Optical sensing technique for Young's modulus measurements in piezoelectric materials

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In the design of microsensors using microelectromechanical system (MEMS) technology, it is necessary to know the elastic properties of the materials employed in their fabrication. Reliable mechanical properties of the materials are critical to the safety and correct functioning of these microdevices. Mechanical testing of microstructures that are only a few microns thick requires novel techniques and specialized procedures for preparation and handling. In this paper a simplified optic sensing is used to measure the Young's modulus in piezoelectric cantilever. This optical technique was chosen because it is the most appropriate when working with small devices, besides being easily implemented and low cost.

Keywords: Young's Modulus; piezoelectric cantilever.

En el diseño de micro sensores utilizando la tecnología de sistemas micro electromecánicos (MEMS por sus siglas en inglés), es necesario conocer las propiedades mecánicas de los materiales empleados. Las propiedades mecánicas fidedignas son críticas para el funcionamiento correcto y seguro de estos micro dispositivos. Someter a prueba a micro estructuras cuyo grosor es de una cuantas micras requiere técnicas nuevas y procedimientos especializados para su preparación y manejo. En este trabajo se utiliza un sensado óptico simplificado para medir el módulo de Young en trampolines piezoeléctricos. Esta técnica óptica se eligió debido a que es la mas apropiada cuando se trabaja con dispositivos pequeños; además es fácil de instrumentar y de bajo costo.

Descriptores: Módulo de Young; trampolín piezoeléctrico.

PACS: 62.20.de; 81.70Fy; 7.10Cm

1. Introduction

Several techniques have been used to determine the Young's Modulus of thin films. The nanoindentation technique is used particularly for thin film [1], and consists in measuring the force needed to make these small indentations. This technique has also been used to measure small deflections in micro-structures, such as cantilevers, bridges and membranes, made by a micromachining process [2], bulge test technique [3], resonance method [4,5] and others [6]. The resonance frequency method was chosen because of its relative simplicity and also because an optical sensing can be made [7].

Piezoelectric cantilevers were used to simplify the excitation; their dimensions were between 6000 and 12000 microns in length, 1600 microns in width and 600 microns in thickness. We implemented an experimental setup that, in spite of its simplicity, provides the same results that other more expensive methods.

2. Fundamentals

The differential equation of a beam supported at one end (cantilever) is given by Eq. (1). Figure 1 shows a reference to the coordinate planes.

$$EI\frac{\partial^4 z\,(x;t)}{\partial x^4} = -\lambda_m\frac{\partial^2 z\,(x,t)}{\partial t^2} \tag{1}$$

where

$\lambda_m = \rho A-$ Linear mass density

$E-$ Young's Modulus

$I-$ Moment of inertia

$L-$ Length of cantilever

The solution to Eq. (1) can be seen in Ref. 8 but is not discussed in this paper. By splitting Eq. (1) into two equations we find a separation constant:
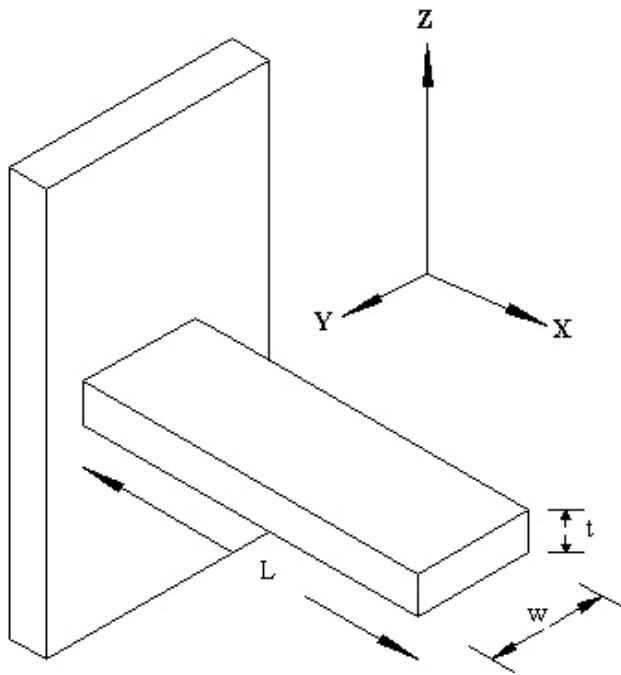
$$k_n^4 = \frac{\omega_n^2 \lambda_m}{EI} \tag{2}$$

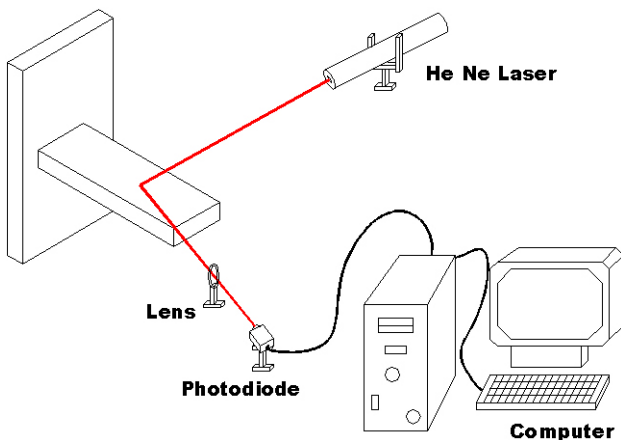FIGURE 1. Cantilever with length L, width w and thickness t.

FIGURE 2. Experimental setup.

To determine k_n we need to solve Eq. (3), which is also obtained from Eq. (1):

$$\cos\left(k_n L\right)\cosh\left(k_n L\right) = -1 \qquad (3)$$

Using a MATLAB-based program [9] we can obtain the solutions to Eq. (3), and for the first vibration mode $k_1 L = 1.875$. Using this and Eq. (2) we have:

$$\omega_1 = \frac{\left(1.875\right)^2}{L^2}\sqrt{\frac{EI}{\lambda_m}} \qquad (4)$$

The moment of inertia of a cantilever with width w and thickness t is:

$$I = \frac{wt^3}{12} \qquad (5)$$

Substituting Eq. (5) into Eq. (4):

$$f_1 = \frac{(1.875)^2 t}{4\pi L^2}\sqrt{\frac{E}{3\rho}} \qquad (6)$$

From Eq. (6) and knowing the cantilever dimensions and density we can determine the Young's Modulus by simply measuring the resonance frequency of the first vibration mode of the cantilever.

3. Experimental setup

There are several ways to detect the vibration of a cantilever, but the most accepted are the optical ones, because of their advantages [7]. Figure 2 shows the experimental setup used in this paper. The Laser beam strikes near the free end of the cantilever. The reflected beam passes through a convergent lens in order to reduce its dispersion and to focus the spot on the active area of the photodiode. In this case, the beam reflection angle is not important because of the way the measurements are taken (only changes in light power are measured, but not the position of the beam spot). Finally, and because the photodiode functions as a current source, a resistor is used to convert the signal as a voltage source. The signal is then acquired with the sound card of a conventional PC.

An InGaAs photodiode with a bandwidth of 800 MHz and a wide sensing area (350 nm) was used as a detector (Perkin Elmer C30618G). As a light source, a HeNe Laser was used, with a 532 nm wavelength and 5 mJ power. In order to achieve a better focus, a lens was used between the cantilever and the photodiode. This experimental array is similar to the one used in Ref. 10, but without the gain/phase analyzer.

The excitation signal was given by the sound card of a personal computer. A program was developed in MATLAB for generating the excitation signal (input of the system), capturing the photodiode signal (output of the system), and for the analysis after the measurement. Two excitation signals were used in this experiment:

a) a sinusoidal signal with a linear frequency sweep (chirp) and

b) random noise (white).

In this program, the user can choose the bandwidth of the signal (initial and final frequency of the sweep), the duration, and sampling frequency. Finally, the input and output data can be saved together with the sampling frequency and the resonance frequency. The PC used was a P4 @ 2.8 GHz with 1.5 GB RAM.

In order to verify the correct operation of the program and the experimental setup, the resonance frequency of a loudspeaker was measured and then compared to the measurement obtained by the conventional method. This comparison showed a difference of less than 1 Hz between the two measurements. Another comparison was made, but this time

between the measurements of the resonance frequency of a sample cantilever with the program and with a spectral analyzer (Bruel & Kjaer mod. 2034), Fig. 3.

4. Measurements

To reduce the error in the measurement of the Young's Modulus, an array of eleven cantilevers with different lengths made up the experimental setup for which it was calculated, instead of calculating it for each cantilever. All the cantilevers were made of the same material. Rewriting Eq. (6) we obtain a proportional relation between the resonance frequency and $1/L^2$, namely:

$$f_1 = m \cdot L^{-2} \tag{7}$$

Plotting the resonance frequency versus $1/L^2$ a line is obtained with a slope:

$$m = \frac{(1.875)^2 t}{4\pi} \left(\frac{E}{3\rho}\right)^{1/2} \tag{8}$$

And so:

$$E = \frac{48\pi^2 \rho}{(1.875)^4 t^2} m^2 \tag{9}$$

The material of the eleven identical cantilevers was PbTiO$_3$, but after mounting them as cantilevers their length changed from 6000 to 12000 nm. Two signals were used for excitation of the cantilevers: the chirp signal (sine wave whose frequency increases at a linear rate with time) and

broadband noise. The duration of both excitation signals was 20 sec. The chirp was made from 1 Hz to 20 kHz. In addition, a spectral analyzer B & K 2034 was employed for comparison purposes.

The resonance frequencies for each cantilever obtained with the two excitation signals and the spectrum analyzer B & K 2034 are shown in Table I. Also, from this data a lineal regression analysis was implemented; Fig. 4 shows an example of this regression (best fit) line. The Young's Modulus was calculated from the slopes of this regression line using Eq. (9). The results are summarized in Table II.
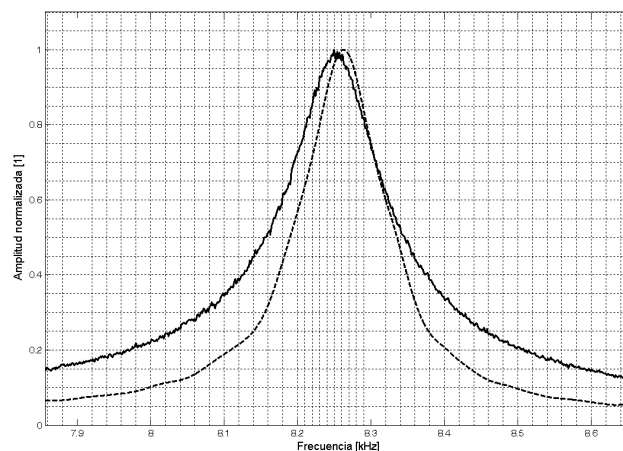

FIGURE 3. Comparison of resonance frequency measurements; with spectral analyzer (continuous curve), with the development program (dot curve).

TABLE I. Resonance frequencies for each method.

Cantilever	Length [μm]	Resonance frequency [Hz]		
		Chirp	Noise	Analyzer
A	12000	1821.1	1837	1829.2
B	10000	2569.3	2619.7	2623
C	12000	1591.1	1632.2	1694.4
D	6000	6949.2	7010.8	7011
E	6000	8198.8	8262.9	8249.6
F	10000	2399.4	2436.3	2430.9
G	8000	4565	4645.4	4641
H	8000	4004.1	4082.3	4096.1
I	8000	4507.3	4554.7	4546.5
J	10000	2450.5	2474.8	2463.2
K	12000	1760.3	1772.2	1761.5

TABLE II. Young's Modulus measured with different excitation methods.

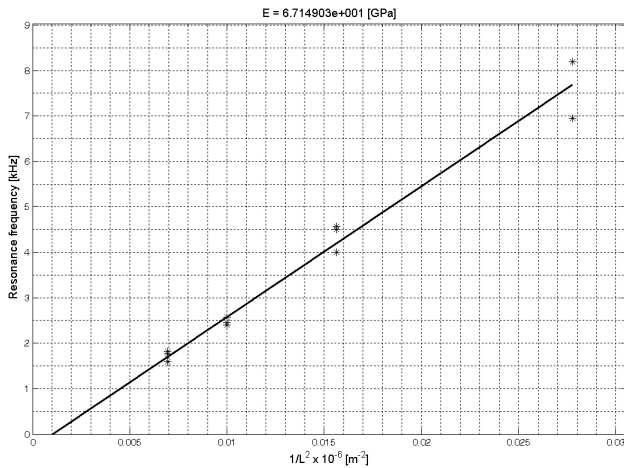| Method	m slope from fitting data	Young's Moduli [GPa]
Chirp	259130	67.149
White noise	260940	68.091
Analyzer	260260	67.737

FIGURE 4. Example of regression line for data measurement resonance frequencies.

5. Error analysis

Error in the measurement was analyzed using Eqs. (8) and (9). We obtained the following equation:

$$\frac{\Delta \hat{E}}{\hat{E}} = \sqrt{\left(\frac{\Delta \rho}{\rho}\right)^2 + \left(\frac{2\Delta f_1}{f_1} + \frac{4\Delta L}{L}\right)^2 + \left(\frac{-2\Delta t}{t}\right)^2} \quad (10)$$

where:

$\frac{\Delta \hat{E}}{\hat{E}}$ Relative uncertainty in Young's modulus

$\frac{\Delta \rho}{\rho}$ Relative uncertainty in density

$\frac{\Delta f_1}{f_1}$ Relative uncertainty in resonance frequency

$\frac{\Delta L}{L}$ Relative uncertainty in cantilever length

$\frac{\Delta t}{t}$ Relative uncertainty in cantilever thickness

The greatest inaccuracies occurred in the measurement of the resonance frequencies and the lengths of the cantilevers. A total error of 35% was calculated; the uncertainty in resonance frequency determination was 1.25 %; the uncertainty of the length measurement, the most significant (Eq. 10), was 8.3 %. The error of the length measurement is due to the type of cantilever assembly, which was mounted on a frame, and the uncertainty in the fixed length.

6. Conclusions

The result shown in Table II, are within the range of the values reported by the manufacturers, between 6.63 and 7.5×10^{10}Pa (but with a density between 7800 and 7900 kg/m^3) [11], and by other authors 6.6 ×10^{10} [12].

In this experiment the error is high, due to the uncertainty in the fixed length of the cantilever, but the objective of implementing a Young's Modulus measurement technique was very well achieved. This result will allow us to design an experimental setup to determine the Young's Modulus of microstructures, which is the work we are engaged in at present.

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