Two techniques for generating a secondary electromagnetic source with desired statistical properties

A.S. Ostrovsky, M.A. Olvera-Santamaría, and C. Rickenstorff-Parrao
Facultad de Ciencias Físico Matemáticas, Universidad Autónoma de Puebla,
Puebla 72000, México,
e-mail: andreyo@fcfm.buap.mx

G. Martínez-Niconoff and V. Arrizón
Departamento de Óptica, Instituto Nacional de Astrofísica Óptica y Electrónica,
Puebla 72000, México.

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Two alternative techniques for generating a secondary electromagnetic source with the desired degree of polarization and transverse coherence length are considered and compared. The first technique is based on the changes of coherence and polarization in propagation, while the second one makes use of the coherence and polarization modulation by a random phase screen. The dependence of the results on the employed definition of electromagnetic coherence is discussed.

Keywords: Cross-spectral density matrix; degree of coherence; degree of polarization.

Se consideran y comparan dos técnicas alternativas para generar una fuente electromagnética secundaria con el grado deseado de polarización y longitud de coherencia transversal. La primera técnica está basada en los cambios de coherencia y polarización en propagación, mientras que la segunda hace uso de la modulación de coherencia y polarización por una pantalla de fase aleatoria. Se discute la dependencia de los resultados de la definición de coherencia electromagnética empleada.

Descriptores: Matriz de densidad espectral cruzada; grado de coherencia; grado de polarización.

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1. Introduction

In many optical applications it is necessary to create a secondary electromagnetic plane source with the desired values of the degree of polarization and the transverse coherence length (see, *e.g.* Refs. 1-4). Two opposite approaches can be used to solve this problem. In the first approach one can start from a completely incoherent and completely unpolarized primary source and try to get the desired result using the changes of coherence and polarization in propagation [5,6]. In the second approach the result can be obtained by transmitting the radiation of completely coherent and completely polarized source through a random phase screen [7,8]. In the present paper we consider and compare two alternative techniques which realize these approaches. To provide an easier comprehension of these techniques, we start with a brief summary of the basic electromagnetic coherence theory definitions.

2. Basic definitions

As well known [9], the second-order statistical properties of a random planar (primary or secondary) electromagnetic source can be completely described by the 2×2 cross-spectral density matrix $\mathbf{W}(\mathbf{x}_1,\mathbf{x}_2)$ (for brevity we omit the explicit dependence of the considered quantities on frequency. Using this matrix, the following three fundamental statistical characteristics of the source can be defined: the power spectrum

$$S(\mathbf{x}) = Tr\mathbf{W}(\mathbf{x},\mathbf{x})\,, \tag{1}$$

the spectral degree of coherence

$$\eta(\mathbf{x}_1,\mathbf{x}_2) = \frac{\mathrm{Tr}\mathbf{W}(\mathbf{x}_1,\mathbf{x}_2)}{[\mathrm{Tr}\mathbf{W}(\mathbf{x}_1,\mathbf{x}_1)\mathrm{Tr}\mathbf{W}(\mathbf{x}_2,\mathbf{x}_2)]^{\,1/2}}, \tag{2}$$

and the spectral degree of polarization

$$P(\mathbf{x}) = \left(1-\frac{4\mathrm{Det}\mathbf{W}(\mathbf{x},\mathbf{x})}{[\mathrm{Tr}\mathbf{W}(\mathbf{x},\mathbf{x})]^2}\right)^{1/2}. \tag{3}$$

In Eqs. (1)-(3) Tr stands for the trace and Det denotes the determinant. In practice, to characterize the coherence properties of the source, it is often sufficient to specify the effective width of function $\eta(\xi)$, known as the transverse coherence length, which is defined as

$$\Delta\xi = 2\int_0^\infty |\,\eta(\xi)|^2\,d\xi\ , \tag{4}$$

where $\xi = |\mathbf{x}_1-\mathbf{x}_2|$. The larger $\Delta\xi$, the greater is the source coherence.

3. First technique

Let the primary source be characterized by the cross spectral density matrix

$$\mathbf{W}_{PS}(\mathbf{x'}_1,\mathbf{x'}_2)=\frac{1}{2}\left[S_{PS}(\mathbf{x'}_1)S_{PS}(\mathbf{x'}_2)\right]^{1/2}\times\delta(\mathbf{x'}_1-\mathbf{x'}_2)\begin{bmatrix}1&0\\0&1\end{bmatrix},\qquad(5)$$

where $\delta(\mathbf{x})$is the Dirac delta function. It may be readily verified that in this case $\eta_{PS}(\mathbf{x}_1,\mathbf{x}_2)=0$ and $P_{PS}(\mathbf{x})=0$, *i.e.* that such a source is completely incoherent and completely unpolarized. Let us assume that the radiation from this source passes through a Mach-Zehnder interferometer with two orthogonally aligned polarizers and two attenuators, whose intensity transmittances have the ratio $\alpha:(1-\alpha)$, at its opposite arms. Then, using the vector version of the van Cittert-Zernike theorem [5,6], we find that the cross-spectral density matrix of the secondary source in some plane at a distance z from the output of the interferometer is given by

$$\mathbf{W}_{SS}(\mathbf{x}_1,\mathbf{x}_2)=\frac{1}{2(\lambda z)^2}\exp\left[-i\frac{\pi}{\lambda z}(\mathbf{x}_1^2-\mathbf{x}_2^2)\right]\times\int\limits_{(z=0)}S_{PS}(\mathbf{x'})\exp\left(-i\frac{2\pi}{\lambda z}\mathbf{x'}\cdot\Delta\mathbf{x}\right)d\mathbf{x'}\times\begin{bmatrix}\alpha&0\\0&1-\alpha\end{bmatrix},\qquad(6)$$

where $\Delta\mathbf{x}=\mathbf{x}_1-\mathbf{x}_2$. On substituting from Eq. (6) into Eqs. (2) and (3), we obtain, respectively

$$\eta_{SS}(\mathbf{x}_1,\mathbf{x}_2)=\exp\left[-i\frac{\pi}{\lambda z}(\mathbf{x}_1^2-\mathbf{x}_2^2)\right]\times\frac{\int\limits_{(z=0)}S_{PS}(\mathbf{x'})\exp\left(-i\frac{2\pi}{\lambda z}\mathbf{x'}\cdot\Delta\mathbf{x}\right)d\mathbf{x'}}{\int\limits_{(z=0)}S_{PS}(\mathbf{x'})\,d\mathbf{x'}},\qquad(7)$$

$$P_{SS}(\mathbf{x})=|\,2\alpha-1\,|\,.\qquad(8)$$

As can be seen from Eq. (8), when α varies from 0.5 to 1, the degree of polarization changes in the full range from 0 to 1. To examine the behavior of the degree of coherence given by Eq. (7), we consider a particular case, when the primary source has the Gaussian power spectrum

$$S_{PS}(\mathbf{x'})=S_0\exp\left(-\frac{\mathbf{x'}^2}{4\varepsilon^2}\right),\qquad(9)$$

where S_0 and ε are positive constants. It may be readily shown (see, *e.g.*, Ref. 1) that in this case Eq. (7) takes the form

$$\eta_{SS}(\mathbf{x}_1,\mathbf{x}_2)=\exp\left[-i\frac{\pi}{\lambda z}(\mathbf{x}_1^2-\mathbf{x}_2^2)\right]\times\exp\left[-\frac{(2\pi\varepsilon)^2\xi^2}{(\lambda z)^2}\right],\qquad(10)$$

where ξ has the meaning defined after Eq. (4). Then, substituting from Eq. (10) into Eq. (4), after straightforward calculations we obtain

$$\Delta\xi=0.2\,\frac{\lambda z}{\varepsilon}\,.\qquad(11)$$

Hence, varying the distance z, one can change the transverse coherence length of the secondary source.

Concluding this section, we note that, as follows from Eq. (11), to generate the secondary source with an arbitrary length of transverse coherence, one must choose the distance z of order ε^2/λ. In practice it means that this technique can be successfully used only for small-size primary (and, hence, secondary) sources. Such a situation meets some applications in remote optical sensing and communication optics. Of course, in this case, it is appropriately to use the fiber optics wave guide to transmit the radiation of the primary source at a large distance z. In addition it may be also noted that the only purpose of the Mach-Zehnder interferometer used in the considered technique is to alter the degree of polarization.

4. Second technique

Now let the primary source be characterized by the cross spectral density matrix

$$\mathbf{W}_{PS}(\mathbf{x}_1,\mathbf{x}_2)=\frac{1}{2}\left[S_{PS}(\mathbf{x}_1)S_{PS}(\mathbf{x}_2)\right]^{1/2}\times\begin{bmatrix}1&1\\1&1\end{bmatrix}.\qquad(12)$$

It may be readily verified that in this case $\eta_{PS}(\mathbf{x}_1,\mathbf{x}_2)=1$ and $P_{PS}(\mathbf{x})=1$, *i.e.* that such a source is completely coherent and completely (linearly) polarized. Let us assume that the radiation from this source passes through a system which consists of two crossed zero-twisted nematic liquid-crystal spatial light modulators controlled by computer. It has been shown [8] that under certain conditions the transmittance of such a system can be represented as

$$\mathbf{T}(\mathbf{x})=\exp(i\phi_0)\begin{bmatrix}\exp[-i\phi(\mathbf{x})]&0\\0&\exp[i\phi(\mathbf{x})]\end{bmatrix},\qquad(13)$$

where ϕ_0 is a constant and $\phi(\mathbf{x})$ is assumed to be a random variable with the probability density given by function

$$\frac{1}{\sqrt{2\pi}\,\sigma_\phi}\exp\left[-\frac{\phi^2(\mathbf{x})}{2\sigma_\phi^2}\right],\qquad(14)$$

and the cross-correlation for two points $\mathbf{x}_1$ and $\mathbf{x}_2$given by function

$$\sigma_\phi^2\exp\left(-\frac{\xi^2}{2\gamma_\phi^2}\right).\qquad(15)$$

Then, using the Jones calculus, we find that the cross-spectral density matrix of the secondary source formed at the output of modulators is given by [8]

$$\mathbf{W}_{SS}(\mathbf{x}_1,\mathbf{x}_2)=\frac{1}{2}\,\left[S_{PS}(\mathbf{x}_1)S_{PS}(\mathbf{x}_2)\right]^{1/2}\exp(-\sigma_\phi^2)\begin{bmatrix}\exp\left[\sigma_\phi^2\exp\left(-\frac{\xi^2}{2\gamma_\phi^2}\right)\right] & \exp\left[-\sigma_\phi^2\exp\left(-\frac{\xi^2}{2\gamma_\phi^2}\right)\right]\\ \exp\left[-\sigma_\phi^2\exp\left(-\frac{\xi^2}{2\gamma_\phi^2}\right)\right] & \exp\left[\sigma_\phi^2\exp\left(-\frac{\xi^2}{2\gamma_\phi^2}\right)\right]\end{bmatrix}. \tag{16}$$

On substituting from Eq. (16) into Eqs. (2) and (3), we obtain, respectively

$$\eta_{SS}(\mathbf{x}_1,\mathbf{x}_2)=\exp\left\{-\sigma_\phi^2\left[1-\exp\left(-\frac{\xi^2}{2\gamma_\phi^2}\right)\right]\right\}, \tag{17}$$

$$P_{SS}(\mathbf{x})=\exp(-2\sigma_\phi^2)\,. \tag{18}$$

As can be seen from Eq. (18), varying the parameter σ_ϕ, which is the variance of the variable $\phi(\mathbf{x})$, one can change the degree of polarization P_{SS} in a wide range, practically from 0 to 1. To examine the behavior of the degree of coherence, we note that for large values of σ_ϕ Eq. (17) can be well approximated by the expression [7]

$$\eta_{SS}(\mathbf{x}_1,\mathbf{x}_2)=\exp\left(-\frac{\sigma_\phi^2\xi^2}{2\gamma_\phi^2}\right). \tag{19}$$

Then, substituting from Eq. (19) into Eq. (4), we obtain

$$\Delta\xi=1.8\,\frac{\gamma_\phi}{\sigma_\phi}\,. \tag{20}$$

Hence, varying the parameter γ_ϕ, which is the cross-correlation width of the variable $\phi(\mathbf{x})$ for two different points, one can change the transverse coherence length of the secondary source.

Concluding this section, we note that, this time, in contrary to the first technique, the transverse coherence length $\Delta\xi$ does not depend neither on the size of the primary source nor on its wave-length. Hence the second technique can be successfully used for generating the partially coherent and partially polarized secondary source of any size. Such a situation meets many different applications in optical imaging and optical data processing.

5. Discussion

The considered techniques where analyzed using Wolf's definition of the electromagnetic coherence given by Eq. (2). However there are other possible definitions of the same quantity [10-13]. To examine the dependence of the obtained results on the employed definition of coherence, we have repeated the above analysis using an alternative definition of the electromagnetic coherence as the normalized Frobenius norm of the cross-spectral density matrix $\mathbf{W}(\mathbf{x}_1,\mathbf{x}_2)$ [10]. When doing this, we have shown that the replacement of the electromagnetic coherence definition by an alternative one practically does not affect the results given above.

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