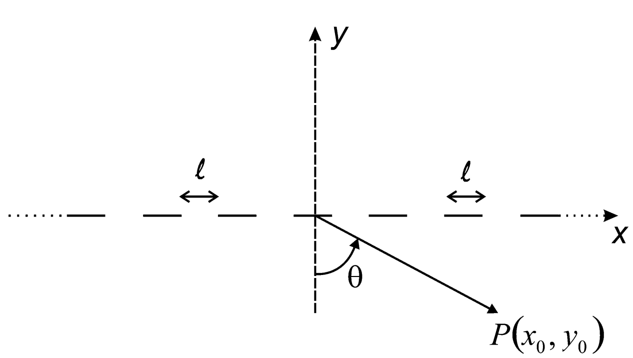

FIGURE 1. A Ronchi ruling made of alternate opaque and transparent zone of widths ℓ. The slit is parallel to the Oz axis.

Then, the intensity $I(\theta)$ diffracted at an angle θ (see Fig. 1) is given by $C\,|f(\theta)|^2$, where C is a constant which will be taken as unity since we are interested only in relative quantities, so that we have

$$I(\theta)=\frac{1}{2\pi}k\cos^2\theta\left|\int_{-\infty}^{+\infty} t(x)E_i(x)\exp(-ik\sin\theta\, x)dx\right|^2. \tag{6}$$

Then the diffracted field can be determined from Eq. (6) if the input field $E_i(x)$ and the transmittance function $f(x)$ are given.

In what follows our attention is focused on the transmitted power P_T and on the normally diffracted energy to the screen $I(0^\circ)$, which will be denoted by E. The normally diffracted energy E is calculated from Eq. (6) with the angle $\theta = o^\circ$ as

$$E = \frac{k}{2\pi}\left|\int_{-\infty}^{+\infty} t(x)E_i(x)dx\right|^2 \tag{7}$$

and the transmitted power P_T is obtained from

$$P_T = \int_{-\pi/2}^{\pi/2} I(\theta)\, d\theta \tag{8}$$

We have compared the results obtained by means of Eqs. (6) to (8) with those calculated by means of the rigorous diffraction theories given in Refs. 16 and 17. We have found that the three theories give practically the same results in the scalar regime[8] where the polarization effects can be neglected. The interested reader is referred to Ref. 18, where the Rayleigh-Sommerfeld theory is compared with a rigorous diffraction theory in the case of a slit in a perfectly conducting screen (see cases $\lambda/\ell < 0.2$).

3. Definition of Hermite-Gaussian beam width

In this paper, the width of a Gaussian mode TEM_{00} is obtained when the intensity drops to $1/e$ and will be denoted by L from now on. There are other definitions for the Gaussian spot size; for instance, the beam width can be calculated using the diameter that covers 86.5% of the energy, and in this case the beam width will be denoted by $L_{86.5}$. As was pointed in Ref. 12, the relationship between the Gaussian widths L and $L_{86.5\%}$ is given by $L_{86.5\%} = 1.057L$, so that the values of $L_{86.5\%}$ are very close to the values of L; in fact, in practice we can consider that $L_{86.5\%} = L$.

We assume the following intensity distribution on the screen ($y = 0$) of the normally incident Hermite-Gaussian beam of order n:

$$I_n(x) = [H_n\{2(x-b)/L\}]^2\exp[-4(x-b)^2/L^2], \tag{9}$$

where $In(x)$ is the intensity as a function of the coordinate x, L is the Gaussian beam width when $n = 0$, and H_n is the Hermite polynomial of order n, some of which are

$H_0(t) = 1$, $H_1(t) = 2t$, $H_2(t) = 4t^2 - 2$, $H_3(t) = 8t^3 - 12t$, and so forth. The position of the incident Hermite-Gaussian beam with respect to the Oy axis is fixed by the parameter b. If the Hermite-Gaussian beam diameter L_n is defined by the 86.5% energy content, then, L_n is related to L by means of a linear relationship. We have found that $L_1 = 1.667L$, $L_2 = 2.122L$, $L_3 = 2.503L$, $L_4 = 2.836L$, and so forth[12]. Therefore, if the parameter L is fixed, the Hermite-Gaussian diameter L_n increases when n increases. In fact, we can consider L as a common parameter for all the Hermite-Gaussian beams; however, it is necessary not to forget that the interesting and practical parameter is L_n.

4. Definitions of K and P

In this paper, we are mainly interested in studying the intensity ratio K defined as follows:

$$K = E_{\min}/E_{\max} \tag{10}$$

and the power ratio P given by

$$P = P_{\min}/P_{\max}, \tag{11}$$

where $E_{\min}$ and $E_{\max}$ are the minimum and maximum values of the normally diffracted energy $I(0°)$, $P_{\min}$ and $P_{\max}$ are the minimum and maximum transmitted power, both of them obtained when the beam is scanned by the ruling.

In Fig. 2, the transmitted power P_T is plotted as a function of the normalized beam position (b/d) for normally incident Hermite-Gaussian beams of order n =0, 1, and 2. The period of the ruling is $d = 1.0$ and the Gaussian width is $L/d = 0.5$. In this case we find that $L_1/d = 0.8335$, $L_2/d = 1.061$, and $L_3/d = 1.418$. The results of Fig. 2 show a periodical behavior with the same period as that of the ruling, for all values of n. We observe that the minima for n =0 are located at the same positions as the maxima for

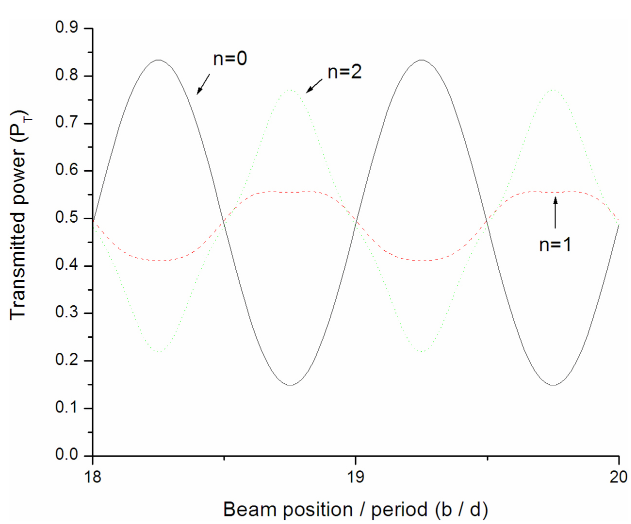

FIGURE 2. Transmitted power as a function of the beam position for Hermite-Gaussian beams of order =0 (solid line), 1 (dash line), and 2 (dot line), with width $L/d = 0.5$, and $\lambda/d = 0.03$.

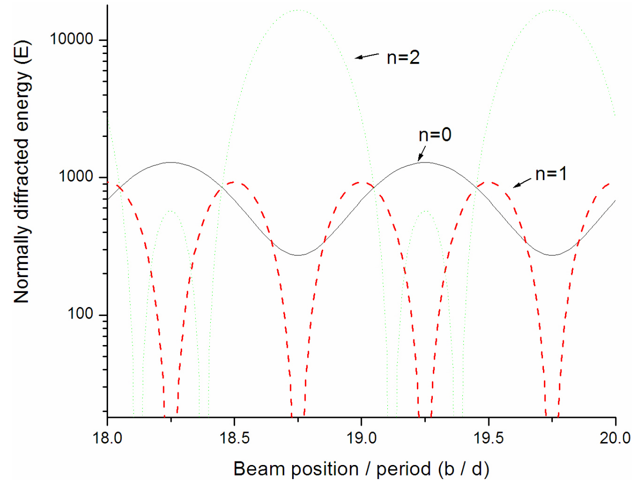

FIGURE 3. Same as Fig. 2 but for the normally diffracted energy E.

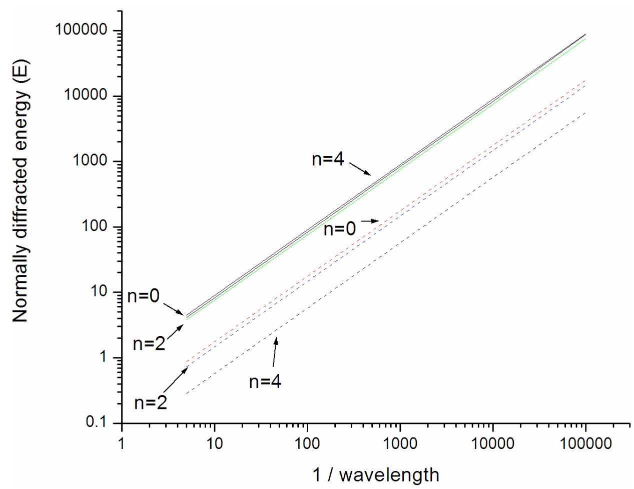

FIGURE 4. Minimum $E_{\min}$ (dash lines) and maximum $E_{\max}$ (solid lines) values of the normally diffracted energy as functions of the inverse of the wavelength for the Hermite-Gaussian beams of order = 0, 2, 4, when ℓ=0.5.

n =1, 2, and the opposite also happens. We have verified, as expected, that the transmission coefficient is independent of the wavelength, so that the ratio power P is also independent of the wavelength. This independence of the wavelength is an important condition in the establishment of a method for determining the spot size of the beams.

Figure 3 is similar to Fig. 2 but for the normally diffracted energy $I(0^0)$. We note that these two figures are very different. We observe also that the behavior of E has the same periodicity as that of the ruling. From the numerical results of Fig. 3 we have obtained that K=0.211 for n =0; however, for n =1, 2 we have $K \approx 0$. We have found that this conclusion is in general true for odd-numbers n.

In Fig. 4 we plot the minimum ($E_{\min}$) and maximum ($E_{\max}$) values of the normally diffracted energy as functions of the inverse of the wavelength. From this figure we get that ($E_{\min}$) and ($E_{\max}$) have the following behavior:

$$E_{\min} \propto \frac{1}{\lambda} \quad \text{and} \quad E_{\max} \propto \frac{1}{\lambda}, \tag{12}$$

so we can conclude from Fig. 4 and other results not shown that the intensity ratio K is also independent of the wavelength for Hermite-Gaussian beams. The results of Eq. (12) are in agreement with the property already presented in Ref. 14 given by $E = N\tau/\lambda$, where N is the number of slits covered by the beam and τ is the transmission coefficient.

5. Pattern diffractions at maximum and minimum power

In Fig. 5 the diffraction patterns are plotted at maximum and minimum transmitted power for a normally incident Gaussian beams ($n = 0$), when $\lambda/d = 0.03$, $\ell/d = 0.5$, and $L/d = 0.5$. We observe a more oscillating behavior for the diffraction pattern at minimum transmitted power. In both cases, practically all the diffracted energy is inside the angular sector $\Delta\theta = 7^0$, so that the diffracted energy is mainly concentrated close to the normal to the screen.

Figure 6 is similar to Fig. 5 but for an Hermite-Gaussian beam of order n=1. We note that the diffraction patterns are more complicated that those of Fig. 5. A null intensity is observed at $\theta = 0^0$, in fact, the central maximum of Fig. 5 is split into two maxima in this figure. This last result is an effect of the Hermite polynomial of order n=1. In this case, practically all the energy is concentrated in the sector $\Delta\theta = 10^0$.

Figure 7 is similar to Figs. 5 and 6, but now for n=2. We note immediately that the central maximum of Fig. 5 is divided into three maxima in this figure; this is a direct result of the Hermite polynomial of order n=2. In this case, the diffracted energy is concentrated in the angular sector

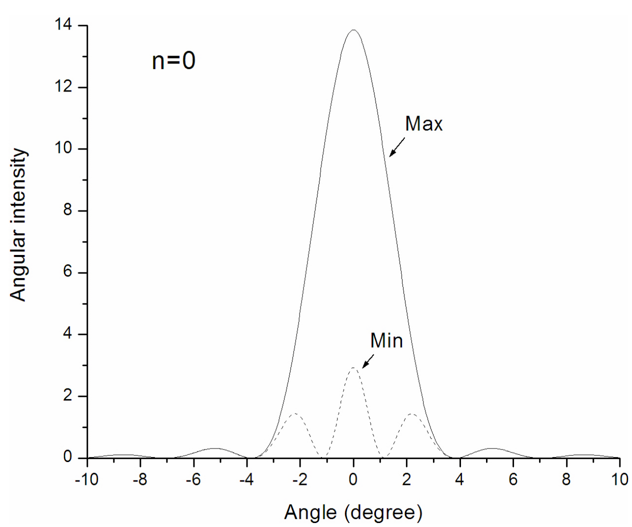

FIGURE 5. Diffraction patterns at maximum (solid line) and minimum (dash line) transmitted power for a normally incident Gaussian beam ($n = 0$), when $\lambda/d = 0.03$, $\ell/d = 0.5$, and $L/d = 0.5$.

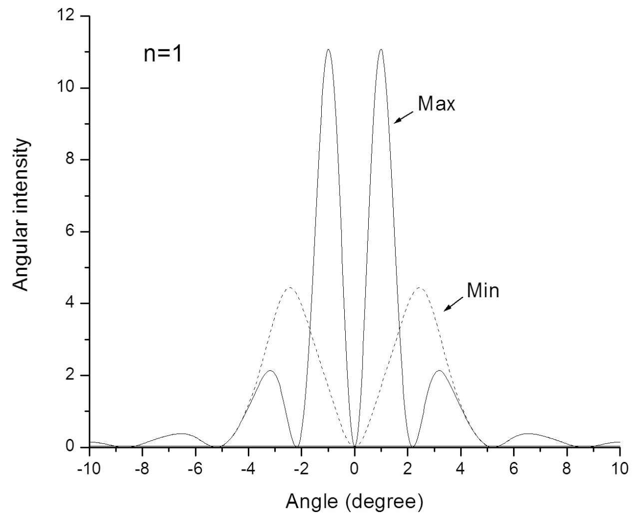

FIGURE 6. Same as Fig. 5 but for $n = 1$.

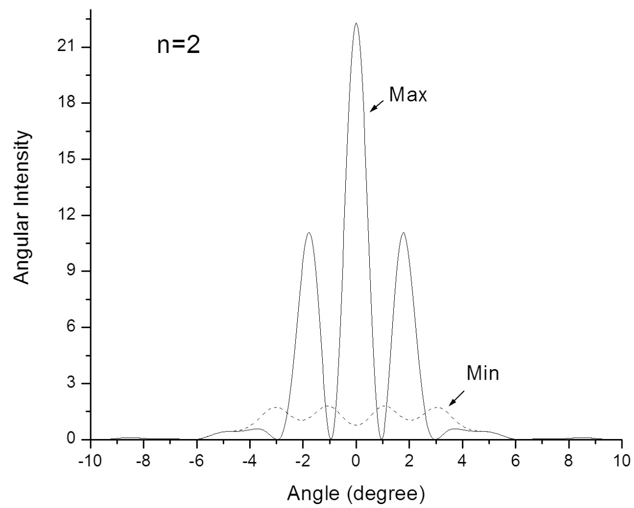

FIGURE 7. Same as Fig.5 but for $n = 2$.

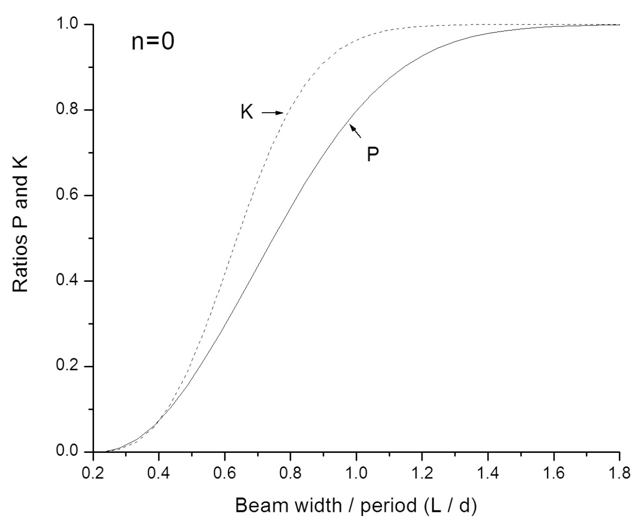

FIGURE 8. Intensity ratio $K = E_{\min}/E_{\max}$ (dashed curve) and $P = P_{\min}/P_{\max}$ (solid curve) for a normally incident Gaussian beam as functions of the normalized beam width L/d, for a Ronchi ruling with period $d = 1.0$.

$\Delta\theta = 8^0$. From the results given in Figs. 6 and 7 we can conclude that the central maximum for a Gaussian beam is split into n+1 maxima for an Hermite-Gaussian beam of order n. This is an interesting result in diffraction theory which shows that some of the accepted ideas must change in considering Hermite-Gaussian beams.

6. Two methods based on ratios K and P

In Fig. 8, we plot the ratios P and K as functions of the Gaussian spot size normalized to the grating period d (L/d) for a normally incident Gaussian beam. This figure is similar but not identical to Fig. 5 of Ref. 1. If the ratios P and K are experimentally determined, the corresponding spot width can be obtained as long as 0.3< L/d <1.6.

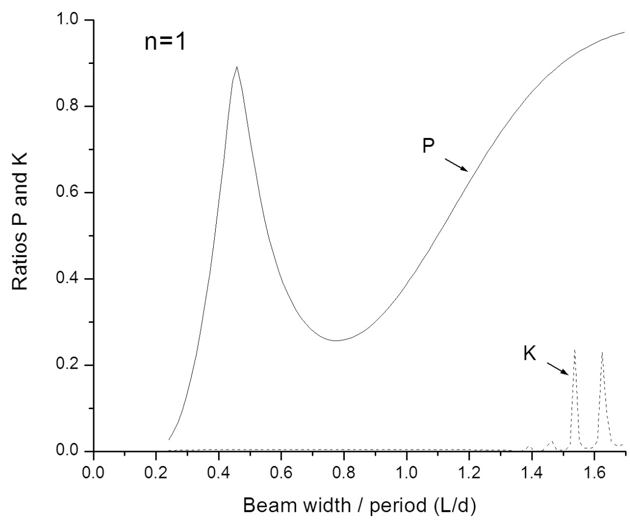

FIGURE 9. Same as Fig. 8 but for $n = 1$.

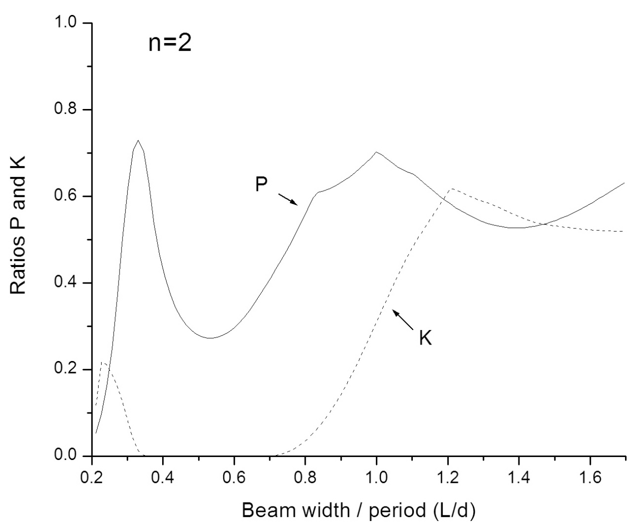

FIGURE 10. Same as Fig. 8 but for $n = 2$.

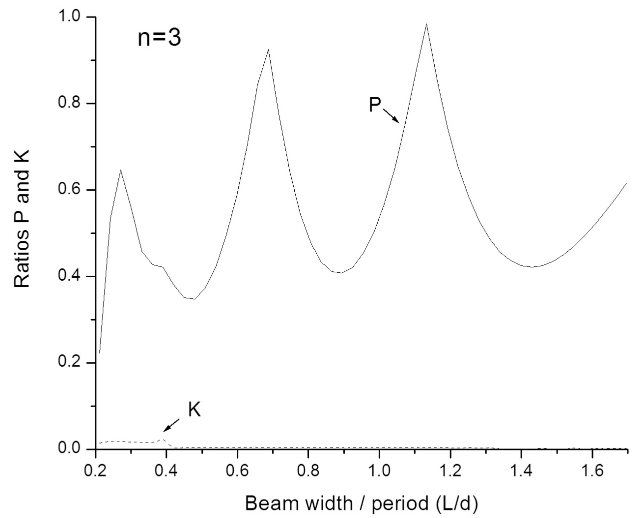

FIGURE 11. Same as Fig. 8 but for $n = 3$.

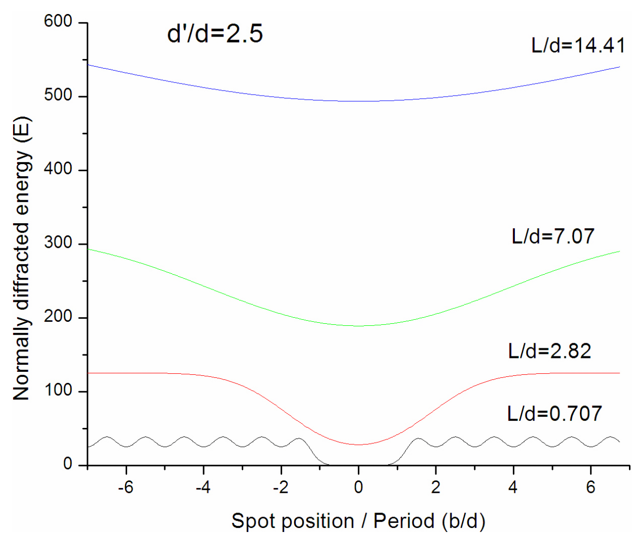

FIGURE 12. Normally diffracted energy as a function of the spot position (b/d) for a normally incident Gaussian beam on an aperiodic ruling with d=1.0 and d'=2.5, with wavelength $\lambda = 0.01$.

Figures 9 to 11 are similar to Fig. 8 but for =1, 2, and 3. We note that, instead of the growing behavior of P and K in Fig. 8, we have an oscillating behavior in these figures. From these figures we can conclude that the two methods for determining the width of Gaussian beams cannot be applied any more to Hermite-Gaussian beams ($n \neq 0$). However, from the point of view of diffraction theory, these results are very interesting. We observe for ratio P that the number of maxima is increased when the order of the Hermite-Gaussian beam is also increased. Moreover, the ratio K is not always null forn-odd numbers; there are certain values of L/d where K can be seen without difficulty in Figs. 9 and 11.

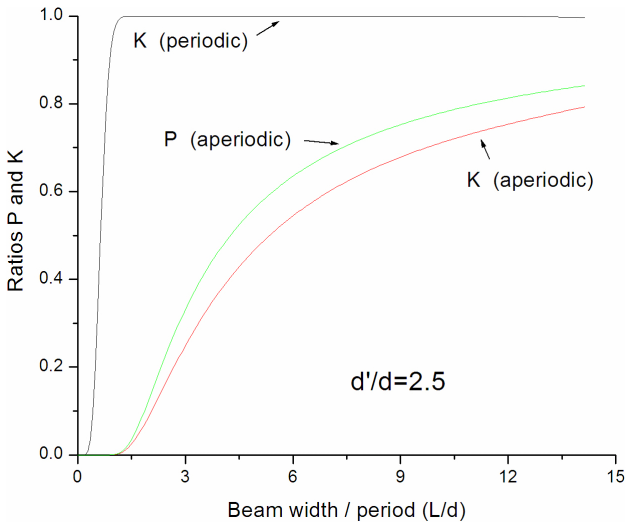

FIGURE 13. Ratios P and K as functions of the normalized Gaussian beam width L/d, when $d'/d = 2.5$ and $d = 1.0$.

7. The aperiodic Ronchi ruling

In this section we shall consider the case of an aperiodic ruling made of alternate transparent and opaque zones. The period of the ruling is d, but with a central opaque zone of width d' which could be equal to or different from d, $i.e.$, we have a central discontinuity in the ruling. In the case where $d' = d$, the conventional Ronchi ruling is recovered. This aperiodic ruling has been studied by Uppal *et al.* in Ref. 15. The Oz-axis will be placed halfway through the central opaque zone.

In Fig. 12 we plot the normally diffracted energy as a function of the spot position (b/d) for a normally incident Gaussian beam on an aperiodic ruling, with d=1.0 and d'=2.5. From these results and others not shown, we have observed a small depression at the centre of the discontinuity and a constant value far from this discontinuity when$L/d > 2.5$. This depression and this constant value are very important in the determination of the Gaussian spot size as we shall see below. It is important to say that we have gotten the same conclusions when the transmitted power is plotted as a function of the spot position; this case was already considered by Uppal[15] and is not shown here.

In Fig. 13 we plot the ratios P and K as functions of the normalized beam width L/d, when $d'/d = 2.5$ and $d = 1.0$. The results for P have been published by Uppal *et al.* in Ref. 15. Our contribution in this direction is to show that K can also be used to determine the width of Gaussian beams. We note that the values of P and K are very close but they are not identical. For comparison, we have also plotted the results of K obtained with the periodic ruling when $d = d'$. This comparison shows that the aperiodic ruling extends the values of L/d up to 14. More research in this direction is in process.

8. Conclusions

The diffraction of Hermite-Gaussian beams by Ronchi and aperiodic rulings was studied by means of the transmitted power and the normally diffracted energy. We have shown that the two methods for determining the width of Gaussian beams with Ronchi rulings previously used in Ref. 1 cannot be applied to Hermite-Gaussian beams. The main contribution of this paper is to show that the normally diffracted energy can be used to determine the width of Gaussian beams with aperiodic rulings.

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