Finite-time thermoeconomic optimization of a non endoreversible heat engine model

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Within the context of Finite-Time Thermodynamics (FTT), we study the thermoeconomics of a simplified non-endoreversible thermal power plant model (the so-called Novikov engine). In our study, we use different heat transfer laws: the so called Newton's law of cooling, the Stefan-Boltzmann radiation law, the Dulong-Petit's law and another phenomenological heat transfer law. We use two FTT optimization criteria: the maximum power regime (MP) and the so-named modified ecological criterion for performance analysis. This last criterion leads the engine model towards a mode of performance that appreciably diminishes the engine's wasted energy. It is shown that under ecological conditions the plant dramatically reduces the amount of heat rejected to the environment, and a loss of profit is translated into a better usage of fuel such that the heat rejected towards the environment is remarkably reduced compared to that of a maximum power regime. Besides, we analyze the effect on the reduction of power output and the optimal efficiencies in terms of an internal irreversibility parameter that comes from the Clausius inequality which characterizes the degree of internal irreversibility.

Keywords: Thermoeconomics; endoreversible cycles; optimization.

Dentro del contexto de la Termodinámica de Tiempos Finitos (TTF) estudiamos la termoeconomia de un modelo simplificado no endorreversible de planta de potencia. (modelo de Novikov). En nuestro estudio, usamos diferentes leyes de transferencia de calor: Ley de enfriamiento de Newton, ley de radiación de Stefan-Boltzmann, ley de Dulong-Petit y una ley fenomenológica de transferencia de calor. Usamos dos criterios de optimización: régimen de operación de Máxima Potencia y el así llamado criterio de operación de máxima Función Ecológica modificada. Este último criterio proporciona un modo de operación que apreciablemente disminuye la energía disipada por la máquina térmica. En este trabajo mostramos bajo condiciones de operación de máxima Función Ecológica que la pérdida en beneficios se ve compensada por un mejor uso en el combustible de modo que la planta de potencia reduce dramáticamente la cantidad de energía disipada al medio ambiente con respecto a la cantidad de energía bajo un régimen de operación de Máxima Potencia. También analizamos el efecto sobre la potencia de salida y la eficiencia del modelo de planta de potencia en términos de un parámetro de irreversibilidad interna que proviene de la desigualdad de Clausius, el cual caracteriza el grado de irreversibilidad interna.

Descriptores: Termo-economía; ciclos térmicos endorreversibles; optimización.

PACS: 44.60.+K; 44.90.+C

1. Introduction

In 1995 [1], De Vos introduced a thermoeconomical analysis of the Novikov plant [2] considering as an objective function the power output (W) per unit running cost of the plant exploitation (C). In his study De Vos assumed that the running cost of the plant consists of two parts: a capital cost that is proportional to the investment and, therefore, to the size of the plant, and a fuel cost that is proportional to the fuel consumption and, therefore, to the heat input rate Q_H. Assuming that $Q_{\max}$ is an appropriate measure for the size of the plant, the running cost of the plant exploitation is defined as [1]

$$C = aQ_{\max} + bQ_H, \tag{1}$$

where the proportionality constants a and b have units of $\$/Joule$ and $Q_{\max}$ is the maximum heat that can be extracted from the heat reservoir without supplying work (see Fig. 1). De Vos considered that the heat input rate is given by the Newton heat transfer law, that is,

$$Q_H = g\left(T_H - T_W\right), \tag{2}$$

$$Q_{\max} = g\left(T_H - T_L\right), \tag{3}$$

where T_H and T_L are the temperatures of the hot and cold thermal reservoirs respectively, T_W is the variable temperature of the working fluid (see Fig. 1), and g is a thermal conductance. Applying the first law of thermodynamics and using Eqs. (1) - (3), the objective function can be expressed as [1]

$$F = \frac{W}{C} = \frac{1}{a}\frac{\left(T_W - T_L\right)\left(T_H - T_W\right),}{T_W\left(T_H - T_L\right) + \beta\left(T_H - T_W\right)}, \tag{4}$$

where $\beta = b/a$, is the economical parameter. The optimization of the objective function F in Eq. (4) is obtained by

$$\left.\frac{dF}{dT_W}\right|_{T_W^*} = 0,$$

where T_W^* is the optimal value for which the objective function F has its maximum. De Vos showed that the value of

TABLE I. Relative cost for several kinds of fuels [1].

Fuel	Fractional fuel cost (%)
Renewable	0
Uranium	25
Coal	35
Gas	50

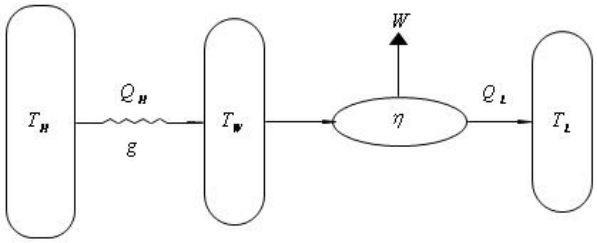

FIGURE 1. Novikov's model for a nuclear power plant.

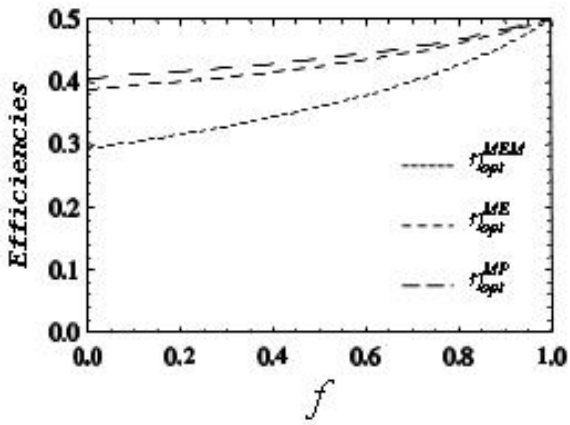

FIGURE 2. Comparison between optimum efficiencies under maximum power and under maximum ecological regimes (the former E function and the modified EM) with $\tau = 1/2$.

the temperature of the working fluid that maximizes Eq. (4) is given by,

$$T_W^* = \sqrt{T_H T_L}\frac{\sqrt{1+\beta}\,(T_H - T_L) - \beta\sqrt{T_H T_L}}{T_H - (1+\beta)\,T_L}, \qquad (5)$$

and the corresponding optimal efficiency is,

$$\eta_{opt} = 1 - \frac{T_L}{T_W^*} = 1 - \sqrt{\frac{T_L}{T_H}}\frac{T_H - (1+\beta)\,T_L}{\sqrt{1+\beta}\,(T_H - T_L) - \beta\sqrt{T_H T_L}}. \qquad (6)$$

The expression for the optimal efficiency can be obtained in terms of the fuel fractional cost, f, which is defined as the ratio of the fuel cost and the total costs of the plant [1], that is,

$$f = \frac{bQ_H}{aQ_{\max} + bQ_H} = \frac{\beta\,(T_H - T_W)}{(T_H - T_L) + \beta\,(T_H - T_W)}. \qquad (7)$$

The value of f for several types of fuel is shown in Table I. Therefore, the De Vos optimal efficiency in terms of the fractional fuel cost is given by [1],

$$\eta_{opt}\,(\tau, f) = 1 - \frac{f}{2}\tau - \frac{\sqrt{4(1-f)\tau + f^2\tau^2}}{2}, \qquad (8)$$

where $\tau = T_L/T_H$. De Vos showed how the optimal efficiency smoothly increases from the MP-efficiency (Curzon-Ahlborn efficiency, η_{CA} [3]) for $f = 0$, corresponding to energy sources where the investment is the preponderant cost up to the Carnot value for $f = 1$, that is, for energy sources where the fuel is the predominant cost; thus, $\eta_{CA}<\eta_{opt}<\eta_C$, as we can observe in Fig. 2 [1]. Eq. (8) gives the optimal efficiency for a Novikov power plant working at maximum power regime in terms of the fractional fuel cost f when the heat fluxes in Fig. 1 are given by a Newtonian heat transfer law. Recently [4,5], we have also studied a Novikov engine following the thermoeconomical approach used by De Vos, but by means of the so-called modified ecological optimization criterion [6,7], which consists in the maximization of the ecological function defined by $E = W - \varepsilon T_L \sigma$, W being the power output, σ the total entropy production and ε a parameter that contains the dependence on the particular heat transfer law used in the Novikov model [7]. In general the maximization of the ecological function leads to an engine performance with a power output around 75% of the maximum power and an entropy production around 25% of the entropy produced under maximum power conditions; this property of the ecological function is called the corollary 75-25 [7,8]. Furthermore, the ecological criterion has another important property: the efficiency at maximum ecological conditions is approximately the semi-sum of the efficiencies corresponding to the maximum power regime (Curzon-Ahlborn efficiency), and the Carnot efficiency [7]. When the heat transfer in the Novikov model is of the Newtonian type, we show that the optimal efficiency under maximum ecological conditions satisfies the following inequality $\eta_{CA}<\eta_{opt}<\eta_{opt}^E<\eta_C$ [5] (Fig. 2 shows the optimal ecological efficiency η_{opt}^E). We found that the optimal efficiency under the ecological regime, with a Newtonian heat transfer law is given by [4,5],

$$\eta_{opt}^E\,(\tau, f) = 1 - \frac{f}{2}\tau - \frac{\sqrt{4(1-f)\tau^{3/2} + f^2\tau^2}}{2}. \qquad (9)$$

Figure 2 also shows how η_{opt}^E smoothly varies with f from the maximum ecological function point with f=0 $\left(\eta^E = 1 - \tau^{3/4}\right)$ [7, 9] up to the Carnot point $(f = 1)$, in an analogous way to the De Vos-efficiency (see Fig. 2). In a similar way to De Vos' study, Sahin and Kodal (SK) [10] made a thermoeconomic analysis of a Curzon-Ahlborn engine. SK maximized a profit function defined by

$$F_{SK} = \frac{W}{C_i + C_f}, \qquad (10)$$

where C_i and C_f are the annual investment and fuel consumption costs, respectively. Sahin and Kodal [10] assumed that the plant's size can be taken proportional to the total heat transfer area, instead of the maximum heat input previously considered by De Vos [1]. Thus, the yearly investment cost of the system can be given as [10],

$$C_i = \gamma_i \left(A_H + A_L\right), \tag{11}$$

where A_H and A_L are the heat transfer areas of the heat exchangers in both the hot and the cold reservoirs and the proportionality coefficient for the investment cost γ_i is equal to the capital recovery factor times investment cost per unit heat transfer area. The annual fuel consumption cost is proportional to the heat rate input, that is [10],

$$C_f = \gamma_f Q_H, \tag{12}$$

where the coefficient γ_f is equal to the annual operation hours times the price per unit of heat input. SK also showed that the variation of the optimal thermal efficiency with respect to the fuel cost parameter $f = \gamma_i/\gamma_i + \gamma_f$, in the interval $0 \leq f \leq 1$, satisfies the inequality $\eta_{MP} < \eta_{opt} < \eta_C$ under maximum power conditions; that is, the Carnot (η_C) and the maximum power (η_{MP}) efficiencies are the upper and lower bounds of the optimum thermal efficiencies. On the other hand, Antar and Zubair [11] performed a finite-time thermoeconomic analysis of a Curzon-Ahlborn engine model, considering the total cost per unit power output as an objective function. They expressed the total cost of conductance in terms of unit cost parameters as $\Gamma = \zeta_H \left(UA\right)_H + \zeta_L \left(UA\right)_L$, where ζ_H and ζ_L are the unit conductance costs at hot and cold end heat exchanger, respectively. Antar and Zubair [11], minimized a function defined as $F = \Gamma/(W/T_H)$ with respect to the temperature ratios T_W/T_H and T_C/T_H and a parameter $G = \zeta_H/\zeta_L$. Antar and Zubair [11], obtained the optimum values of the absolute temperature values and they discussed the influence of G and T_W/T_H on the thermoeconomic performance of the engine model.

The aim of this paper is to extend the thermoeconomical analysis of the Novikov plant model by considering different heat transfer laws as well as to analyze the effects of internal irreversibilities on the optimal performance. The paper is organized as follows: In Sec. 2, following the De Vos' procedure we present a thermoeconomical analysis of the Novikov plant model considering different heat transfer laws and considering two criteria of performance: the maximum power output criterion and the so-called ecological function criterion. In Sec. 3, we analyze the effect of the internal irreversibilities of the model on the environmental impact, the entropy production and the power output of the engine model. Finally, in Sec. 4 we present our conclusions.

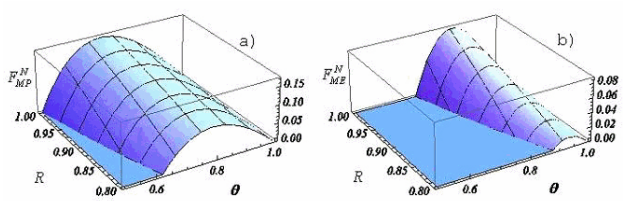

FIGURE 3. Profit function versus the parameter of internal irreversibility and the reduced temperature θ for the Newtonian heat transfer law, with $\tau = 1/2$. a) Maximum power regime and b) Maximum ecological function regime.

2. Thermoeconomic optimization

From the first law of thermodynamics, the power output for the engine shown in Fig. 1 is given by

$$W = Q_H - Q_L, \tag{13}$$

where Q_H and Q_L are the heat transfer supplied by the hot source to the working fluid and the heat transfer from the working fluid to cold source, respectively. On the other hand, the internal efficiency of the engine is given by

$$\eta = \frac{W}{Q_H} = 1 - \frac{1}{R}\frac{T_L}{T_W} = 1 - \frac{\tau}{R}\frac{1}{\theta}, \tag{14}$$

where $\theta = T_W/T_H$ (see Fig. 1) and $R = \Delta S_{1w}/|\Delta S_{2w}|$ is the non-endoreversibility parameter [12-14] (which characterizes the degree of internal irreversibility that comes from the Clausius inequality) ΔS_{1w} being the change in the internal entropy along the hot isothermal branch and ΔS_{2w} the entropy change corresponding to the cold isothermal compression. This parameter, which in principle is within the interval $0 < R \leq 1$ ($R = 1$ for the endoreversible limit), can be seen as a measure of the departure from the endoreversible regime [12-14]. If we consider that the heat transferred between the hot source and the working fluid follows a generalized law of the type $Q \propto \Delta T^n$, where n is a heat transfer exponent such that when the heat transfer obeys a Newtons's law (N) $n = 1$, and for a Dulong-Petit heat transfer law (DP) $n = 5/4$, then

$$Q_H = g T_H^n \left(1 - \theta\right)^n. \tag{15}$$

Combining Eqs. (1), (13), (14) and (15), and using the definition by De Vos for the profit function ($F = W/C$), the dimensionless objective functions both at maximum power and at maximum modified ecological function conditions are, respectively,

$$F_{MP}^{N-DP} = \frac{1}{a}\frac{\left(1 - \frac{\tau}{R}\frac{1}{\theta}\right)\left(1-\theta\right)^n}{\left(1-\tau\right)^n + \beta\left(1-\theta\right)^n}, \tag{16}$$

and

$$F_{ME}^{N-DP} = \frac{1}{a}\frac{\left[(1+\varepsilon)\left(1-\frac{\tau}{R}\frac{1}{\theta}\right) - \varepsilon(1-\tau)\right]\left(1-\theta\right)^n}{\left(1-\tau\right)^n + \beta\left(1-\theta\right)^n}, \tag{17}$$

where $\varepsilon = 1/\sqrt{\tau}$ for the case of a Newton heat transfer law ($n = 1$) and

$$\varepsilon = \frac{8 + \tau - \sqrt{\tau\left(\tau + 80\right)}}{\sqrt{\tau\left(\tau + 80\right)} - 9\tau}$$

for the case of a Dulong-Petit heat transfer law $(n = 5/4)$ [7,14]. In previous equations, we also have considered that $Q_{\max}$ is the maximum heat that can be extracted from the heat reservoir without performing work, in the same way as previously considered by De Vos [1], which in this case gives $Q_{\max} = gT_H^n (1-\tau)^n$. In Figs. 3a and 3b we show the behavior of F_{MP}^{N-DP} and F_{ME}^{N-DP} for the Newtonian law case $(n = 1)$ versus the reduced temperature θ and versus the parameter R. As can be seen from Figs. 3a and 3b, there exists an optimal efficiency value which depends on the parameter R and the optimum value of θ. We can also observe in Figs. 3a and 3b how the benefit diminishes as the internal irreversibilities (parameter R) increase. Moreover, the benefits under maximum ecological function conditions are lower than those obtained under maximum power conditions. The maximization of the objective functions given by Eq. (16) and Eq. (17) by

$$\left.\frac{d\left(F^{N-DP}\right)}{d\theta}\right|_{\theta^*} = 0$$

gives the corresponding optimal efficiencies. Therefore, taking the derivatives of F_{MP}^{N-DP} and F_{ME}^{N-DP} with respect to θ and setting them equal to zero we obtain the following equations for both conditions: maximum power output and maximum ecological function, respectively

$$(1-\theta)^{n+1}\tau\beta - \left[(\theta(1-n)-1)\tau + nR\theta^2\right](1-\tau)^n = 0, \quad (18)$$

$$(1-\theta)^{n+1}\beta - \left[(\theta(1-n)-1) + \frac{nR}{\sqrt{\tau}}\theta^2\right](1-\tau)^n = 0. \quad (19)$$

We can solve the above equations to obtain the optimal efficiency in terms of the parameter β (see Eqs. (A.4) and (A.11) in the Appendix, for the case of Newton heat transfer law). However, by using Eq. (7), which in the present case is written as

$$\beta = \frac{f}{1-f}\frac{(1-\tau)^n}{(1-\theta)^n},$$

we can calculate the optimum working fluid temperature $(\theta^* = T_W^*/T_H)$ that maximizes both Eqs. (16) and (17), and the optimal efficiencies can be obtained in terms of the fractional fuel cost. Therefore, the optimal efficiencies at maximum power output and ecological function conditions are respectively given by (see Appendix),

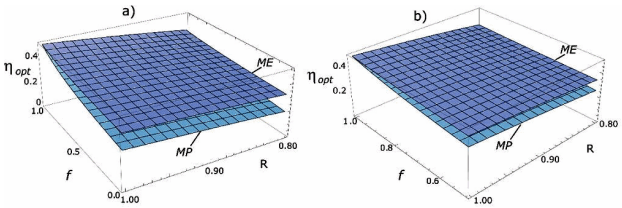

FIGURE 4. Comparison between the optimal efficiencies at maximum power and maximum ecological conditions versus fractional fuel cost and the parameter R with $\tau = 1/2$, for a) Newton heat transfer law and for b) Dulong-Petit heat transfer law.

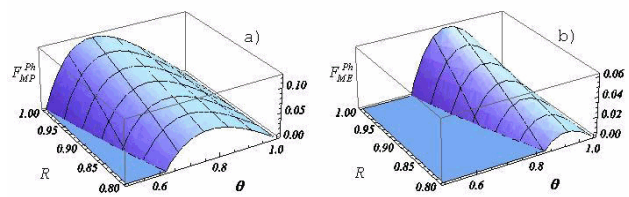

FIGURE 5. Profit function versus the parameter R and the reduced temperature for a phenomenological heat transfer law, with $\tau = 1/2$. a) Maximum power regime and b) Maximum ecological function regime.

$$\eta_{MP}^{N}(f,\tau,R) = 1 - \frac{2(f-1)\tau}{f\tau + \sqrt{4(1-f)R\tau + f^2\tau^2}}, \quad (20)$$

$$\eta_{MP}^{DP}(f,\tau,R) = 1 - \frac{10(f-1)\tau}{(5f-1)\tau + \sqrt{80(1-f)R\tau + (1-5f)^2\tau^2}}, \quad (21)$$

$$\eta_{ME}^{N}(f,\tau,R) = 1 - \frac{2(f-1)\tau}{f\sqrt{\tau} - \sqrt{4(1-f)R\sqrt{\tau} + f^2\tau}}, \quad (22)$$

$$\eta_{ME}^{DP}(f,\tau,R) = 1 - \frac{100(1-f)\tau}{(1-5f)\Lambda + \sqrt{800\Lambda(1-f)R + [(1-5f)\Lambda]^2}}. \quad (23)$$

In Eq. (23), we have defined $\Lambda = \tau + \sqrt{(80+\tau)\tau}$ to simplify the expression for the optimal efficiency under maximum ecological conditions. In Figs. 4a and 4b, we show both optimal efficiencies at maximum power and at maximum ecological conditions for the Newton (see Fig. 4a) and the Dulong-Petit (see Fig. 4b) cases. We can see in Figs. 4a and 4b, how the optimal efficiencies smoothly increase from the maximum point-efficiency, $f = 0$, corresponding to energy sources where the investment is the preponderant cost up to the Carnot value for $f = 1$, that is, for energy sources where the fuel is the predominant cost [1]. Besides, Figs. 4a and 4b, show that the engine with internal irreversibilities has a lower efficiency when is compared to the endoreversible Carnot case $(R = 1)$ [4,5].

Analogously, if we consider in the Novikov model that the heat transfer is a Stefan-Boltzmann radiation law (Muser engine [15]) and another phenomenological heat transfer law [16], then the heat input Q_H and $Q_{\max}$ are given by,

$$Q_H = gT_H^n(1-\theta^n)\,Sign(n), \quad (24)$$

$$Q_{\max} = gT_H^n(1-\tau^n)\,Sign(n), \quad (25)$$

where $Sign(n)$ is the sign function, such that $Sign(n)$=1 if $n > 0$ and $Sign(n) = -1$ if $n < 0$. Therefore, in this case, both objective functions at maximum power and at maximum ecological function conditions respectively are,

$$F_{MP}^{SB-Ph} = \frac{1}{a}\frac{\left(1-\frac{\tau}{R}\frac{1}{\theta}\right)(1-\theta^n)}{(1-\tau^n)+\beta(1-\theta^n)}, \quad (26)$$

and

$$F_{ME}^{SB-Ph} = \frac{1}{a}\frac{\left[2\left(1-\frac{\tau}{R}\frac{1}{\theta}\right)-(1-\tau)\right](1-\theta^n)}{(1-\tau^n)+\beta\,(1-\theta^n)}, \quad (27)$$

In Figs. 5a and 5b we show the behavior of F_{MP}^{SB-Ph} and F_{ME}^{SB-Ph} for the phenomenological heat transfer law case $(n=-1)$ versus the reduced temperature θ and the parameter R. As we can see in Figs. 5a and 5b, the profit functions have a behavior similar to the Newton and the Dulong-Petit cases, that is, there exists an optimal efficiency which depends on the parameter R and the optimum value of θ. We can also observe in Figs. 5a and 5b how the benefit diminishes as the internal irreversibilities (parameter R) increase. In a similar way to the Newton and the Dulong-Petit cases (see Appendix), we can obtain the optimal efficiencies in terms of the parameter β. After taking the derivative of Eq. (26) with respect to θ and setting it equal to zero, for the case of the phenomenological heat transfer law $(n=-1)$, we get,

$$\left[(1-R)\,\tau+R-(1+\beta)\tau^2\right]\theta^2 - 2\tau\left[1-(1+\beta)\tau\right]\theta-\tau^2\beta=0 \quad (28)$$

Solving Eq. (28) for θ yields

$$\theta=\frac{\tau\left[1-(1+\beta)\tau\right]-\tau\sqrt{(\tau-1)(\tau-1)+(\tau-R)\beta}}{(\tau-1)(\tau+R)-\tau^2\beta}. \quad (29)$$

Combining Eqs. (14) and (29) we obtain the optimal efficiency,

$$\eta_{MP}^{Ph}(\beta,\tau,R)=1 - \frac{(1-\tau)(R+\tau)-\beta\tau^2}{R\tau\left[1-(1+\beta)\tau\right]+R(1-\tau)\sqrt{1+\frac{\beta(\tau-R)}{(\tau-1)}}}. \quad (30)$$

Instead of expressing the result in terms of the parameter β, a number that is difficult to obtain from the literature [1], we can also express it in terms of the fractional fuel cost [1]. From Eq. (7), in this case $n=-1$, we obtain the following relation between θ and the parameter β,

$$\theta=\frac{(1-f)\beta\tau}{f(1-\tau)+(1-f)\beta\tau}. \quad (31)$$

After substituting Eq. (31) into Eq. (29) we obtain,

$$\beta=\frac{f(2-f)}{(1-f)^2}\frac{(1-\tau)}{(R-\tau)}. \quad (32)$$

Therefore, after substituting Eq. (32) into Eq. (30), the optimal efficiency in terms of the fractional fuel cost under maximum power conditions is given by

$$\eta_{MP}^{Ph}(f,\tau,R)=\frac{R-\tau}{R\,(2-f)}. \quad (33)$$

From Eq. (33) it can be observed that for $R=1$ (endoreversible case), the result previously obtained by Chen *et al.* is recovered [16]. In Fig. 6a, we show the optimal efficiency at maximum power conditions for the phenomenological heat transfer law case, and it can be seen how the optimal efficiency increases from the maximum point-efficiency, $f=0$, corresponding to energy sources where the investment is the preponderant cost up to the Carnot value for $f=1$.

In a similar way to the maximum power regime, from Eq. (27), taking the derivative of F_{ME}^{Ph} with respect to θ and setting it equal to zero

$$\left(\frac{dF_{ME}^{Ph}}{d\theta}=0\right)$$

yields,

$$\left[(2-R\tau)\,\tau+R-2(1+\beta)\tau^2\right]\theta^2 - 4\tau\left[1-(1+\beta)\tau\right]\theta-2\tau^2\beta=0. \quad (34)$$

Solving Eq. (34) for θ yields

$$\theta=\frac{2\tau\left[1-(1+\beta)\tau\right]-\tau\sqrt{2R(1-\tau^2)\beta+4(1-\tau)\left[1-(1+\beta)\tau\right]}}{R(1-\tau^2)+2\tau\left[1-(1+\beta)\tau\right]}. \quad (35)$$

In a similar way to the maximum power regime, we can obtain a relation between the parameter β and the fractional fuel cost (f) by using Eqs. (31) and (35), and therefore we can obtain the optimal efficiency in terms of the fractional fuel cost under maximum ecological conditions, which is given by

$$\eta_{ME}^{Ph}(f,\tau,R)=\frac{R\left[(f-3)-(f-1)\tau\right]+2\tau}{2R\,(f-2)}. \quad (36)$$

In Eq. (36) it can be observed that for $R=1$ (endoreversible case), the result previously obtained by Barranco-Jiménez and Angulo-Brown is recovered [5]. In Fig. 6a, we also show the optimal efficiency under maximum ecological function conditions for the phenomenological heat transfer law case. On the other hand, when in the Novikov engine model we consider the Stefan-Boltzmann radiation law, from Eqs. (26) and (27) (for $n=4$) we obtain the following equations for the maximum power and maximum ecological regimes, respectively

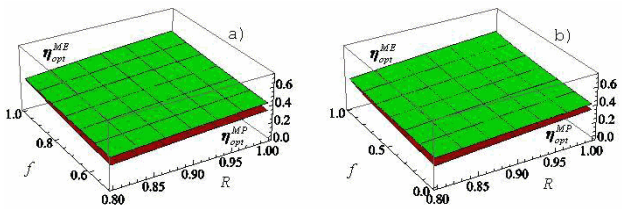

FIGURE 6. Comparison between the optimal efficiencies at maximum power and maximum ecological conditions versus fractional fuel cost and the parameter R with $\tau = 1/2$, for: a) Phenomenological heat transfer law and for b) Stefan-Boltzmann radiation law.

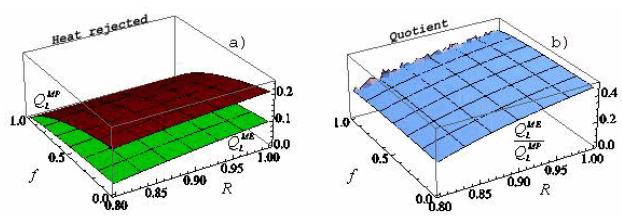

FIGURE 7. a) Ecological rejected heat and maximum power rejected heat versus fractional fuel cost and the parameter R, and b) Quotient between ecological rejected heat and maximum power rejected heat for a Newton heat transfer law

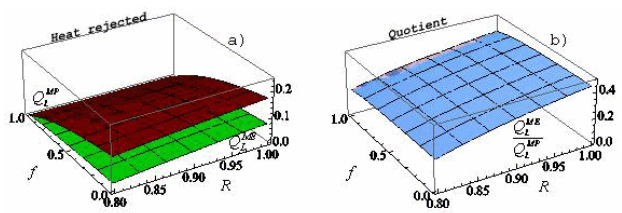

FIGURE 8. a) Ecological rejected heat and maximum power rejected heat versus fractional fuel cost and the parameter R, and b) Quotient between ecological rejected heat and maximum power rejected heat for a DP heat transfer law.

$$\frac{dF_{MP}^{SB}}{d\theta} = \tau\beta\theta^8 - 4R\left(1-\tau^4\right)\theta^5 + \left[3\tau\left(1-\tau^4\right) - 2\beta\tau\right]\theta^4 + \tau\left[(1+\beta)-\tau^4\right] = 0, \quad (37)$$

$$\frac{dF_{ME}^{SB}}{d\theta} = \tau\beta\theta^8 - 2R\left[\left(1-\tau^4\right) + \left(\tau-\tau^5\right)\right]\theta^5 + \left[3\tau\left(1-\tau^4\right) - 2\beta\tau\right]\theta^4 + \tau\left[(1+\beta)-\tau^4\right] = 0. \quad (38)$$

The previous equations can be solved numerically for different values of β; however, using the expression for the fractional fuel cost given by

$$\beta = \frac{f}{1-f}\frac{\left(1-\tau^4\right)}{\left(1-\theta^4\right)},$$

we get

$$4R\left(1-f\right)\theta^5 + \left[\left(4f-3\right)\tau\right]\theta^4 - \tau = 0, \quad (39)$$

$$2R\left[\left(1-f\right)\left(1+\tau\right)\right]\theta^5 + \left[\left(4f-3\right)\tau\right]\theta^4 - \tau = 0. \quad (40)$$

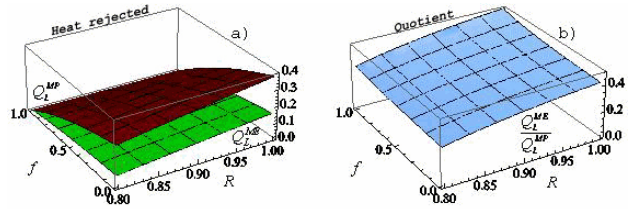

FIGURE 9. a) Ecological rejected heat and maximum power rejected heat versus fractional fuel cost and the parameter R, and b) Quotient between ecological rejected heat and maximum power rejected heat for a phenomenological heat transfer law.

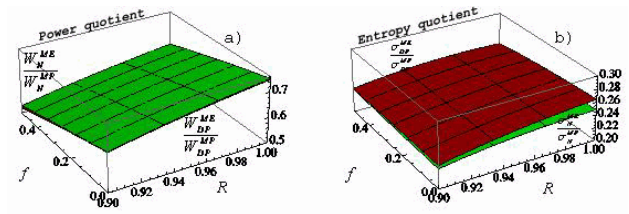

FIGURE 10. a) Quotient between power output at maximum ecological function and at maximum power conditions versus fractional fuel cost and the parameter R, with $\tau = 1/2$. b) Quotient between total entropy production at maximum ecological function and at maximum power conditions versus fractional fuel cost and the parameter R, with $\tau = 1/2$.

Therefore, the optimal efficiencies for the Stefan-Boltzmann radiation law (SB) in both regimes, *i.e* maximum power output and maximum ecological function, can be found by solving numerically Eqs. (39) and (40). In Fig. 6b, we show these optimal efficiencies for several values of the parameters f and R. These optimal efficiencies have a similar behavior to Eqs. (20)-(23), that is, they smoothly increase from the maximum point-efficiency, $f = 0$, corresponding to energy sources where the investment is the preponderant cost up to the Carnot value for $f = 1$, that is, for energy sources where the fuel is the predominant cost [1].

3. Effect on the dissipation, power output and environmental impact

In the previous section, we considered the fractional fuel costs for several kinds of fuels, which range from coal to natural gas (see Table I). We have also shown that the optimal economical point under maximum ecological function conditions provides a higher reduction on the profits than those obtained under maximum power conditions (see Figs. 3 and 5). However, this reduction of profit is concomitant with a better efficiency for a given value of the parameter f that provides a decrease of the energy rejected to the environment by the power plant (see Figs. 4 and 6) [4,5]. Therefore, the ecological criterion seems to be a suitable procedure for the search of insights into an engine's performance with a less aggressive interaction with the environment. In this context, the following simplified analysis is proposed.

Applying the first law of thermodynamics to the engine shown in Fig. 1 we get

$$Q_L = Q_H - W = (1-\eta)\, Q_H, \tag{41}$$

where Q_L is the heat rejected to the environment by the power plant. When the power plant works at maximum power regime for both the Newton and Dulong-Petit cases, Q_L is obtained by using Eqs. (14), (15) and (41):

$$Q_L\left(R,\tau,\theta^*_{MP}\right) = gT_H^n\left[\frac{\tau}{R}\frac{1}{\theta^*_{MP}}\right]\left(1-\theta^*_{MP}\right)^n. \tag{42}$$

In a similar way, when the power plant works under maximum ecological function conditions, we get

$$Q_L\left(R,\tau,\theta^*_{ME}\right) = gT_H^n\left[\frac{\tau}{R}\frac{1}{\theta^*_{ME}}\right]\left(1-\theta^*_{ME}\right)^n. \tag{43}$$

From Eqs. (42) and (43), we can calculate the heat rejected to the environment for each value of f and under different ways of operation of the power plant. In Fig. 7a, it can be seen how the heat rejected to the environment under ecological conditions is lower than the heat rejected under maximum power conditions for the Newton law case. In Fig. 7b we show the ratio between the amounts of rejected heat for each regime: the ecological function and maximum power. In a similar manner, in Figs. 8 and 9, we show both amounts of rejected heat for each regime: maximum power and maximum ecological function, and the ratio between the amounts of rejected heats for the Dulong-Petit heat transfer law and the phenomenological heat transfer law. In addition, we can calculate the total entropy production for the Novikov model for both regimes: ecological function and maximum power. Applying the second law of thermodynamics to the engine model of Fig. 1, the entropy production is given by

$$\sigma = \frac{Q_L}{T_L} - \frac{Q_H}{T_H} = \frac{Q_H - W}{T_L} - \frac{Q_H}{T_H}, \tag{44}$$

which, by using Eqs. (14) and (15) for the Newton and Dulong-Petit cases, becomes

$$\sigma\left(R,\tau,\theta\right) = gT_H^n\left[(1-\tau) - \left(1-\frac{\tau}{R}\frac{1}{\theta}\right)\right](1-\theta)^n. \tag{45}$$

As for Eqs. (42) and (43), by using Eq. (45) we can calculate the quotient of the total entropy production for the Novikov model under both maximum ecological function and maximum power conditions and for different heat transfer laws used in the Novikov model, that is,

$$\frac{\sigma\left(R,\tau,\theta^*_{ME}\right)}{\sigma\left(R,\tau,\theta^*_{MP}\right)}.$$

In a similar way, using Eqs. (13), (14) and (15), for the Newton and Dulong-Petit cases, we can calculate the quotient between the power output of the plant for both ecological function and maximum power conditions, that is,

$$\frac{W\left(R,\tau,\theta^*_{ME}\right)}{W\left(R,\tau,\theta^*_{MP}\right)}.$$

We can observe in Figs. 10a and 10b, that only in the endoreversible limit, the corollary 75 - 25 [7] is maintained for each of the different heat transfer laws used in the Novikov model.

4. Conclusions

In a recent paper, Fischer and Hoffmann [17] showed that a simple endoreversible model (the so-called Novikov engine) can reproduce the complex behavior of a quantitative dynamical simulation of an Otto engine including, but no limited to, effects from losses due to heat conduction, exhaust losses and frictional losses. Also Curto-Risso *et al.* [18] have analyzed a FTT-model for an irreversible Otto cycle suitable for reproducing performance results of a real spark ignition heat engine. In these articles the spirit of FTT is illustrated by emphasizing the virtues and limitations of this methodology. However, the usefulness of the FTT models is shown beyond any doubt. In fact, we can assert that the FTT spirit is concomitant with the spirit of Carnotian thermodynamics in the sense of the search for a certain kind of limit for thermodynamics variables and functionals. In this work, we study the thermoeconomics of a non-endoreversible heat engine model (the so-called Novikov engine). In our study we have used different heat transfer laws: the Newton's law of cooling, the Stefan-Boltzmann radiation law, the Dulong-Petit's law of cooling and a phenomenological heat transfer law. We calculate the thermoeconomical optimal efficiencies under two regimes of performance, namely, the maximum power regime and the so-called ecological regime. We found that when the Novikov model maximizes the ecological function, it reduces the rejected heat to the environment up to about 55% of the rejected heat in the case of a power plant model working under maximum power conditions. In the final part of our paper, we also analyzed the effect of the internal irreversibilities on corollary 75-25 for different relative costs of fuel for several energy sources.

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A Optimal efficiencies under both maximum power and maximum ecological conditions: Newton and Dulong-Petit cases.

From Eq. (16) the profit function under maximum power regime in the case of a Newton heat transfer law $(n = 1)$ is given by

$$F^N_{MP} = \frac{1}{a}\frac{\left(1-\frac{\tau}{R}\frac{1}{\theta}\right)(1-\theta)}{(1-\tau)+\beta(1-\theta)}. \tag{A.1}$$

The derivative of F_{MP}^{N} with respect to θ is

$$\frac{dq_{MP}^{N}}{d\theta}=-\frac{\tau^{2}+R\theta^{2}-\tau\left[1+\beta\left(\theta-1\right)^{2}+R\theta^{2}\right]}{R\left[\beta\left(\theta-1\right)^{2}+\tau-1\right]^{2}\theta^{2}}. \quad \text{(A.2)}$$

Now, solving Eq. (A.2) for θ yields

$$\theta_{MP}^{*}\left(\beta,\tau,R\right) = \frac{\tau\beta-\sqrt{(1-\tau)\tau}\sqrt{R(1+\beta-\tau)-\tau\beta}}{R(\tau-1)+\tau\beta}. \quad \text{(A.3)}$$

After substituting Eq. (A.3) in the expression for efficiency, we get

$$\eta_{MP}^{N}\left(\beta,\tau,R\right)=1-\frac{\tau}{R}\frac{1}{\theta_{MP}^{*}\left(\beta,\tau,R\right)} = 1-\sqrt{\frac{\tau}{R}}\frac{R(1-\tau)-\tau\beta}{(1-\tau)R\sqrt{1+\frac{\beta(R-\tau)}{(1-\tau)R}}-\sqrt{R\tau}\beta}. \quad \text{(A.4)}$$

We can observe in Eq. (A.4) that for $R=1$ the result obtained previously by De Vos [1] is recovered. Besides, when $\beta=0$, we obtain

$$\eta_{opt}=1-\sqrt{\tau/R}$$

which was previously obtained by Wu and Kiang [11], and Arias-Hernández *et al.* [14]. Instead of expressing the result in terms of the parameter β, a number that is difficult to obtain in the literature [1], we can also express it in terms of the fractional fuel cost [1]. In a similar way to the case of a phenomenological heat transfer law, we can express the parameter β as

$$\beta=\frac{f}{1-f}\frac{1-\tau}{1-\theta},$$

by using the relation between the parameter β and the fractional fuel cost, for the case $n=1$ (see Eq. (7)). Therefore, the resultant quadratic equation to be solved for θ is found to be

$$\left(1-f\right)R\theta^{2}+f\tau\theta-\tau=0, \quad \text{(A.5)}$$

with the solution

$$\theta^{*}{}_{MP}^{N}=\frac{f\tau+\sqrt{4(1-f)R\tau+f^{2}\tau^{2}}}{2R(f-1)}, \quad \text{(A.6)}$$

By substituting $\theta^{*}{}_{MP}^{N}$ in the expression for the efficiency, we get

$$\eta_{MP}^{N}\left(f,\tau,R\right)=1-\frac{\tau}{R}\frac{1}{\theta^{*}{}_{MP}^{N}} = 1-\frac{2\left(f-1\right)\tau}{f\tau+\sqrt{4(1-f)R+f^{2}\tau^{2}}}. \quad \text{(A.7)}$$

From Eq. (17), the profit function under maximum ecological function regime is given by

$$F_{ME}^{N}=\frac{1}{a}\frac{\left(1-\frac{\tau}{R}\frac{1}{\theta}\right)\left[\left(1+\frac{1}{\sqrt{\tau}}\right)+\sqrt{\tau}\left(1-\frac{1}{\tau}\right)\right]}{\left(1-\tau\right)+\beta\left(1-\theta\right)}. \quad \text{(A.8)}$$

In the above equation, we have substituted $\varepsilon=1/\sqrt{\tau}$ for the case of a Newton heat transfer law $(n=1)$ [7]. If we calculate the derivative of F_{ME}^{N} with respect to θ we obtain in this case,

$$\frac{dq_{ME}^{N}}{d\theta}=\frac{(1+\sqrt{\tau})\left[\sqrt{\tau}\left(1+\beta\left(\theta-1\right)^{2}\right)+R(1-\tau)\theta^{2}-\tau^{3/2}\right]}{R\left[(\theta-1)\beta+\tau-1\right]^{2}\theta^{2}}. \quad \text{(A.9)}$$

Solving Eq. (A.9) for θ yields

$$\theta_{ME}^{*}\left(\beta,\tau,R\right)=\frac{\sqrt{\tau}\beta-\tau^{1/4}\sqrt{1-\tau}\sqrt{R\left(1+\beta-\tau\right)-\sqrt{\tau}\beta}}{R(\tau-1)+\sqrt{\tau}\beta}. \quad \text{(A.10)}$$

After substituting Eq. (A.10) in the expression for efficiency, we get

$$\eta_{ME}^{N}\left(\beta,\tau,R\right)=1-\frac{\tau}{R}\frac{1}{\theta_{ME}^{*}\left(\beta,\tau,R\right)}=1-\frac{\tau^{3/4}\left[R(1-\tau)-\sqrt{\tau}\beta\right]}{R\left[\sqrt{1-\tau}\sqrt{(R(1-\tau)+\beta)-\sqrt{\tau}\beta}-\tau^{1/4}\beta\right]}. \quad \text{(A.11)}$$

In Eq. (A.11), when we calculate

$$\lim_{\beta\to 0}\left[\eta_{ME}^{N}\left(\beta,\tau,R\right)\right],$$

we obtain

$$\eta_{ME}\left(\tau,R\right)=1-\frac{\sqrt{1-\tau}}{\sqrt{R-R\tau}}\tau^{3/4},$$

and it can be observed that for $\beta = 0$ (endoreversible case) we obtain $\eta_{opt} = 1 - \tau^{3/4}$, which was previously obtained by Velasco *et al.* [9] and also by Angulo Brown and Arias-Hernández [7].

In a similar way to the maximum power conditions, we can calculate the derivative of F_{ME}^{N} with respect to θ and by the appropriate substitution of the parameter β in terms of the fractional fuel cost; in this case, the resultant quadratic equation to be solved for θ is found to be

$$(1-f)\,R\theta^2 + f\sqrt{\tau}\theta - \sqrt{\tau} = 0, \tag{A.12}$$

with solution

$$\theta^{*}{}_{ME}^{N} = \frac{f\sqrt{\tau} - \sqrt{4(1-f)R\sqrt{\tau} + f^2\tau}}{2R(f-1)}. \tag{A.13}$$

After substituting $\theta^{*}{}_{ME}^{N}$ in the expression for the efficiency, we get

$$\eta_{ME}^{N}(f,\tau,R) = 1 - \frac{\tau}{R}\frac{1}{\theta^{*}{}_{ME}^{N}} = 1 - \frac{2\,(f-1)\,\tau}{f\sqrt{\tau} - \sqrt{4(1-f)R\sqrt{\tau} + f^2\tau}}, \tag{A.14}$$

From Eq. (16) the profit function under maximum power regime for the Dulon-Petit heat transfer law $(n = 5/4)$ is given by

$$F_{MP}^{DP} = \frac{1}{a}\frac{\left(1 - \frac{\tau}{R}\frac{1}{\theta}\right)(1-\theta)^{5/4}}{(1-\tau)^{5/4} + \beta\,(1-\theta)^{5/4}}. \tag{A.15}$$

Similarly to the Newton heat transfer law, we can calculate the derivative of F_{MP}^{DP} with respect to θ and by the appropriate substitution of the parameter

$$\beta = \frac{f}{1-f}\frac{(1-\tau)^{5/4}}{(1-\theta)^{5/4}};$$

in this case, the resultant equation to be solved for θ is found to be,

$$\frac{dq_{MP}^{DP}}{d\theta} = -\frac{(f-1)\,(1-\theta)^{1/4}\left[5(f-1)R\theta^2 + \tau(4+\theta-5f\theta)\right]}{4R\,(1-\tau)^{5/4}\,\theta^2} = 0, \tag{A.16}$$

with solution

$$\theta^{*}_{MP^{DP}} = \frac{(5f-1)\tau + \sqrt{-\tau}\sqrt{80(f-1)R - (1-5f)^2\tau}}{10R(f-1)}, \tag{A.17}$$

By substituting $\theta^{*}_{MP^{DP}}$ in the expression for the efficiency we get

$$\eta_{MP}^{DP}(f,\tau,R) = 1 - \frac{\tau}{R}\frac{1}{\theta^{*}_{MP^{DP}}} = 1 - \frac{10R(f-1)}{(5f-1)\tau + \sqrt{-\tau}\sqrt{80(f-1)R - (1-5f)^2\tau}}, \tag{A.18}$$

From Eq. (17) the profit function under maximum ecological regime for the Dulong-Petit heat transfer law $(n = 5/4)$ is given by,

$$F_{ME}^{DP} = \frac{1}{a}\frac{\left[\left(1 + \frac{8+\tau-\sqrt{\tau(\tau+80)}}{\sqrt{\tau(\tau+80)}-9\tau}\right)\left(1 - \frac{\tau}{R}\frac{1}{\theta}\right) - \frac{8+\tau-\sqrt{\tau(\tau+80)}}{\sqrt{\tau(\tau+80)}-9\tau}(1-\tau)\right](1-\theta)^{5/4}}{(1-\tau)^{5/4} + \beta\,(1-\theta)^{5/4}}. \tag{A.19}$$

Likewise, we can calculate the derivative of F_{ME}^{DP} with respect to θ and by the appropriate substitution of the parameter

$$\beta = \frac{f}{1-f}\frac{(1-\tau)^{5/4}}{(1-\theta)^{5/4}};$$

in this case, the resultant equation to be solved for θ is found to be,

$$\frac{dq_{ME}^{DP}}{d\theta} = \left[5R\,(1-f)\,\sqrt{\tau(80+\tau)} + 5R(f-1)\tau\right]\theta^2 + \tau(40f-8)\theta - 32\tau = 0, \tag{A.20}$$

with solution,

$$\theta^*_{ME^{DP}} = \frac{(1-5f)\Lambda + \sqrt{800(1-f)R\Lambda + (1-5f)^2\Lambda^2}}{100(1-f)R}, \tag{A.20}$$

with $\Lambda = \tau + \sqrt{(80+\tau)\,\tau}$. Finally, after substituting $\theta^*_{ME^{DP}}$ in the expression for efficiency we get

$$\eta^{DP}_{ME}\,(f,\tau,R) = 1 - \frac{\tau}{R}\frac{1}{\theta^*_{ME^{DP}}} = 1 - \frac{100\,(1-f)\,\tau}{(1-5f)\,\Lambda + \sqrt{800\Lambda(1-f)R + [(1-5f)\,\Lambda]^2}}. \tag{A.20}$$

1. A. De Vos, *Energy. Convers. Mgmt* **36** (1995) 1.
2. I.I. Novikov, the efficiency of atomic power stations (a review). *Atomimaya Energiya* **3** (1957); 409: *in English translation; I. Nuclear Energy* **7** (1958) 125.
3. F. Curzon and B. Ahlborn, *Am. J. Phys.* **43** (1975) 22.
4. M. A. Barranco-Jiménez and F. Angulo-Brown *Rev. Mex. Fís.* **51** (S2), (2005) 49.
5. M. A. Barranco-Jiménez and F. Angulo-Brown, *J Energy Inst.* **80** (2007) 96.
6. F. Angulo-Brown, *J. Appl. Phys.* **69** (1991) 7465.
7. F. Angulo-Brown and L.A. Arias-Hernández, *J. Appl. Phys.* **89** (2001) 1520.
8. G. Lebon, D. Jou and J. Casas-Vazquez, "*Understanding Non-equilibrium thermodynamics* (Springer-Verlag Berlin Heidelberg. 2008).
9. S. Velasco, J.M.M. Roco, A. Medina, J.A. White, and A. Calvo-Hernández, *J. Phys. D*, **33** D (2000) 355.
10. B. Sahin and A. Kodal, *Energy. Convers. Management* **45** (2004) 1941.
11. M.A. Antar and S.M. Zubair, *Energy. Convers. Management* **42** (2001) 1169.
12. C. Wu and R.L. Kiang, *Energy* **1** (1992) 1173.
13. J Chen, *J. Phys. D: Appl. Phys.* **27** (1994) 1144.
14. L.A. Arias-Hernández, G. Ares de Parga and F. Angulo-Brown, *Open Sys. & Information Dyn.* **10** (2003) 351.
15. A. De Vos, "*Endorreversible Thermodynamics of Solar Energy Conversion*", (Oxford University Press. 1992).
16. L. Chen, F. Sun and C. Wu, *Appl. Energy.* **81** (2005) 388.
17. M. Fisher and K.H. Hoffmann, *J. Non-Equil. Thermodyn.* **29** (2004) 9.
18. P.L. Curto-Risso, A. Medina and A. Calvo-Hernández., *J. Appl. Phys.* **104** (2008) 094911.