On the decay of an accelerated proton

A. Queijeiro
Departamento de Física, Escuela Superior de Física y Matemáticas,
Edificio 9, Unidad Profesional Adolfo López Mateos,
Instituto Politécnico Nacional Ciudad de México 07738, México.
e-mail: aquei@esfm.ipn.mx

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We compute the decay rate width of the strong decay $p \to n + \pi^+$ for a linearly accelerated proton in both the inertial frame and in the coaccelerated proton frame. In this last reference system we use the Unruh effect, where the proton sees a bath of thermal particles at the temperature $T = a/2\pi$, where a is proton's acceleration. Analytical results agree, thus giving a simpler example where the Unruh effect is necessary to keep the consistency between inertial and Rindler frame calculations of a physical observable.

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It is known that the lifetime of a particle can be manipulated by exposing it to a large acceleration. For instance, among the various decays studied in the literature, the most interesting is the weak decay of a proton with acceleration a,

$$p \to n + e^+ + \nu_e, \tag{1}$$

which is forbidden for an inertial proton. This problem was first considered by Müller in a toy model [1], assuming all particles involved are scalars. Then, Matsas and Vanzella [2], within a semiclassical approach in a two-dimensional spacetime, took the $e^\pm$ and $\overset{(-)}{\nu}_e$ as Dirac particles, and the p and n as a classical current . This last assumption is suitable as far the nucleons are energetic enough to have a well defined trajectory. Besides doing this they computed the total decay rate in the coaccelerated proton frame, where according to the Unruh effect [3] the Minkowski vacuum corresponds to a thermal state of Rindler particles at the Unruh temperature $T = a/2\pi$, from where the particles e^- and $\overline{\nu}_e$ (or both) are absorbed by the proton to transform into a neutron and emitting a ν_e and e^+ (or none), respectively. The decay rates in both reference frames were shown to agree by a numerical computation. Later, Suzuki and Yamada [4] did the same calculation analytically and in a 4-dimensional spacetime, confirming the result in [2].

Another interesting decay for an accelerated proton are the strong processes

$$\begin{aligned} &p \to p + \pi^0, \\ &p \to n + \pi^+ \end{aligned} \tag{2}$$

when the proton is in the presence of a very intense magnetic field [5], or in circular motion under the influence of gravitational fields [5]. In these works the emphasis was the study of emission of cosmic and gamma rays from compact stellar objects associated with strong magnetic fields.

In the present work we revisited the strong decay $p \to n + \pi^+$ for a linearly uniformly accelerated proton, under the same set of assumptions as in [2]: (i) the nucleons constitute a two level quantum system described by a semiclassical current, (ii) the neutron velocity does not change with respect to the proton, the so-called no-recoil condition, and (iii) that proton acceleration $a \ll M_1$, M_2,where $M_1$and M_2 are proton and neutron masses, respectively. We calculate the life-time of the proton in the inertial system of reference and in the non-inertial proton reference system, and we will show that the life-time is the same in both frames. The calculation turns out to be simpler than the one in [2], since no Dirac spinors and $\gamma-$matrices in curved spacetime are involved. The only ingredient required is the solution of the Klein-Gordon equation in the accelerated frame. In what follows we present our calculations, first in the inertial frame and then in the non-inertial one, where the proton can absorb a pion present in the thermal bath [6]. We follow the procedure developed in [2] and [4].

We consider motion in one spatial dimension and constant acceleration in the z-direction. In terms of the Rindler coordinates, the path of the proton is given by

$$u = 1/a \tag{3}$$

The participation of a scalar particle requires to take as a semiclassical baryonic current the expression

$$j(x) = \widehat{q}(\tau)\delta(u - a^{-1}) \tag{4}$$

where $\widehat{q}(\tau)$ is an Hermitian operator [7], with $\widehat{q}(\tau) = e^{i\widehat{H}_0\tau}q_0e^{-i\widehat{H}_0\tau}$, and $\widehat{H}_0$ the proper Hamiltonian of the proton-neutron system. This current is suitable for describing the strong conversion of a proton into a neutron, assuming that the back reaction on the neutron is negligible. In this sense the model consider the pair of nucleons as excited and unexcited states of the nucleon two-level system. They are eigenstates of the Hamiltonian $\widehat{H}_0$ with eigenvalues M_1 and M_2, the masses of the proton and neutron, respectively. The corresponding interaction is given by

$$S_I = \int j(x)[\Phi^\dagger(x) + \Phi(x)]d^2x \tag{5}$$

where the scalar field $\Phi(x)$ is described by annihilation $\widehat{a}_k$ and creation $\widehat{c}_k$ operators

$$\Phi(x) = \int d^3k[\widehat{a}_k\phi_k^{(+\omega)}(x) + \widehat{c}_k\phi_{-k}^{(-\omega)}(x) \quad (6)$$

Then, the transition amplitude, from proton state $|p_1\rangle$ to neutron state $|p_2\rangle$, is

$$A_k = \langle p_2 \,| * \langle g_k |\, S_I \,|0\rangle * |\, p_1\rangle \quad (7)$$

and the differential transition rate is given as

$$\frac{dP}{dk} = |A_k|^2 \quad (8)$$

After substitution of (5) and (6) into (7) we obtain

$$A_k = g_{ef}a \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dz \frac{e^{i\Delta M\tau}}{\sqrt{1+a^2t^2}} \times \langle g_k \mid \Phi(x) \mid 0\rangle \delta(z - \sqrt{1+a^2t^2})$$

where $\Delta M = M_1 - M_2$, and τ is proper time of the proton, and $g_{ef} = |\langle p\,|\widehat{q}(0)|\,n\rangle|$ is the effective strong coupling constant. The integral over z is immediate, and using $dt = \gamma d\tau$, where

$$\gamma = (1 - v^2)^{-1/2} \quad (9)$$

with v being proton's velocity

$$v = \frac{at}{\sqrt{1+a^2t^2}}, \quad (10)$$

we obtain

$$A_k = \frac{g_{ef}}{(2\pi)^{1/2}\sqrt{2\omega}} \int_{-\infty}^{\infty} d\tau e^{i\Delta M\tau + ik\cdot x} \quad (11)$$

In terms of the other Rindler coordinate $\mathrm{v} = a\tau$, we write $k\cdot x = \widetilde{\omega}\sinh \mathrm{v} - \widetilde{k}\cosh \mathrm{v}$, where $\widetilde{\omega} = \omega/a$ and $\widetilde{k} = k/a$; then (8) is given by

$$\frac{dP}{dk} = |A_k|^2 = \frac{g_{ef}^2}{2\pi}\frac{1}{2\omega a^2} \int_{-\infty}^{\infty} dv \int_{-\infty}^{\infty} dv' \times e^{i\Delta\widetilde{M}(v-v') + i[\widetilde{\omega}(\sinh v - \sinh v') - \widetilde{k}(\cosh v - \cosh v')]} \quad (12)$$

where $\Delta\widetilde{M} = (M_1 - M_2)/a$. A change in the integration variables gives

$$\frac{dP}{d\widetilde{k}} = \frac{g_{ef}^2}{2\pi}\frac{1}{\widetilde{\omega}} \int_{-\infty}^{\infty} d\sigma \int_{-\infty}^{\infty} ds \times e^{i\Delta\widetilde{M}\sigma + 2i(\widetilde{\omega}\cosh s - \widetilde{k}\sinh s')\sinh\sigma} \quad (13)$$

Next, we perform a rotation

$$\widetilde{\omega}' = \widetilde{\omega}\cosh s - \widetilde{k}\sinh s,$$
$$\widetilde{k} = -\widetilde{\omega}\sinh s - \widetilde{k}\cosh s \quad (14)$$

which gives

$$\frac{1}{\mathfrak{T}}\frac{dP}{d\widetilde{k}'} = \frac{g_{ef}^2}{2\pi}\frac{1}{\widetilde{\omega}' a^2} \int_{-\infty}^{\infty} d\sigma \times e^{2i(\Delta\widetilde{M}\sigma + \widetilde{\omega}'\sinh\sigma)} \quad (15)$$

and

$$\mathfrak{T} = \int_{-\infty}^{\infty} ds$$

is the total proper time of proton. At this point the integral can be evaluated giving

$$\frac{1}{\mathfrak{T}}\frac{dP}{d\widetilde{k}'} = \frac{g_{ef}^2}{2\pi a^2} e^{-\pi\Delta\widetilde{M}} \frac{1}{\widetilde{\omega}'} K_{2i\Delta\widetilde{M}}(2\widetilde{\omega}'). \quad (16)$$

Here $K_{2i\Delta\widetilde{M}}(2\widetilde{\omega}')$ is the modified Bessel function of index $2i\Delta\widetilde{M}$. Total transition rate is obtained integrating, using the Mathematica program, over pion energy,

$$\Gamma = \frac{g_{ef}^2}{2\pi a^2} e^{-\pi\Delta\widetilde{M}} \int_{\widetilde{m}}^{\infty} \sqrt{\widetilde{\omega}'^2 - \widetilde{m}^2} K_{2i\Delta\widetilde{M}}(2\widetilde{\omega}') d\widetilde{\omega}'$$
$$= \frac{g_{ef}^2\,\widetilde{m}}{8\pi^{1/2}a^2} e^{-\pi\Delta\widetilde{M}} G_{1,3}^{3,0} \times \left(\widetilde{m}^2 \,\middle|\, \begin{matrix} 0 \\ -\frac{1}{2}, -\frac{1}{2} + i\Delta\widetilde{M}, -\frac{1}{2} - i\Delta\widetilde{M} \end{matrix}\right) \quad (17)$$

where $\widetilde{m} = m/a$, with m the mass of the pion. An identity for the Meijer function $G_{1,3}^{3,0}$ in terms of the modified Bassel function [8] gives

$$\Gamma = \frac{g_{ef}^2}{4\pi a^2} e^{-\pi\Delta\widetilde{M}} \left[K_{i\Delta\widetilde{M}}(\widetilde{\omega})\right]^2 \quad (19)$$

Now, from the point of view of a uniformly accelerating particle, empty space contains a gas of particles at a temperature proportional to acceleration. In the accelerated proton reference system, the decay process is seen as one in which the proton captures, from the particle bath he sees, a pion and then turning into a neutron: $p + \pi^- \to n$. The necessary temperature for having pions in the bath is greater than $10^{12}K$, corresponding to an acceleration a of order $10^{31}g$, where g is the standard gravitational acceleration, and since proton mass m_p is equivalent to an acceleration of $10^{32}g$, we still have $a/m_p < 1$.

The transition rate is computed in the following way. First, amplitude is given by

$$A_k = g_{ef} \int d^2x e^{i\Delta M\tau} \phi_k^{\omega}(x^0, x^3). \quad (20)$$

Here $\phi_k^\omega(x^0, x^3)$ is the pion function, solution of the Klein-Gordon equation in the accelerated frame of reference (by using the equivalence principle: a uniform acceleration is equivalent to a constant gravitational field, where the components of the metric tensor are given in terms of proton's acceleration [9]),

$$\phi_k^\omega(x^0, x^3) = \frac{1}{\sqrt{2\pi}} a^{-1/2} e^{-i\omega\tau} \times \sinh^{1/2}(\pi\widetilde{\omega}) K_{i\widetilde{\omega}}(\widetilde{m}). \quad (18)$$

where $K_{i\widetilde{\omega}}(\widetilde{m})$ is the modified Bessel function of index $i\widetilde{\omega}$ and argument $\widetilde{m} = m/a$, with m the mass of the pion. Then,

$$A_k = \frac{g_{ef}}{\sqrt{2\pi} a^{3/2}} \sinh^{1/2}(\pi\widetilde{\omega}) K_{i\widetilde{\omega}}(\widetilde{m}) \delta(\widetilde{\omega} - \Delta\widetilde{M}) \quad (19)$$

The differential transition rate is given by

$$\frac{1}{\mathfrak{T}} \frac{dP}{d\widetilde{\omega}} = |A_k|^2 \, n_B(\omega). \quad (20)$$

where $n_B(\omega)$ is the bosonic thermal factor associated with the thermal bath and, as before, $\mathfrak{T}$ is the proton proper time. According to Unruh [7], bath temperature, in units where Boltzmann constant k_B, reduced Planck constant $\hbar$ and speed of light c all take value one, is

$$T = \frac{a}{2\pi} \quad (21)$$

Substitution of $n_B(\omega) = (e^{\omega/T} - 1)^{-1}$ with this temperature, gives

$$\frac{1}{\mathfrak{T}} \frac{dP}{d\widetilde{\omega}} = \frac{g_{ef}^2}{4\pi a^2} e^{-\pi\widetilde{\omega}} \left|K_{i\widetilde{\omega}}(\widetilde{m})\right|^2 \delta(\widetilde{\omega} - \Delta\widetilde{M}) \quad (22)$$

Now, we integrate over pion energy to obtain finally the transition rate in the non-inertial reference frame

$$\Gamma_{ni} = \frac{g_{ef}^2}{4\pi a^2} \int_{\widetilde{m}}^{\infty} e^{-\pi\widetilde{\omega}} \left|K_{i\widetilde{\omega}}(\widetilde{m})\right|^2 \delta(\widetilde{\omega} - \Delta\widetilde{M}) d\widetilde{\omega}$$
$$= \frac{g_{ef}^2}{4\pi a^2} e^{-\pi\Delta\widetilde{M}} \left|K_{i\Delta\widetilde{M}}(\widetilde{m})\right|^2. \quad (23)$$

In summary, we have computed the transition width for the strong decay $p \to n + \pi^+$, for the uniformly accelerated proton, in an inertial frame and in proton's frame. The analytical results Eq. (19) and Eq. (26) respectively, shows explicitly that they agree, providing a further theoretical check for the Unruh effect. The point of view in this work, following [2], is that the Unruh effect is essential to obtain the proper decay rate in the uniformly accelerated frame.

The result in Eq. (19) can be compared to the calculation made by Ren and Weinberg [10] for emission from an accelerated scalar source. In this case $\Delta\widetilde{M} = 0$, then Eq. (19) reduces to

$$\Gamma = \frac{g_{ef}^2}{4\pi a^2} \left[K_0(\widetilde{\omega})\right]^2$$

which is of the same form than their result (3.17) for the total emission probability, with the effective coupling g_{ef} replaced by the corresponding coupling constant q and recalling that we work in a two-dimensional spacetime.

Another interesting case is the same decay for protons in circular motion under the influence of an intense gravitational field, as the one considered by Fregolente *et al* [11]. However, as was demonstrated by Letaw and Pfautsch [12] the spectrum of vacuum fluctuations in the non-inertial frame, is composed by a thermal energy plus a non-thermal contribution arising from the observer's acceleration, which make the calculation quite complicated. This latter task is under current investigation.

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