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Linearized five dimensional Kaluza-Klein theory as a gauge theory

G. Atondo-Rubio, J.A. Nieto*, L. Ruiz, and J. Silvas
Facultad de Ciencias Físico-Matemáticas, Universidad Autónoma de Sinaloa,
80000, Culiacán, Sinaloa, México,
**e-mail: nieto@uas.uasnet.mx*

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We develop a linearized five-dimensional Kaluza-Klein theory as a gauge theory. By perturbing the metric around flat and de Sitter backgrounds, we first discuss linearized gravity as a gauge theory in any dimension. In the particular case of five dimensions, we show that in using the Kaluza-Klein mechanism, the field equations of our approach imply both linearized gauge gravity and Maxwell theory in flat and de Sitter scenarios. As a possible further development of our formalism, we also discuss an application in the context of gravitational polarization scattering by means of the analogue of the Mueller matrix in optical polarization algebra. We argue that this application can be of particular interest in gravitational wave experiments.

Keywords: Kaluza-Klein theory; linearized gravity; Mueller matrix

Desarrollamos una teoría Kaluza-Klein linealizada en cinco dimensiones como una teoría de norma. Discutimos primero gravedad linealizada como una teoría de norma en cualquier dimensión perturbando las métricas de fondo plana y la de de Sitter. Demostramos que usando el mecanismo de Kaluza-Klein en el caso particular de cinco dimensiones, las ecuaciones de campo de nuestra aproximación implican tanto gravedad linealizada de norma como la teoría de Maxwell en escenarios planos y de de Sitter. Como otro posible desarrollo de nuestro formalismo, también discutimos una aplicación en el contexto de dispersión gravitacional polarizada, por medio de una matriz análoga a la de Mueller usada en el algebra de polarización óptica. Argumentamos que esta aplicación puede ser de interés particular en los experimentos de ondas gravitacionales.

Descriptores: Teoría de Kaluza-Klein; gravedad linealizada; matriz de Mueller.

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1. Introduction

It is known that linearized gravity can be considered to be a gauge theory [1]. In this context, one may be interested in the idea of a unified theory of linearized gravity and Maxwell theory. This idea is, however, not completely new since in fact the quest for a unified theory of gravity and electromagnetism has a long history [2]. One can mention, for instance, the five-dimensional Kaluza-Klein theory [3], which is perhaps one of the most interesting proposals. The central idea in this case is to incorporate electromagnetism into a geometrical five-dimensional gravitational scenario. The gauge properties arise as a result of broken general covariance via a mechanism called "spontaneous compactification". Symbolically, one may describe this process through the transition $M^5 \to M^4 \times S^1$, where M^5 and M^4 are five- and four-dimensional manifolds respectively and S^1 is a circle. Thus, after compactification the fiber-bundle $M^4 \times S^1$ describes the Kaluza-Klein scenario. Let us picture this attempt of unification as

$$em \to g, \tag{1}$$

where em means electromagnetism and g gravity.

In the case of linearized gravity theory, the scenario looks different because it can be understood as a gauge theory rather than a pure geometrical structure. Therefore, a unified theory in this case may be understood as an idea for incorporating linearized gravity in a Maxwell gauge context. In other words, one may start from the beginning with a generalized fiber-bundle $M^4 \times B$, with B as a properly chosen compact space. Thus, we have that this case can be summarized by the heuristic picture

$$em \leftarrow g. \tag{2}$$

Our aim in this paper is to combine the two scenarios (1) and (2) in the form

$$em \longleftrightarrow g. \tag{3}$$

Specifically, we start with linearized gravity in five-dimensions and apply the Kaluza-Klein compactification mechanism. We probe that the resultant theory can be understood as a gauge theory of linearized gravity in five dimensions. Furthermore, we show that, by using this strategy, one can derive an unified theory of gravity and electromagnetism with a generalized gauge field strength structure. As an advantage of our formalism, we outline the possibility that optical techniques can be applied to both gravity and electromagnetic radiation in a unified context. Thus, we argue that our results may be of particular interest in the detection of gravitational waves.

Technically this article is organized as follows. In Sec. 2, we develop linearized gravity in any dimension. In Sec. 3, we discuss linearized gravity in a 5-dimensional Kaluza-Klein theory and in Sec. 4 we generalize our procedure to a de Sitter background. In Sec. 5, we outline a possible application of our formalism of unified framework of electromagnetic and gravitational radiation to an optical geometry via the Mueller matrix. In Appendix A we present a generalization to any dimension of the Novello and Neves work [4].

2. Linearized gravity in any dimension

Let us consider a $1+d$-dimensional manifold M^{1+d}, with the associated metric $\gamma_{AB}(x^C)$. We shall assume that γ_{AB} can be written in the form

$$\gamma_{AB} = \eta_{AB} + h_{AB}, \tag{4}$$

where $\eta_{AB} = diag(-1, 1, \ldots, 1)$ and $h_{AB}(x^C)$ is a small perturbation, that is

$$|h_{AB}| \ll 1. \tag{5}$$

To first order in h_{AB}, the inverse of the metric γ_{AB} becomes

$$\gamma^{AB} = \eta^{AB} - h^{AB}. \tag{6}$$

Using (4) and (6) we find that the Christoffel symbols and Riemann curvature tensor are

$$\Gamma^A_{CD} = \frac{1}{2}\eta^{AB}(h_{BC,D} + h_{BD,C} - h_{CD,B}) \tag{7}$$

and

$$R_{ABCD} = \partial_A \mathcal{F}_{CDB} - \partial_B \mathcal{F}_{CDA}, \tag{8}$$

respectively. Here, the symbol $\mathcal{F}_{CDB}$ means

$$\mathcal{F}_{CDB} = \frac{1}{2}(h_{BC,D} - h_{BD,C}). \tag{9}$$

Observe that $\mathcal{F}_{CDB}$ is antisymmetric in the indices C and D.

In terms of the quantity $\mathcal{F}_A$ defined by

$$\mathcal{F}_A = \eta^{CB}\mathcal{F}_{ACB}, \tag{10}$$

and the symbol $\mathcal{F}_{ADB}$, the Ricci tensor R_{BD} reads

$$R_{BD} = \partial^A \mathcal{F}_{ADB} + \partial_B \mathcal{F}_D. \tag{11}$$

Thus, we find that the Ricci scalar R is given by

$$R = 2\partial^A \mathcal{F}_A. \tag{12}$$

Substituting (11) and (12) into the Einstein weak field equations in $1+d$ dimensions

$$R_{BD} - \frac{1}{2}\eta_{BD}R = \frac{8\pi G_{1+d}}{c^2}T_{BD}, \tag{13}$$

we find

$$\partial^A \mathcal{F}_{ADB} + \partial_B \mathcal{F}_D - \eta_{BD}\partial^A \mathcal{F}_A = \frac{8\pi G_{1+d}}{c^2}T_{BD}, \tag{14}$$

where G_{1+d} is the Newton gravitational constant in $1+d$ dimensions.

Let us now define the symbol

$$F_{ADB} \equiv \mathcal{F}_{ADB} + \eta_{BA}\mathcal{F}_D - \eta_{BD}\mathcal{F}_A, \tag{15}$$

which has the property

$$F_{ADB} = -F_{DAB}. \tag{16}$$

Thus, by using expression (15) we find that field equations (14) are simplified in the form

$$\partial^A F_{ABD} = \frac{8\pi G_{1+d}}{c^2}T_{BD}. \tag{17}$$

Since $\partial^A F_{ADB} = \partial^A F_{ABD}$, field equations (17) can also be written as

$$\partial^A F_{A(BD)} = \frac{16\pi G_{1+d}}{c^2}T_{BD}, \tag{18}$$

where the bracket (BD) means symmetrization of the indices B and D. It is worth mentioning that in a $1+3$-dimensional spacetime field equations (18) are reduced to those proposed by Novello and Neves [4].

Furthermore, it is not difficult to show that field equations (17) can be derived from the action

$$S = \int d^{1+d}x\{\mathcal{F}^{ADB}\mathcal{F}_{ADB} - 2\mathcal{F}^A\mathcal{F}_A - L_{matter}\}, \tag{19}$$

where L_{matter} denotes a Lagrangian associated with matter fields. In fact, by varying action (19) with respect to h_{AB} and and assuming

$$\frac{\delta L_{matter}}{\delta h_{BD}} = \frac{32\pi G_{1+d}}{c^2}T^{BD},$$

one obtains the field equations (17).

3. Linearized gravity in a five-dimensional Kaluza-Klein theory

In a 5-dimensional spacetime the weak field metric tensor $\gamma_{AB} = \eta_{AB} + h_{AB}$, where $\eta_{AB} = diag(-1, 1, 1, 1, 1)$, can be written in the block-matrix form

$$\gamma_{AB} = \begin{pmatrix} \eta_{\mu\nu} + h_{\mu\nu} & h_{4\mu} \\ h_{4\nu} & 1 + h_{44} \end{pmatrix}, \tag{20}$$

with $\mu, \nu = 0, 1, 2, 3$. If one adopts the Kaluza-Klein ansatz, with

$$h_{\mu\nu} = h_{\mu\nu}(x^\alpha), \tag{21}$$

$$h_{4\mu} = A_\mu(x^\alpha) \tag{22}$$

and

$$h_{44} = 0, \tag{23}$$

where $A_\mu(x^\alpha)$ is identified with the electromagnetic potential, we discover that the only nonvanishing terms of $\mathcal{F}_{DAB}$ are

$$\mathcal{F}_{\mu\nu\alpha} = \frac{1}{2}(h_{\alpha\mu,\nu} - h_{\alpha\nu,\mu}), \tag{24}$$

$$\mathcal{F}_{4\nu\alpha} = \frac{1}{2}\partial_\nu A_\alpha, \tag{25}$$

and

$$\mathcal{F}_{\mu\nu 4} = -\frac{1}{2}F_{\mu\nu}, \tag{26}$$

where $F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}$ is the electromagnetic field strength. Thus, from (15) we find that the nonvanishing components of F_{DAB} are

$$F_{\mu\nu\alpha} = \mathcal{F}_{\mu\nu\alpha} + \eta_{\alpha\mu}\mathcal{F}_\nu - \eta_{\alpha\nu}\mathcal{F}_\mu, \tag{27}$$

$$F_{4\nu\alpha} = \frac{1}{2}(\partial_\nu A_\alpha - \eta_{\alpha\nu}\partial^\beta A_\beta), \tag{28}$$

$$F_{\mu\nu 4} = -\frac{1}{2}F_{\mu\nu}, \tag{29}$$

and

$$F_{\mu 44} = -\mathcal{F}_\mu. \tag{30}$$

Now, since all fields are independent of the coordinate x^4, we see that field equations (17) can be written as

$$\partial^\mu F_{\mu BD} = \frac{8\pi G_{1+4}}{c^2}T_{BD}. \tag{31}$$

These field equations can be separated as follows:

$$\partial^\mu F_{\mu\nu\alpha} = \frac{8\pi G_{1+4}}{c^2}T_{\nu\alpha}, \tag{32}$$

$$\partial^\mu F_{\mu 4\nu} = \frac{8\pi G_{1+4}}{c^2}T_{4\nu}, \tag{33}$$

$$\partial^\mu F_{\mu\nu 4} = \frac{8\pi G_{1+4}}{c^2}T_{\nu 4}, \tag{34}$$

and

$$\partial^\mu F_{\mu 44} = \frac{8\pi G_{1+4}}{c^2}T_{44}. \tag{35}$$

Using (28) and (29), we find that (33) and (34) lead to exactly the same field equations, namely

$$\partial^\mu F_{\nu\mu} = 4\pi J_\nu, \tag{36}$$

where $J_\nu=(4G_{1+4}/c^2)T_{\nu 4}$ is the electromagnetic current. Of course, field equations (32) and (36) correspond to linearized gravity and Maxwell field equations, respectively. If we set $T_{44} = 0$, then one can see that field equation (35) is a pure gauge expression. In fact, if one assumes the transverse traceless gauge in five dimensions

$$\begin{aligned} h^{AB}{}_{,B} &= 0, \\ h = \eta^{AB}h_{AB} &= 0, \end{aligned} \tag{37}$$

one discovers that $F_{\mu 44}$ is identically equal to zero.

For completeness let us observe that (37) leads to

$$\begin{aligned} h^{\mu\nu}{}_{,\nu} &= 0, \\ h = \eta^{\mu\nu}h_{\mu\nu} &= 0, \end{aligned} \tag{38}$$

and the Lorentz gauge for A_μ

$$A^\mu{}_{,\mu} = 0. \tag{39}$$

Consequently one finds that, in the gauge (38) and (39), the field equations (32) and (36) are reduced to

$$\Box^2 h_{\mu\nu} = -\frac{16\pi G}{c^2}T_{\mu\nu} \tag{40}$$

and

$$\Box^2 A^\mu = -4\pi J^\mu, \tag{41}$$

respectively, where $\Box^2 = \eta^{\mu\nu}\partial_\mu\partial_\nu$ is the d'Alembertian operator. Thus, we have found a framework in which the gravitational and electromagnetic waves can be treated on the same footing.

4. The de Sitter generalization

In order to generalize the formalism described in the previous section to a de Sitter scenario, we shall replace the flat metric $\eta_{\mu\nu}$ by the de Sitter metric $f_{\mu\nu}(x^\alpha)$. In this case the perturbed Kaluza-Klein metric γ_{AB} takes the form

$$\gamma_{AB} = \begin{pmatrix} f_{\mu\nu} + h_{\mu\nu} & A_\mu \\ A_\nu & 1 \end{pmatrix}, \tag{42}$$

whose inverse is given by

$$\gamma^{AB} = \begin{pmatrix} f^{\mu\nu} - h^{\mu\nu} & -A^\mu \\ -A^\nu & 1 \end{pmatrix}. \tag{43}$$

By combining the results from Sec. 2 and Appendix A it is not difficult to obtain the generalized field equations

$$D^A F_{ABD} - \frac{3}{2l^2}(h_{BD} - hf_{BD}) = \frac{8\pi G_{1+d}}{c^2}T_{BD}. \tag{44}$$

Thus, using (42) and considering that in five dimensions $d = 4$ and $\Lambda = 6/l^2$ we find

$$D^\alpha F_{\alpha\mu\nu} - \frac{\Lambda}{4}(h_{\mu\nu} - hf_{\mu\nu}) = \frac{8\pi G_{1+4}}{c^2}T_{\mu\nu}, \tag{45}$$

and

$$D^\nu F_{\mu\nu} - \frac{\Lambda}{4}A_\mu = 4\pi J_\mu. \tag{46}$$

Here, we have used the fact that $D^4 F_{4AB} = 0$ and $f_{4D} = 0$. Observe that in this case the electromagnetic field strength $F_{\mu\nu}$ is given by

$$F_{\mu\nu} = D_\mu A_\nu - D_\nu A_\mu.$$

The field equations (45) and (46) are remarkable because if we set

$$m^2 = \frac{\Lambda}{4}, \tag{47}$$

one discovers that, up to factors, both the graviton and the photon have the same mass m and even more intriguing is the fact that such a mass is proportional to the square root of the cosmological constant.

5. Final remarks

The present work may have a number of interesting developments. In particular, as Novello and Neves [4] have shown, Eq. (15) can be derived from the formula

$$\partial^{A*} F_{A(BD)} = 0, \tag{48}$$

where

$${}^*F^{A(BD)} = \varepsilon^{ABEF} F^D_{EF} + \varepsilon^{ADEF} F^B_{EF}, \tag{49}$$

with ε^{ABEF} the completely antisymmetric symbol. Thus, one may consider an alternative approach [5] for duality aspects of linearized gravity [6] (see Refs. 7 to 11) as in the case of Maxwell theory (see Refs. 12 and 13).

Another source of physical interest of the present formalism is a possible connection with the Randall-Sundrum brane world scenario [14,15], with gravitational wave formalism (see Refs. 16 and references therein) and with quantum linearized gravity [17,18]. Moreover, our work may also be useful in clarifying some aspects regarding the relation between the mass of the graviton and the cosmological constant, which has been the subject of some controversy [19,20].

Aside from theoretical developments, the present work also opens the possibility of making a number of applications of linearized gravity arising from the Maxwell theory itself. Let us outline just one possibility. In Maxwell theory the concept of polarization scattering is of considerable interest in optical physics (see Ref. 21 and references therein). The subject of interest in this arena is to describe the interaction of polarized waves with a target in a complex setting. It turns out that the useful mathematical tool in the scattering radiation process is the so-called Mueller matrix [22] (see also Ref. 23 and references therein). What appears interesting about such a matrix is that its elements refer to intensity measurements only. Let us recall the main ideas for the construction of the Mueller matrix.

A polarized radiation field may be represented by a 2-dimensional complex field

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix}. \tag{50}$$

From the components of this vector, one may define the Hermitian coherency matrix

$$\mathbf{C} = \begin{pmatrix} E_x E_x^* & E_x E_y^* \\ E_x^* E_y & E_y E_y^* \end{pmatrix} \tag{51}$$

and the vector

$$\mathbf{g} = \begin{pmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{pmatrix}, \tag{52}$$

where

$$g_0 = E_x E_x^* + E_y E_y^*, \tag{53}$$

$$g_1 = E_x E_x^* - E_y E_y^*, \tag{54}$$

$$g_2 = E_x^* E_y + E_x E_y^*, \tag{55}$$

and

$$g_3 = i(E_x^* E_y - E_x E_y^*). \tag{56}$$

The Jones matrix J and the Mueller matrix M apply to $\mathbf{C}$ and the vector $\mathbf{g}$, respectively, as follows:

$$\mathbf{C}' = \mathbf{JC} \tag{57}$$

and

$$\mathbf{g}' = \mathbf{Mg}. \tag{58}$$

It turns out that $\mathbf{J}$ can be identified with a SU(2) matrix, while $\mathbf{M}$ is a 4×4 augmented matrix form of O_3. Of course the matrices $\mathbf{J}$ and $\mathbf{M}$ must be related:

$$M = \frac{1}{2} Tr J^{*T} \sigma J \sigma, \tag{59}$$

where σ denotes the four Pauli matrices (see Ref. 22 for details).

Let us make the identification

$$F_{0i\alpha} = E_{i\alpha}. \tag{60}$$

Here $F_{0i\alpha}$ denotes some of the components of F_{ADB} according to the expression (15). The idea is now to consider generalization of (50)

$$\begin{pmatrix} E_{x\alpha} \\ E_{y\alpha} \end{pmatrix} \tag{61}$$

and to consider the analogue of (53)-(56), namely

$$G_0 = E_x^\alpha E_{x\alpha}^* + E_y^\alpha E_{y\alpha}^*, \tag{62}$$

$$G_1 = E_x^\alpha E_{x\alpha}^* - E_y^\alpha E_{y\alpha}^*, \tag{63}$$

$$G_2 = E_x^{*\alpha} E_{y\alpha} + E_x^\alpha E_{y\alpha}^* \tag{64}$$

and

$$G_3 = i(E_x^{*\alpha} E_{y\alpha} - E_x^\alpha E_{y\alpha}^*). \tag{65}$$

Thus, we can apply the Mueller matrix M as in (58): $\mathbf{G}' = \mathbf{MG}$. In fact, since $\mathbf{M}$ contains the information of the intensities of both gravitational and electromagnetic radiation via the quantity $F_{0i\alpha}$ one should expect a broad range of applications of $\mathbf{M}$ in a gravitational wave context.

Appendix A

In this appendix we shall study the linearized gravity with f_{AB} as a metric background. Consider the perturbed metric

$$\gamma_{AB} = f_{AB} + h_{AB}, \tag{66}$$

where h_{AB} is a small perturbation, that is $|h_{AB}| \ll 1$. To first order in h_{AB} one finds

$$\gamma^{AB} = f^{AB} - h^{AB}. \tag{67}$$

The Christoffel symbols can be written in the interesting form

$$\Gamma^A_{CD} = \Omega^A_{CD} + H^A_{CD}, \tag{68}$$

where Ω^A_{CD} are the Christoffel symbols associated with f_{AB} and H^A_{CD} is defined by

$$H^A_{CD} = \frac{1}{2} f^{AE}(D_C h_{DE} + D_D h_{CE} - D_E h_{CD}). \tag{69}$$

Here, the symbol D_A denotes covariant derivatives with Ω^A_{CD} as a connection.

By using (68) it is straightforward to check that the Riemann tensor can be written in the form

$$R^A_{BCD} = \mathcal{R}^A_{BCD} + D_C H^A_{BD} - D_D H^A_{BC}, \tag{70}$$

where $\mathcal{R}^A_{BCD}$ is the Riemann tensor associated with Ω^A_{CD}.

With the help of the commutation relation

$$D_C D_D h_{AB} - D_D D_C h_{AB} = -\mathcal{R}^E_{ACD} h_{EB} - \mathcal{R}^E_{BCD} h_{AE} \tag{71}$$

we find that the Riemann curvature tensor (70) can also be written as

$$\begin{aligned} R_{ABCD} &= D_A \mathcal{F}_{CDB} - D_B \mathcal{F}_{CDA} + \mathcal{R}_{ABCD} \\ &+ \frac{1}{2}\mathcal{R}^E_{DAB} h_{EC} - \frac{1}{2}\mathcal{R}^E_{CAB} h_{ED}. \end{aligned} \tag{72}$$

Here, the symbol $\mathcal{F}_{CDB}$ takes the form

$$\mathcal{F}_{CDB} = \frac{1}{2}(D_D h_{BC} - D_C h_{BD}). \tag{73}$$

Observe that $\mathcal{F}_{CDB}$ can be obtained from (9) by replacing ordinary partial derivatives by covariant derivatives. From (72) we get the Ricci tensor

$$\begin{aligned} R_{BD} &= D^A \mathcal{F}_{ADB} + D_B \mathcal{F}_D + \mathcal{R}_{BD} \\ &- \frac{1}{2}\mathcal{R}_{ABCD} h^{AC} + \frac{1}{2}\mathcal{R}_{EB} h^E_D \end{aligned} \tag{74}$$

where

$$\mathcal{F}_A = f^{CB}\mathcal{F}_{ACB}. \tag{75}$$

Thus, we find that the Ricci scalar R is given by

$$R = 2D^A \mathcal{F}_A + \mathcal{R} - \mathcal{R}_{BD} h^{BD}. \tag{76}$$

Substituting (74) and (76) into the Einstein weak field equations in $1 + d$ dimensions, with cosmological constant

$$\begin{aligned} R_{BD} - \frac{1}{2}(f_{BD} + h_{BD})R + (f_{BD} + h_{BD})\Lambda \\ = \frac{8\pi G_{1+d}}{c^2} T_{BD,} \end{aligned} \tag{77}$$

we obtain

$$\begin{aligned} D^A \mathcal{F}_{ADB} + D_B \mathcal{F}_D - f_{BD} D^A \mathcal{F}_A - \frac{1}{2}\mathcal{R}_{ABCD} h^{AC} \\ + \frac{1}{2}\mathcal{R}_{EB} h^E_D + \frac{1}{2} f_{BD}\mathcal{R}_{EF} h^{EF} - \frac{1}{2} h_{BD}\mathcal{R} + h_{BD}\Lambda \\ = \frac{8\pi G_{1+d}}{c^2} T_{BD}. \end{aligned} \tag{78}$$

Here, we used the fact that

$$\mathcal{R}_{BD} - \frac{1}{2} f_{BD}\mathcal{R} + f_{BD}\Lambda = 0. \tag{79}$$

Since we have

$$\mathcal{R}_{ABCD} = \frac{1}{l^2}(f_{AC} f_{BD} - f_{AD} f_{BC}) \tag{80}$$

and

$$\Lambda = \frac{d(d-1)}{2l^2}, \tag{81}$$

we discover that (78) is reduced to

$$\begin{aligned} D^A \mathcal{F}_{ADB} + D_B \mathcal{F}_D - f_{BD} D^A \mathcal{F}_A \\ - \frac{1}{2l^2}(d-1)(h_{BD} - h f_{BD}) = \frac{8\pi G_{1+d}}{c^2} T_{BD}. \end{aligned} \tag{82}$$

Thus, by defining the symbol

$$F_{ADB} \equiv \mathcal{F}_{ADB} + f_{BA}\mathcal{F}_D - f_{BD}\mathcal{F}_A, \tag{83}$$

we find that the field equations (82) are simplified in the form

$$D^A F_{ABD} - \frac{1}{2l^2}(d-1)(h_{BD} - h f_{BD}) = \frac{8\pi G_{1+d}}{c^2} T_{BD}. \tag{84}$$

In four dimensions $d = 3$ and $\Lambda = 3/l^2$. Therefore (84) becomes

$$D^A F_{ABD} - \frac{\Lambda}{3}(h_{BD} - h f_{BD}) = \frac{8\pi G_{1+3}}{c^2} T_{BD}, \tag{85}$$

which are the field equations obtained by Novello and Neves [4].

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