Effect of heat transfer on the performance of a thermoelectric heat pump driven by a thermoelectric generator

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A model of a thermoelectric heat pump driven by a thermoelectric generator with external heat transfer irreversibility is proposed. The performance of the combined thermoelectric heat pump device obeying Newton's heat transfer law is analyzed using the combination of finite time thermodynamics and non-equilibrium thermodynamics. Two analytical formulae for heating load versus working electrical current, and the coefficient of performance (COP) versus working electrical current, are derived. For a fixed total heat transfer surface area of four heat exchangers, the allocations of the heat transfer surface area among the four heat exchangers are optimized for maximizing the heating load and the COP of the combined thermoelectric heat pump device. For a fixed total number of thermoelectric elements, the ratio of the number of thermoelectric elements of the generator to the total number of thermoelectric elements is also optimized for maximizing both the heating load and the COP of the combined thermoelectric heat pump device. The influences of thermoelectric element allocation and heat transfer area allocation are analyzed by detailed numerical examples. The optimum working electrical currents for maximum heating load and maximum COP at different total numbers of thermoelectric elements and different total heat transfer areas are provided, respectively.

Keywords: Combined thermoelectric device; thermoelectric generator; thermoelectric heat pump; heat transfer; finite-time thermodynamics; non-equilibrium thermodynamics.

En el presente trabajo se propone un modelo de una bomba de calor termoeléctrica controlada por un generador termoeléctrico con transferencia de calor externa irreversible. Se analiza el desempeño de la bomba de calor combinada, la cual obedece a la ley de Newton de transferencia de calor, usando la combinación de termodinámica de tiempo finito y termodinámica fuera de equilibrio. Se obtienen dos fórmulas analíticas: para la carga de calor y para el coeficiente de desempeño, ambas en función del trabajo de corriente eléctrica. Se realiza una optimización de la posición de la superficie de transferencia de calor entre cuatro intercambiadores para maximizar la carga de calor y el coeficiente de desempeño de la bomba de calor termoeléctrica combinada. Para este mismo fin, se optimiza también la razón entre el número de elementos termoeléctricos del generador y el total. Se analiza, mediante ejemplos numéricos detallados, la influencia entre las posiciones del elemento termoeléctrico y del área de transferencia de calor.

Descriptores: Dispositivo termoeléctrico combinado; generador termoeléctrico; bomba de calor termoeléctrica; transferencia de calor; termodinámica de tiempo finito; termodinámica fuera de equilibrio.

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1. Introduction

Semiconductor thermoelectric power generation, based on the Seebeck effect, and semiconductor thermoelectric cooling, based on the Peltier effect, have very interesting capabilities with respect to conventional power generation, cooling systems and heating systems [1-3]. The absence of moving components results in an increase in reliability, a reduction in maintenance, and an increase in system life; the modularity allows for application in a wide-scale range without significant losses in performance; the absence of a working fluid prevents dangerous environmental leakage; and the noise reduction appears also to be an important feature. Thermoelectric generators, refrigerators, and heat pumps have been used in military, aerospace, instruments, and industrial or commercial products, as power generating, cooling or heating devices for specific purposes. Many researchers are concerned with the physical properties of thermoelectric material and the manufacturing technique of thermoelectric modules. In addition to the improvement of the thermoelectric material and module, the system analysis and optimization of thermoelectric generators, refrigerators and heat pumps are equally important in designing high-performance thermoelectric generators, refrigerators and heat pumps.

In general, conventional non-equilibrium thermodynamics [1,4] is used to analyze the performance of single-stage one- or multiple-element thermoelectric generators [5-13] and heat pumps [14-18]. All objects of this research are independent thermoelectric devices, that is, they generate direct-current power source for users (thermoelectric generator) or need a direct-current power source to provide direct current (thermoelectric heat pump). In some special fields, the heat rejected from the thermal machine may drive a thermoelectric refrigerator or a thermoelectric heat pump through the use of a thermoelectric generator, so that the thermoelectric refrigerator or heat pump does not need an independent power source. This type of new system is different from the traditional thermoelectric systems merely consisting of a thermoelectric generator, a thermoelectric refrigerator or a heat pump. These systems dispense with complicated pipelines and heat insulation, so they can be used in many special fields such as aircraft, submarine, and so on. Chen *et al.* [19] and Khattab *et al.* [20] built a model of this kind of combined system, *i.e.* single-stage thermoelectric refrigerator driven by single-stage thermoelectric generator, and analyzed the performance of the device. Meng *et al.* [21] built a model of a single-stage thermoelectric heat pump driven by single-stage thermoelectric generator, and analyzed the performance of the device.

The theory of finite-time thermodynamics or entropy generation minimization [22-40] is a powerful tool for performance analysis and optimization of practice thermodynamic processes and devices. Some authors have investigated the performances of thermoelectric generators [41-55] and heat pumps [56-60] using the combination of finite-time thermodynamics and non-equilibrium thermodynamics. They analyzed the effects of finite-rate heat transfer between the thermoelectric device and its external heat reservoirs on the performances of single-stage single-element thermoelectric generators [42-48] and heat pumps [56,57]. They also investigated the characteristics of single-stage multi-element thermoelectric generators [49-54] and heat pumps [58,59] and those of two-stage multi-element thermoelectric generators [55] and heat pumps [60] with the irreversibility of finite-rate heat transfer, Joulean heat inside the thermoelectric device, and the heat leakage through the thermoelectric couple leg. However, all of those were performed only for independent thermoelectric devices. There has been no study on performance analysis and optimization for a single-stage thermoelectric heat pump driven by a single-stage thermoelectric generator published in the open literature using the combination of finite-time thermodynamics and non-equilibrium thermodynamics.

On the basis of the exo-reversible model of a single-stage thermoelectric heat pump driven by a single-stage thermoelectric generator without external irreversibility built in Ref. 21, a model of a thermoelectric generator-driven thermoelectric heat pump with internal irreversibilities and external heat transfer irreversibility is built. The performance of the combined thermoelectric device obeying Newton's heat transfer law is analyzed and optimized using the combination of finite-time thermodynamics and non-equilibrium thermodynamics. Two analytical formulae for heating load versus working electrical current, and the coefficient of performance (COP) versus working electrical current, are derived. For a fixed total heat transfer surface area of four heat exchangers, the allocations of the heat transfer surface areas among the four heat exchangers are optimized for maximizing the heating load and the COP of the combined thermoelectric device. For a fixed total number of thermoelectric elements, the ratio of the number of thermoelectric elements of the generator to the total number of thermoelectric elements is also optimized for maximizing both the heating load and the COP of the combined thermoelectric device. The influences of thermoelectric element allocation and heat transfer area allocation are analyzed by detailed numerical examples. The optimum working electrical currents for maximum heating load and maximum COP at different total numbers of thermoelectric elements and different total heat transfer areas, respectively, are provided.

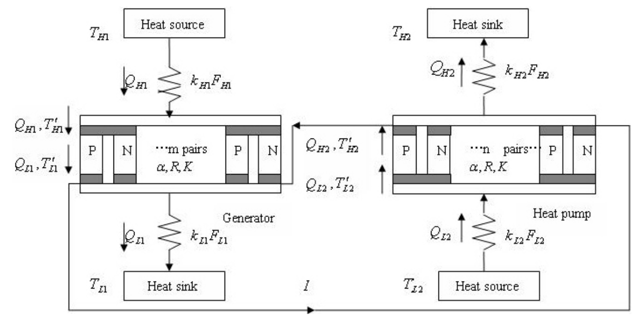

FIGURE 1. Schematic diagram of the combined thermoelectric device

2. Finite-time thermodynamic model of a thermoelectric heat pump driven by a thermoelectric generator device

A schematic diagram of a combined thermoelectric device, *i.e.* a thermoelectric heat pump driven by a thermoelectric generator, is shown in Fig. 1. The device consists of an irreversible single-stage multi-element thermoelectric generator and an irreversible single-stage multi-element thermoelectric heat pump in series with internal and external irreversibilities. The direct-current power source of the heat pump is the direct-current power output of the generator.

The irreversible thermoelectric generator is composed of m pairs of thermoelectric elements. Each element is composed of P-type and N-type semiconductor legs. The thermoelectric power generation element is assumed to be insulated, both electrically and thermally, from its surroundings, except at the junction-reservoir contacts. The internal irreversibility is caused by Joulean electrical resistive loss and heat conduction loss through the semiconductor between the hot and cold junctions. The Joulean loss generates an internal heat I^2R, where R is the total internal electrical resistance of the semiconductor couple and I is the electrical current generating from the semiconductor couple. The conduction heat loss is $K(T_{H1}^{'} - T_{L1}^{'})$, where K is the thermal conductance of the semiconductor couple, $T_{H1}^{'}$ is the hot junction temperature, and $T_{L1}^{'}$ is the cold junction temperature. Finite-rate heat transfers, *i.e.* the temperature differences $(T_{H1} - T_{H1}^{'})$ and $(T_{L1}^{'} - T_{L1})$, where T_{H1} and T_{L1} are the temperatures of the heat source and heat sink of the thermoelectric generator, respectively, cause the external irreversibility. For the thermoelectric generator, the rate of heat transfer at the hot junction is Q_{H1}, and the rate of heat transfer at the cold junction is Q_{L1}.

The irreversible thermoelectric heat pump is composed of n pairs of thermoelectric elements. The Joulean loss generates an internal heat I^2R. The conduction heat loss is $K(T_{H2}^{'} - T_{L2}^{'})$, where $T_{H2}^{'}$ is the hot junction temperature and $T_{L2}^{'}$ is the cold junction temperature. Finite-rate heat transfers, *i.e.* the temperature differences $(T_{H1} - T_{H1}^{'})$ and $(T_{L2}^{'} - T_{L2})$, where T_{H2} and T_{L2} are the temperatures of the heat sink and heat source of the thermoelectric heat pump, respectively, cause the external irreversibility. For the thermoelectric heat pump, the rate of heat transfer at the hot junction is Q_{H2}, and the rate of heat transfer at the cold junction is Q_{L2}.

Assume that the four heat exchangers among the hot and cold junctions of the thermoelectric heat pump, thermoelectric generator and their respective reservoirs are counter-flow, and the heat conductances (product of heat transfer coefficient and heat transfer surface area) of the heat exchangers are $k_{H1}F_{H1}$, $k_{L1}F_{L1}$, $k_{H2}F_{H2}$ and $k_{L2}F_{L2}$, respectively, where k_{H1}, k_{L1}, k_{H2}, and k_{L2} are the heat transfer coefficients of the four heat exchangers, respectively, and F_{H1}, F_{L1}, F_{H2} and F_{L2} are the heat transfer surface areas of the four heat exchangers, respectively.

The total number (M) of thermoelectric element pairs of the irreversible combined thermoelectric device is finite, $M = m + n$. The total heat transfer surface area (F) of the four heat exchangers of the irreversible combined thermoelectric device is finite, $F = F_1 + F_2 + F_3 + F_4$.

Assuming that the heat transfers among the hot and cold junctions of the thermoelectric generator and the thermoelectric heat pump and their respective reservoirs obey Newton's law gives [49,54,58,59]

$$Q_{H1} = k_{H1}F_{H1}\left(T_{H1} - T^{'}_{H1}\right) = m\left[\alpha I T_{H1} + K\left(T^{'}_{H1} - T^{'}_{L1}\right) - \frac{1}{2}I^2R\right] \tag{1}$$

$$Q_{L1} = k_{L1}F_{L1}\left(T^{'}_{L1} - T_{L1}\right) = m\left[\alpha I T_{L1} + K\left(T^{'}_{H1} - T^{'}_{L1}\right) + \frac{1}{2}I^2R\right] \tag{2}$$

$$Q_{H2} = k_{H2}F_{H2}\left(T^{'}_{H2} - T_{H2}\right) = n\left[\alpha I T_{H2} - K\left(T^{'}_{H2} - T^{'}_{L2}\right) + \frac{1}{2}I^2R\right] \tag{3}$$

$$Q_{L2} = k_{L2}F_{L2}\left(T_{L2} - T^{'}_{L2}\right) = n\left[\alpha I T_{L2} - K\left(T^{'}_{H2} - T^{'}_{L2}\right) - \frac{1}{2}I^2R\right] \tag{4}$$

where $\alpha = \alpha_P - \alpha_N$, α_P and α_N are the Seebeck coefficients of the P- and N- type semiconductor legs for each thermoelectric power generation and heating element.

3. Analytical formulae for heating load and COP

Combining Eq. (1) with Eq. (2) gives the hot junction temperature $T^{'}_{H1}$ and the cold junction temperature $T^{'}_{L1}$ of the thermoelectric generator.

$$T^{'}_{H1}=\frac{0.5\alpha Rm^2I^3-(0.5m+m^2K)RI^2+m\alpha T_{H1}I-mK(k_{H1}F_{H1}T_{H1}+k_{L1}F_{L1}T_{L1})-k_{H1}F_{H1}k_{L1}F_{L1}T_{H1}}{m^2\alpha^2I^2+m\alpha(k_{H1}F_{H1}-k_{L1}F_{L1})I-mK(k_{H1}F_{H1}+k_{L1}F_{L1})-k_{H1}F_{H1}k_{L1}F_{L1}} \tag{5}$$

$$T^{'}_{L1}=\frac{-0.5\alpha Rm^2I^3-(0.5m+m^2K)RI^2-m\alpha T_{H1}I-mK(k_{H1}F_{H1}T_{H1}+k_{L1}F_{L1}T_{L1})-k_{H1}F_{H1}k_{L1}F_{L1}T_{H1}}{m^2\alpha^2I^2+m\alpha(k_{H1}F_{H1}-k_{L1}F_{L1})I-mK(k_{H1}F_{H1}+k_{L1}F_{L1})-k_{H1}F_{H1}k_{L1}F_{L1}} \tag{6}$$

Combining Eq. (3) with Eq. (4) gives the hot junction temperature $T^{'}_{H2}$ and the cold junction temperature $T^{'}_{L2}$ of the thermoelectric heat pump.

$$T^{'}_{H2}=\frac{-0.5\alpha Rn^2I^3-(0.5n+n^2K)RI^2-n\alpha T_{H2}I-nK(k_{H2}F_{H2}T_{H2}+k_{L2}F_{L2}T_{L2})-k_{H2}F_{H2}k_{L2}F_{L2}T_{H2}}{n^2\alpha^2I^2+n\alpha(k_{L2}F_{L2}-k_{H2}F_{H2})I-nK(k_{H2}F_{H2}+k_{L2}F_{L2})-k_{H2}F_{H2}k_{L2}F_{L2}} \tag{7}$$

$$T^{'}_{L2}=\frac{-0.5\alpha Rn^2I^3-(0.5n+n^2K)RI^2+n\alpha T_{H2}I-nK(k_{H2}F_{H2}T_{H2}+k_{L2}F_{L2}T_{L2})-k_{H2}F_{H2}k_{L2}F_{L2}T_{H2}}{n^2\alpha^2I^2+n\alpha(k_{L2}F_{L2}-k_{H2}F_{H2})I-nK(k_{H2}F_{H2}+k_{L2}F_{L2})-k_{H2}F_{H2}k_{L2}F_{L2}} \tag{8}$$

Substituting Eqs. (5)- (8) into Eqs. (1)- (4) yields

$$\begin{aligned} Q_{H1} = k_{H1}F_{H1}\{T_{H1} - [&0.5\alpha Rm^2I^3 - (0.5m + m^2K)RI^2 + m\alpha T_{H1}I \\ &- mK(k_{H1}F_{H1}T_{H1} + k_{L1}F_{L1}T_{L1}) - k_{H1}F_{H1}k_{L1}F_{L1}T_{H1}]/[m^2\alpha^2I^2 \\ &+ m\alpha(k_{H1}F_{H1} - k_{L1}F_{L1})I - mK(k_{H1}F_{H1} + k_{L1}F_{L1}) - k_{H1}F_{H1}k_{L1}F_{L1}]\} \end{aligned} \tag{9}$$

$$\begin{aligned}Q_{L1} &= k_{L1}F_{L1}\{[-0.5\alpha R m^2 I^3 - (0.5m + m^2 K)RI^2 - m\alpha T_{H1} I \\ &\quad - mK(k_{H1}F_{H1}T_{H1} + k_{L1}F_{L1}T_{L1}) - k_{H1}F_{H1}k_{L1}F_{L1}T_{H1}]/[m^2\alpha^2 I^2 \\ &\quad + m\alpha(k_{H1}F_{H1} - k_{L1}F_{L1})I - mK(k_{H1}F_{H1} + k_{L1}F_{L1}) - k_{H1}F_{H1}k_{L1}F_{L1}] - T_{L1}\} \qquad (10)\\ Q_{H2} &= k_{H2}F_{H2}\{T_{H2} - [-0.5\alpha R n^2 I^3 - (0.5n + n^2 K)RI^2 - n\alpha T_{H2} I - nK(k_{H2}F_{H2}T_{H2} \\ &\quad + k_{L2}F_{L2}T_{L2}) - k_{H2}F_{H2}k_{L2}F_{L2}T_{H2}]/[n^2\alpha^2 I^2 + n\alpha(k_{L2}F_{L2} - k_{H2}F_{H2})I \\ &\quad - nK(k_{H2}F_{H2} + k_{L2}F_{L2}) - k_{H2}F_{H2}k_{L2}F_{L2}]\} \qquad (11)\\ Q_{L2} &= k_{L2}F_{L2}\{[-0.5\alpha R n^2 I^3 - (0.5n + n^2 K)RI^2 + n\alpha T_{H2} I - nK(k_{H2}F_{H2}T_{H2} \\ &\quad + k_{L2}F_{L2}T_{L2}) - k_{H2}F_{H2}k_{L2}F_{L2}T_{H2}]/[n^2\alpha^2 I^2 + n\alpha(k_{L2}F_{L2} - k_{H2}F_{H2})I \\ &\quad - nK(k_{H2}F_{H2} + k_{L2}F_{L2}) - k_{H2}F_{H2}k_{L2}F_{L2}] - T_{L2}\} \qquad (12)\end{aligned}$$

The overall system is a closed loop circuit, and the heat flow of the system is in equilibrium; one has

$$Q_{H1} + Q_{L2} = Q_{L1} + Q_{H2}. \qquad (13)$$

Substituting Eqs. (9)- (12) into Eq. (13) and re-arranging the results yields the equation that the system stable working electrical current should be satisfied:

$$A_4 I^4 + A_3 I^3 + A_2 I^2 + A_1 I + A_0 = 0 \qquad (14)$$

where

$$\begin{aligned}A_4 &= \alpha^3 m^2 n^2 R(k_{L1}F_{L1} - k_{H1}F_{H1} - k_{L2}F_{L2} + k_{H2}F_{H2}) \qquad (15)\\ A_3 &= \alpha^2 mn[mn(2k_{H2}F_{H2}KR + 2k_{H2}F_{H2}T_{H2}\alpha^2 + 2k_{L2}F_{L2}KR + 2k_{H1}F_{H1}KR \\ &\quad + 2k_{L2}F_{L2}T_{L2}a^2 + 2\alpha^2 k_{L1}F_{L1}T_{L1} + 2k_{L1}F_{L1}KR + 2\alpha^2 k_{H1}F_{H1}T_{H1}) + m(k_{H2}F_{H2}Rk_{H1}F_{H1} \\ &\quad - k_{H2}F_{H2}Rk_{L1}F_{L1} + k_{L2}F_{L2}Rk_{L1}F_{L1} - k_{L2}F_{L2}Rk_{H1}F_{H1} + 2k_{L2}F_{L2}k_{H2}F_{H2}R) \\ &\quad + n(-k_{H2}F_{H2}Rk_{L1}F_{L1} + 2k_{H1}F_{H1}k_{L1}F_{L1}R - k_{L2}F_{L2}Rk_{H1}F_{H1} \\ &\quad + k_{L2}F_{L2}Rk_{L1}F_{L1} + k_{H2}F_{H2}Rk_{H1}F_{H1})] \qquad (16)\\ A_2 &= \alpha[m^2 n(-3k_{L1}F_{L1}KRk_{H2}F_{H2} - 2\alpha^2 k_{L1}F_{L1}T_{L1}k_{H2}F_{H2} + 2\alpha^2 k_{L1}F_{L1}T_{L1}k_{L2}F_{L2} \\ &\quad + 2\alpha^2 k_{H1}F_{H1}T_{H1}k_{L2}F_{L2} + k_{L1}F_{L1}KRk_{L2}F_{L2} - 2\alpha^2 k_{H1}F_{H1}T_{H1}k_{H2}F_{H2} \\ &\quad - 2k_{L2}F_{L2}T_{L2}\alpha^2 k_{H2}F_{H2} + 2k_{H2}F_{H2}T_{H2}\alpha^2 k_{L2}F_{L2} - k_{H1}F_{H1}KRk_{H2}F_{H2} \\ &\quad + 3k_{H1}F_{H1}KRk_{L2}F_{L2}) + mn^2(-k_{L1}F_{L1}KRk_{L2}F_{L2} + 2\alpha^2 k_{L2}F_{L2}T_{L2}k_{H1}F_{H1} \\ &\quad + 3k_{H1}F_{H1}KRk_{L2}F_{L2} - 2\alpha^2 k_{H1}F_{H1}T_{H1}k_{L1}F_{L1} + 2\alpha^2 k_{H2}F_{H2}T_{H2}k_{H1}F_{H1} \\ &\quad - k_{L2}F_{L2}T_{L2}\alpha^2 k_{L1}F_{L1} + k_{H1}F_{H1}KRk_{H2}F_{H2} - 3k_{L1}F_{L1}KRk_{H2}F_{H2} \\ &\quad + 2\alpha^2 k_{L1}F_{L1}T_{L1}k_{H1}F_{H1} - 2k_{H2}F_{H2}T_{H2}\alpha^2 k_{L1}F_{L1}) \\ &\quad + m^2(-k_{L1}F_{L1}Rk_{H2}F_{H2}k_{L2}F_{L2} - k_{H1}F_{H1}Rk_{H2}F_{H2}k_{L2}F_{L2}) \\ &\quad + n^2(k_{L2}F_{L2}Rk_{L1}F_{L1}k_{H1}F_{H1} - k_{H2}F_{H2}Rk_{L1}F_{L1}k_{H1}F_{H1}) \\ &\quad + mn(2k_{L2}F_{L2}Rk_{L1}F_{L1}k_{H1}F_{H1} + 2k_{L1}F_{L1}Rk_{H2}F_{H2}k_{L2}F_{L2} \\ &\quad + 2k_{H2}F_{H2}Rk_{L1}F_{L1}k_{H1}F_{H1} + 2k_{H1}F_{H1}Rk_{H2}F_{H2}k_{L2}F_{L2})] \qquad (17)\end{aligned}$$

$$
\begin{aligned}
A_1 = &-2m^2nK(k_{L2}F_{L2}+k_{H2}F_{H2})(k_{L1}F_{L1}KR+Kk_{H1}F_{H1}R+\alpha^2k_{L1}F_{L1}T_{L1}+\alpha^2k_{H1}F_{H1}T_{H1})\\
&-2m\,n^2K(k_{H1}F_{H1}+k_{L1}F_{L1})(KRk_{L2}F_{L2}+KRk_{H2}F_{H2}+\alpha^2k_{H2}F_{H2}T_{H2}+\alpha^2k_{L2}F_{L2}T_{L2})\\
&-2m^2k_{L2}F_{L2}k_{H2}F_{H2}(KRk_{L1}F_{L1}+KRk_{H1}F_{H1}+\alpha^2k_{L1}F_{L1}T_{L1}+\alpha^2k_{H1}F_{H1}T_{H1})\\
&-2n^2k_{L1}F_{L1}k_{H1}F_{H1}(KRk_{L2}F_{L2}+KRk_{H2}F_{H2}+a^2k_{H2}F_{H2}T_{H2}+\alpha^2k_{L2}F_{L2}T_{L2})\\
&+2mn[\alpha^2(k_{H1}F_{H1}k_{L1}F_{L1}k_{H2}F_{H2}T_{H1}-k_{L2}F_{L2}k_{H2}F_{H2}k_{H1}F_{H1}T_{L2}+k_{L1}F_{L1}k_{H1}F_{H1}k_{L2}F_{L2}T_{L1}\\
&-k_{H2}F_{H2}k_{L2}F_{L2}k_{L1}F_{L1}T_{H2}-k_{L1}F_{L1}k_{H1}F_{H1}k_{H2}F_{H2}T_{L1}+k_{L2}F_{L2}k_{H2}F_{H2}k_{L1}F_{L1}T_{L2}\\
&-k_{H1}F_{H1}k_{L1}F_{L1}k_{L2}F_{L2}T_{H1}+k_{H2}F_{H2}k_{L2}F_{L2}k_{H1}F_{H1}T_{H2})-RK(k_{H2}F_{H2}k_{L1}F_{L1}k_{H1}F_{H1}\\
&+k_{L2}F_{L2}k_{H2}F_{H2}k_{H1}F_{H1}+k_{L2}F_{L2}k_{L1}F_{L1}k_{H1}F_{H1}+k_{L1}F_{L1}k_{L2}F_{L2}k_{H2}F_{H2})]\\
&-2mRk_{H1}F_{H1}\,k_{L1}F_{L1}k_{H2}F_{H2}k_{L2}F_{L2}-2nRk_{L2}F_{L2}\,k_{H2}F_{H2}k_{L1}F_{L1}k_{H1}F_{H1} \qquad (18)
\end{aligned}
$$

$$
\begin{aligned}
A_0 = &\,2mn(Kk_{L1}F_{L1}k_{H1}F_{H1}T_{H1}k_{H2}F_{H2}-Kk_{L1}F_{L1}k_{H1}F_{H1}T_{L1}k_{H2}F_{H2}\\
&+Kk_{L1}F_{L1}k_{H1}F_{H1}T_{H1}k_{L2}F_{L2}-Kk_{L1}F_{L1}k_{H1}F_{H1}T_{L1}k_{L2}F_{L2}\\
&-Kk_{L2}F_{L2}k_{H2}F_{H2}T_{H2}k_{L1}F_{L1}+Kk_{L2}F_{L2}k_{H2}F_{H2}T_{L2}k_{H1}F_{H1}\\
&+Kk_{L2}F_{L2}T_{L2}k_{H2}F_{H2}k_{L1}F_{L1}-Kk_{L2}F_{L2}k_{H2}F_{H2}T_{H2}k_{H1}F_{H1})\\
&+2m(k_{H1}F_{H1}k_{L1}F_{L1}k_{H2}F_{H2}k_{L2}F_{L2}T_{H1}-k_{L1}F_{L1}k_{H1}F_{H1}k_{H2}F_{H2}k_{L2}F_{L2}T_{L1})\\
&+2n(k_{L2}F_{L2}T_{L2}k_{H2}F_{H2}k_{L1}F_{L1}k_{H1}F_{H1}-k_{H2}F_{H2}k_{L2}F_{L2}k_{L1}F_{L1}k_{H1}F_{H1}T_{H2}) \qquad (19)
\end{aligned}
$$

The coefficients of the equation are functions of the parameters of thermoelectric element pairs and parameters concerning heat transfer, so the parameters influence the coefficients of the equation, and then influence the working electrical current and the performance of the device.

For the fixed parameters, one can obtain four theoretical solutions by solving Eq. (14). Analysis showed that there is only one solution I_s, which satisfies $I>0$, $Q_{H1}>0$, $Q_{L1}>0$, and $Q_{L2}>0$. On top of that, one has the cooling load and COP of the combined thermoelectric device as follows:

$$
\begin{aligned}
Q_{H2} = &\,k_{H2}F_{H2}\{T_{H2}-[-0.5\alpha Rn^2I^3-(0.5n+n^2K)RI^2-n\alpha T_{H2}I-nK(k_{H2}F_{H2}T_{H2}\\
&+k_{L2}F_{L2}T_{L2})-k_{H2}F_{H2}k_{L2}F_{L2}T_{H2}]/[n^2\alpha^2I^2+n\alpha(k_{L2}F_{L2}-k_{H2}F_{H2})I\\
&-nK(k_{H2}F_{H2}+k_{L2}F_{L2})-k_{H2}F_{H2}k_{L2}F_{L2}]\} \qquad (20)
\end{aligned}
$$

$$
\beta=\frac{Q_{H2}}{Q_{H1}}=\frac{\begin{array}{c}k_{H2}F_{H2}\{T_{H2}-[-0.5\alpha Rn^2I^3-(0.5n+n^2K)RI^2-n\alpha T_{H2}I-nK(k_{H2}F_{H2}T_{H2}\\+k_{L2}F_{L2}T_{L2})-k_{H2}F_{H2}k_{L2}F_{L2}T_{H2}]/[n^2\alpha^2I^2+n\alpha(k_{L2}F_{L2}-k_{H2}F_{H2})I\\-nK(k_{H2}F_{H2}+k_{L2}F_{L2})-k_{H2}F_{H2}k_{L2}F_{L2}]\}\end{array}}{\begin{array}{c}k_{H1}F_{H1}\{T_{H1}-[0.5\alpha Rm^2I^3-(0.5m+m^2K)RI^2+m\alpha T_{H1}I\\-mK(k_{H1}F_{H1}T_{H1}+k_{L1}F_{L1}T_{L1})-k_{H1}F_{H1}k_{L1}F_{L1}T_{H1}]/[m^2\alpha^2I^2\\+m\alpha(k_{H1}F_{H1}-k_{L1}F_{L1})I-mK(k_{H1}F_{H1}+k_{L1}F_{L1})-k_{H1}F_{H1}k_{L1}F_{L1}]\}\end{array}} \qquad (21)
$$

Obviously, besides the performance parameters of thermoelectric element pairs (α, R and K), parameters concerning the heat transfer (k_{H1}, k_{L1}, k_{H2}, k_{L2}, F_{H1}, F_{L1}, F_{H2} and F_{L2}) would influence the device performance.

If $k_{H1}F_{H1}=k_{L1}F_{L1}=k_{H2}F_{L2}=k_{L2}F_{L2}.\!\rightarrow\!.\infty$, $T_{H1}=T'_{H1}$, $T_{L1}=T'_{L1}$, $T_{H2}=T'_{H2}$ and $T_{L2}=T'_{L2}$, Eqs. (20) and (21) become the results of conventional non-equilibrium thermodynamic analysis [21].

4. Numerical examples

4.1. Design variables and fixed parameters

For an independent thermoelectric generator or thermoelectric heat pump device, there is an optimum heat transfer area allocation between the high temperature side and the low temperature side heat exchangers for the fixed total heat transfer area

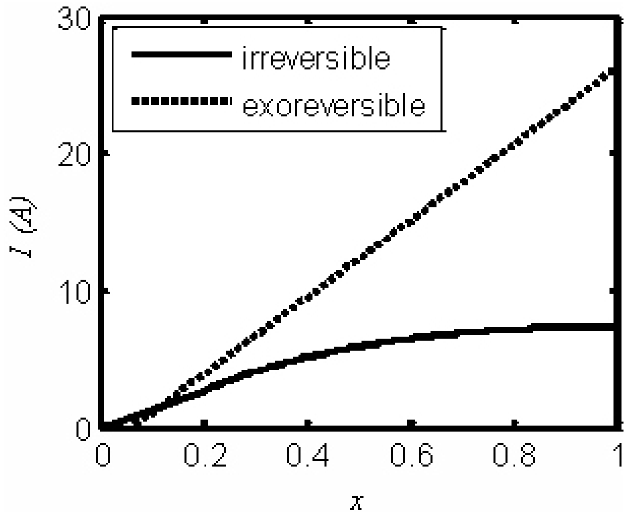

FIGURE 2. Working electrical current vs. the ratio of number of thermoelectric elements.

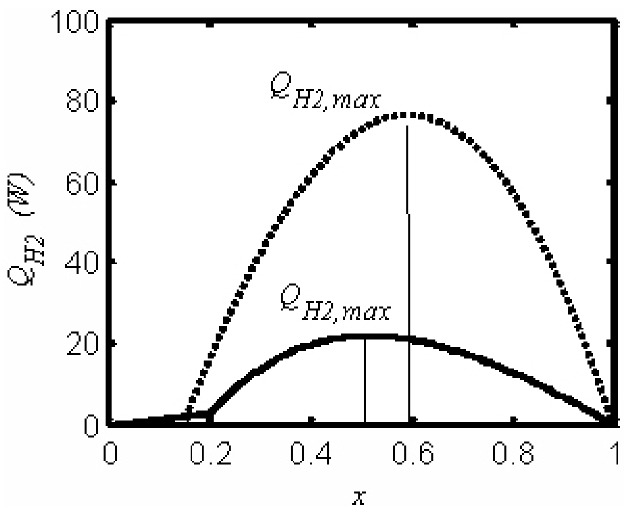

FIGURE 3. Heating load vs. the ratio of number of thermoelectric elements.

of the thermoelectric generator [54] or thermoelectric heat pump [59]. For the combined thermoelectric device discussed here, besides optimum heat transfer surface area allocation between the high temperature side and the low temperature side, there is also an optimum heat transfer surface area allocation between the thermoelectric generator and the thermoelectric heat pump for the fixed total heat transfer surface area of the whole device.

In order to describe the allocations of the heat transfer area, three ratios of heat transfer surface area are defined: the total heat transfer surface area ratio $f = F_1/F$, *i.e.* the ratio of the heat transfer surface area of the thermoelectric generator to the total heat transfer surface area of the combined irreversible device; the generator heat transfer surface area ratio $f_1 = F_{H1}/F_1$, *i.e.* the ratio of the heat transfer surface area of the high-temperature side heat exchanger of the

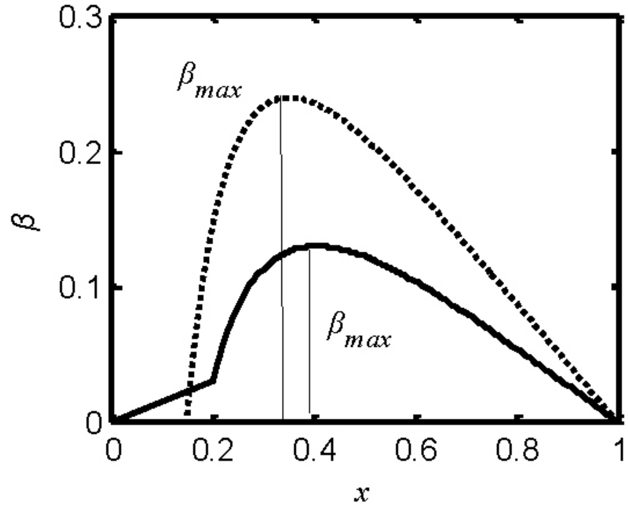

FIGURE 4. COP vs. the ratio of number of thermoelectric elements.

thermoelectric generator to the total heat transfer surface area of the thermoelectric generator; and the heat pump heat transfer surface area ratio $f_2 = F_{H2}/F_2$, *i.e.* the ratio of the heat transfer surface area of the high-temperature side heat exchanger of the thermoelectric heat pump to the total heat transfer surface area of the thermoelectric heat pump. Then, one has

$$F_{H1} = ff_1F, \qquad F_{L1} = f(1-f_1)F,$$
$$F_{H2} = (1-f)f_2F \quad \text{and} \quad F_{L2} = (1-f)(1-f_2)F.$$

In order to describe the allocations of the thermoelectric element pairs, one ratio of numbers of thermoelectric element pairs is defined: $x = m/M$, *i.e.* the ratio of the number of thermoelectric element pairs of the thermoelectric generator to the total number of thermoelectric element pairs of the combined irreversible device. Then, one has $m = xM$ and $n = (1-x)M$.

Obviously, the ranges of the variables x, f, f_1, and f_2 are [0,1].

Numerical calculations are performed in order to analyze and optimize the performance of the thermoelectric heat pump driven by a thermoelectric generator. In the calculations, T_{H1}=$450K$, T_{L1}=$300K$, T_{H2}=$320K$, T_{L2}=$300K$, k_{H1}=$60W/K$, k_{L1}=$15W/K$, k_{H2}=$240W/K$, k_{L2}=$120W/K$, α=$2.1 \times 10^{-4}V/K$, K=$1.6 \times 10^{-2}W/K$, and R=$1.2 \times 10^{-3}\Omega$ are set [53,54,58,59].

4.2. Effects of total number and allocations of thermoelectric elements

The system working electrical current (I), heating load (Q_{H2}) and COP (β) vs. ratio of numbers of thermoelectric elements (x) are shown in Figs. 2-4 by solid lines, respectively. In the calculations, $M = 200$, $F = 1m^2$, $f = 0.6$,

$f_1 = 0.7$, and $f_2 = 0.4$ are set. There exist two optimum working electrical currents corresponding to maximum heating load ($Q_{H2,\max}$) and maximum COP ($\beta_{\max}$). The maximum heating load and maximum COP vs. the total number of thermoelectric elements (M) are shown in Figs. 5 and 6, respectively, by solid lines. In order to compare the results obtained for the irreversible device by using the combination of finite time thermodynamics and non-equilibrium thermodynamics with those obtained for the exoreversible device by using non-equilibrium thermodynamics, the non-equilibrium thermodynamic results, *i.e.* the effects of heat transfer are not taken into account, are also shown in Figs. 2-6 by dotted lines. One can see that the effects of heat transfer are obvious and should be considered in the performance analysis and optimization of the combined irreversible thermoelectric devices.

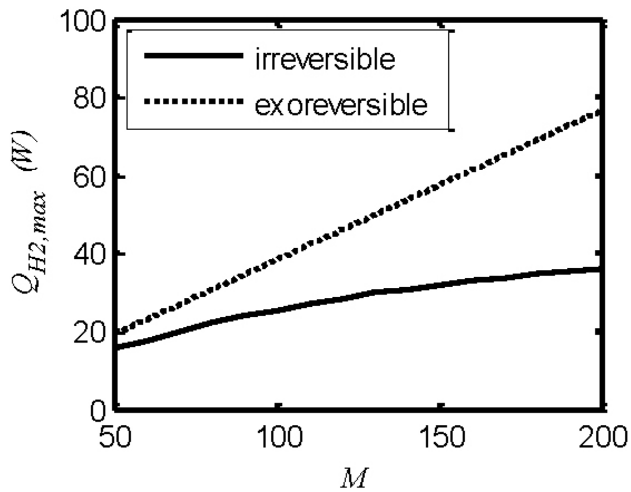

FIGURE 5. Maximum heating load vs. total number of thermoelectric elements.

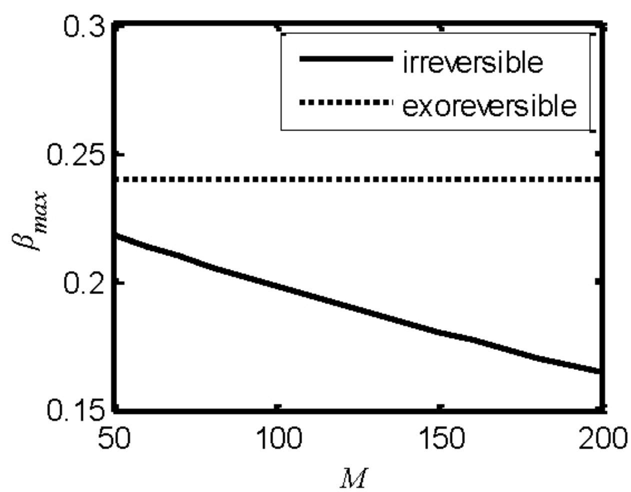

FIGURE 6. Maximum COP vs. total number of thermoelectric elements.

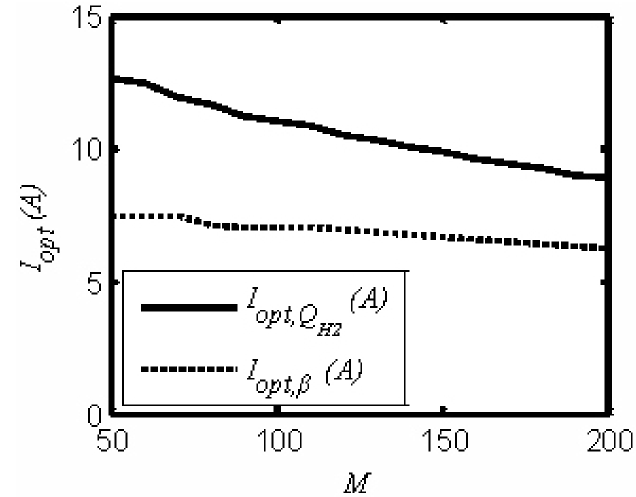

FIGURE 7. Optimum working electrical currents vs. total number of thermoelectric elements.

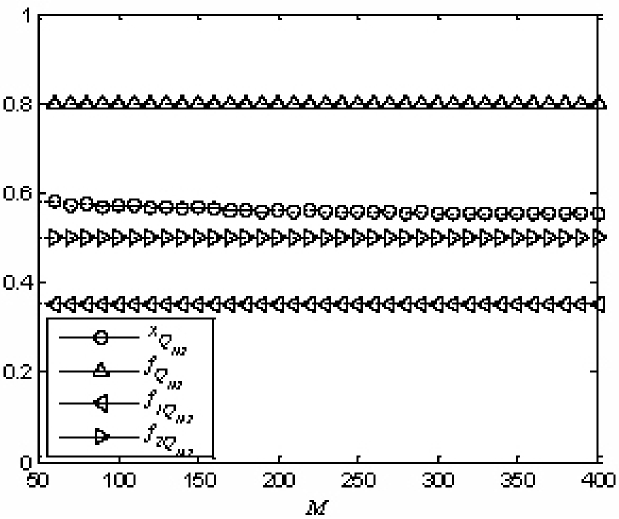

FIGURE 8. Optimum variables at maximum heating load vs. total heat transfer surface area.

The optimum working electrical current ($I_{opt,Q_{H2}}$) corresponding to the maximum cooling load ($Q_{H2,\max}$) and the optimum working electrical current ($I_{opt,\beta}$) corresponding to the maximum COP ($\beta_{\max}$) versus the total number of thermoelectric element pairs (M) are shown in Fig. 7.

The optimum variables corresponding to maximum cooling load ($Q_{H2,\max}$) versus the total number of thermoelectric element pairs (M) are shown in Fig. 8.

One can see that the working electrical current (I) is smaller and no longer proportional to x when effects of heat transfer are taken into account. I increases monotonically and the slope of the curve decreases monotonically with the increase of x. I changes little when x is near 1.

Practical cooling load (Q_{H2}) and COP (β) are smaller. The minimum x, *i.e.* the ratio of number of thermoelectric elements corresponding to a zero cooling load and zero COP is greater when the effects of heat transfer are taken into account. However, heat transfer has little influence on optimum x, *i.e.* the ratios of the number of thermoelectric elements at the maximum heating load and the maximum COP.

The maximum heating load ($Q_{H2,\max}$) is smaller and no longer proportional to M when the effects of heat transfer are taken into account. $Q_{H2,\max}$ increases monotonically and the slope of the curve decreases monotonically with the increase in M. The maximum COP ($\beta_{\max}$) is no longer constant and decreases monotonically with the increase in M.

Both the optimum working electrical currents $I_{opt,Q_{H2}}$ and $I_{opt,\beta}$ decrease with the increase in the total number of thermoelectric elements, and $I_{opt,Q_{H2}} \geq I_{opt,\beta}$ holds. When the total number of thermoelectric elements changes, the optimum variables x, f, f_1 and f_2 remain constant approximately.

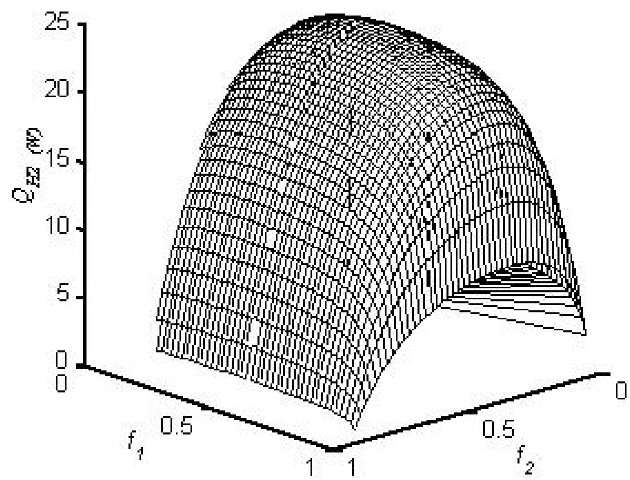

FIGURE 9. Heating load vs. ratios of heat transfer surface areas.

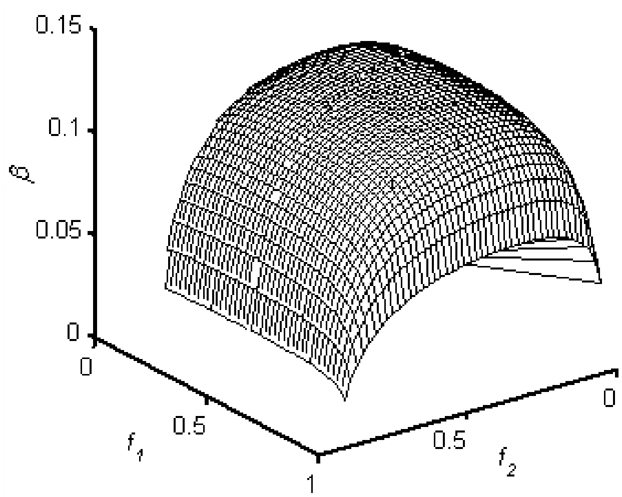

FIGURE 10. COP vs. ratios of heat transfer surface areas.

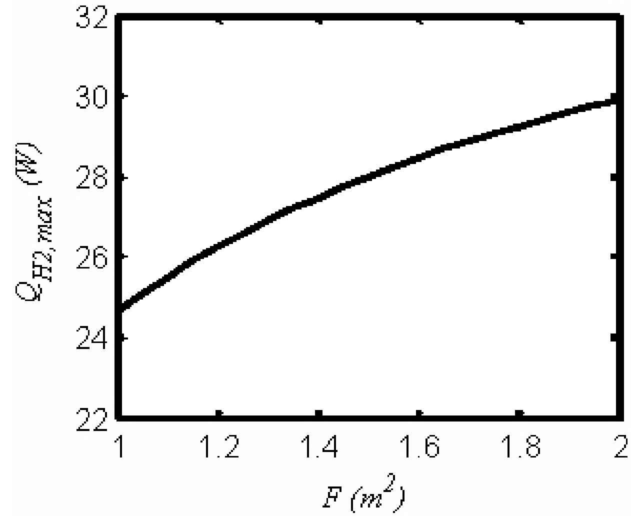

FIGURE 11. Maximum heating load vs. total heat transfer surface area.

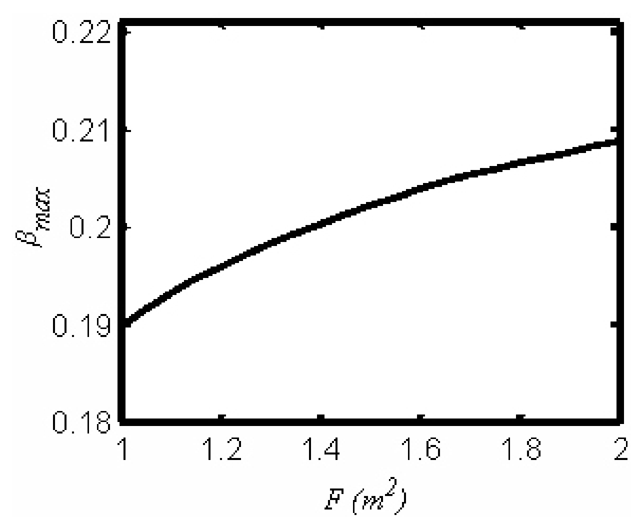

FIGURE 12. Maximum COP vs. total heat transfer surface area.

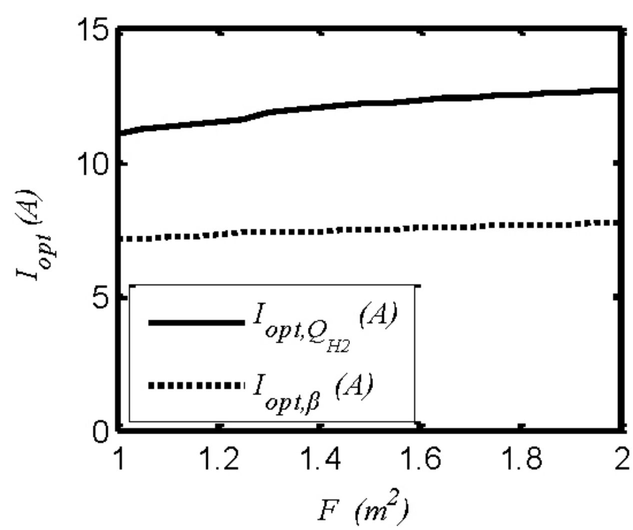

FIGURE 13. Optimum working currents vs. total heat transfer surface area.

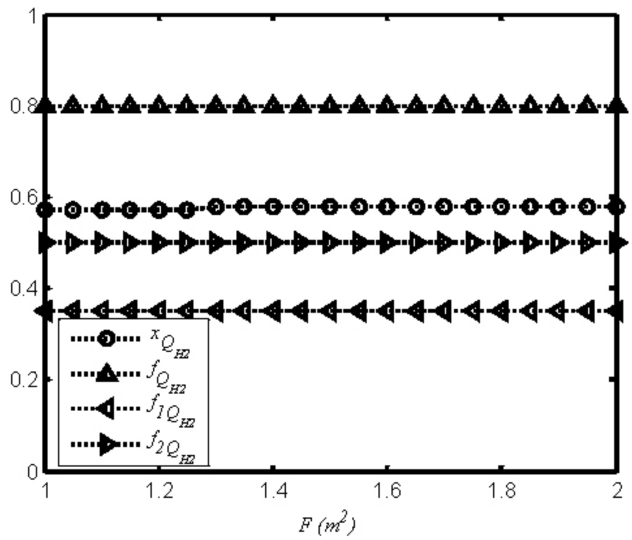

FIGURE 14. Optimum variables at maximum heating load vs. total heat transfer surface area.

4.3. Effects of total heat transfer surface area and area allocations of heat exchangers

The heating load Q_{H2} and the COP β versus heat transfer surface area ratios f_1 and f_2 are shown in Figs. 9 and 10, respectively, with $M = 100$, $F = 1m^2$, and$f = 0.6$.

One can see that there are an optimum f_1 and an optimum f_2 corresponding to the maximum heating load ($Q_{H2,\max}$) and maximum COP ($\beta_{\max}$), respectively. In fact, for a fixed total number of thermoelectric elements (M) and total heat transfer area (F), there are optimum variables x, f, f_1 and f_2 corresponding to the maximum heating load ($Q_{H2,\max}$) and the maximum COP ($\beta_{\max}$), respectively.

Figures. 11-12 show the maximum heating load $Q_{H2,\max}$ and the maximum COP $\beta_{\max}$ versus the total heat transfer surface area F, respectively, with $M = 100$. Fig. 13 shows the optimum working electrical current ($I_{opt,Q_{H2}}$) corresponding to the maximum heating load and the optimum working electrical current ($I_{opt,\beta}$) corresponding to the maximum COP versus the total heat transfer surface area F, respectively, with $M = 100$. One can see that both the maximum heating load and the maximum COP increase monotonically with the increase in F.

The optimum variables corresponding to maximum cooling load ($Q_{H2,\max}$) versus the total heat transfer area (F) are shown in Fig. 14.

The optimum working electrical currents $I_{opt,Q_{H2}}$ increase with the increase in total heat transfer area F while $I_{opt,\beta}$ changes little and $I_{opt,Q_{H2}} \geq I_{opt,\beta}$ holds. When the total heat transfer area changes, the optimum variables x, f, f_1 and f_2 remain constant approximately.

5. Conclusion

A model of an internal and external irreversible thermoelectric heat pump driven by an internal and external irreversible thermoelectric generator is presented in this paper by using a combination of finite-time thermodynamics and non-equilibrium thermodynamics. Two analytical formulae describing the heating load versus working electrical current, and the COP versus working electrical current are derived. The performance optimization of the combined device is performed by searching for the optimum allocations of heat transfer surface areas of the four heat exchangers and the optimum allocations of the number of thermoelectric element pairs based on the optimization of the working electrical current. All parameters should be considered in the design and application of practice thermoelectric devices in order to obtain the maximum economy benefit. The results show that when effects of heat transfer are taken into account, the working electrical current is smaller than that by non-equilibrium thermodynamics. The heating load is smaller and no longer proportional to the total number of thermoelectric elements while the COP is no longer a constant and decreases monotonically with the increase in the total number of thermoelectric elements. For the fixed total number of thermoelectric elements and total heat transfer surface area, there are different optimum variables x, f, f_1 and f_2 corresponding to the maximum cooling load and the maximum COP, respectively. The results obtained here may provide guidelines for the design and application of practical combined thermoelectric devices.

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1. S.W. *Angrist Direct Energy Conversion.* 4^{th} ed. (Boston: Allyn and Bacon Inc., 1992).
2. F.J. di Salvo *Thermoelectric cooling and power generation* **285** (1999) 703.
3. Ma X and S.B. Riffat, *Appl. Thermal Engng.* **23** (2003) 913.
4. A. Bejan, *Advanced Engineering Thermodynamics,* 2^{nd} ed. (New York: Wiley, 1997).

5. A. Sisman and H. Yavuz, *The Int. J.* **20** (1995) 573.
6. J. Chen and Z. Yan, *J. Appl. Phys.* **79** (1996) 8823.
7. J. Chen, Z. Yan, and L. Wu, *The Int. J.* **22** (1997) 979.
8. D.M. Rowe and G. Min *J. Power Sources* **73** (1998) 193.
9. S.A. Omer, and D.G. Infield, *Sol. Energy Mater. & Sol. Cell* **53** (1998) 67.
10. I.D. Mayergoyz, and D. Andrel, *J. Appl. Phys.* **90** (2001) 3019.
11. M. Naji, M. Alata, and M.A. Al-Nimr, *J. Power & Energy* **217** (2003) 615.
12. R.Y. Nuwayhid, A. Shihadeh, and N. Ghaddar, *Energy Convers. Mgnt*, **46** (2005) 1631.
13. J.L. Yu, H. Zhao, *J. Power Sources* **172** (2007) 428.
14. A.J. Mortlock, *Am. J. Phys.* **33** (1965) 813.
15. S.B. Riffat, X. Ma, and G. Qiu, *Int. J. Ambient Energy*, (2004) **25** 177.
16. S.B Riffat and X. Ma, *Int. J. Energy Res.* **28** (2004) 1231.
17. S.B Riffat, Ma X, and R. Wilson, *Appl. Thermal Engng.* **26** (2006) 494.
18. M. Cosnier, G. Fraisse, and L. Luo, *Int. J. Refrig.* (2008) **31** 1051.
19. X. Chen, B. Lin, and J. Chen, *Appl. Energy*, **83** (2006) 681.
20. N.M. Khattab and E.T. El Shenawy, *Energy Convers. Mgmt.* (2006) **47** 407.
21. F. Meng, L. Chen, F. Sun, and C. Wu *Int. J. Ambient Energy* in press.
22. B. Andresen, R.S. Berry, M.J. Ondrechen ,and P. Salamon, *Acc. Chem. Res.* **17**(1984) 266.
23. F. Angulo-Brown, *J. Appl. Phys.* (1991) **69** 7465.
24. M. Feidt, *Thermodynamique et Optimisation Energetique des Systems et Procedes* 2^{nd} Ed. (Paris: Technique et Documentation, Lavoisier,in French, 1996).
25. A. Bejan, *J. Appl. Phys.* **79** (1996) 1191.
26. L.A. Arias-Hernandez and F. Angulo-Brown, *J. Appl. Phys.* **81** (1997) 2973.
27. F. Angulo-Brown, L.A. Arias-Hernandez and Paez-Hernandez. *J. Phys. D: Appl. Phys.* **32** (1999) 1415.
28. R.S. Berry, V.A. Kazakov, S. Sieniutycz, Z. Szwast and A.M. Tsirlin, (Thermodynamic Optimization of Finite Time Processes. Chichester: Wiley, 1999).
29. L. Chen, C. Wuand, and F. Sun, *J. Non-Equilib. Thermodyn.* **24** (1999) 327.
30. J.M Gordon and K.C. Ng Cool, *Thermodynamics.* Cambridge: (Cambridge Int. Science Publishers, UK, 2000).
31. D. Ladino-Luna, *Rev. Mex. Fis.* **48** (2002) 575.
32. A. Durmayaz, O.S. Sogut, B. Sahin and H. Yavuz, *Prog. Energy & Combus. Sci.* **30** (2004) 175.
33. L. Chen, F. Sun, *Advances in Finite Time Thermodynamics: Analysis and Optimization*. (New York: Nova Science Publishers, 2004).
34. D. Ladino-Luna and R.T. Paez-Hernandez *Rev. Mex. Fís.* (2005) 5154.
35. G. Aragon-Gonzalez, A. Canales-Palma, A. Lenon-Galicia, and M. Musharrafie-Martinez, *Rev. Mex. Fis.* **51** (2005) 32.
36. R. Páez-Hernández, F. Angulo-Brown, and M. Santillán, *Engine. J. Non-Equilibrium Thermodynamics* (2006) **31** 173.
37. M.A. Barranco-Jiménez, J.C. Chimal-Eguía, and F. Angulo-Brown, *Rev. Mex. Fís.* (2006) **52** 205.
38. G. Aragon-Gonzalez, A. Canales-Palma, A. Leon-Galicia, and J.R. Morales-Gomez, *Rev. Mex. Fís.* **52** (2006) 309.
39. C.A. Herrera, M.E. Rosillo, and L. Castano, *Rev. Mex. Fís.* **54** (2008) 118.
40. M.A. Barranco-Jimenez, N. Sanchez-Salas, and F. Angulo-Brown, *Rev. Mex. Fís.* **54** (2008) 284.
41. F. Sun, W. Chen, and L. Chen, *Chin. J. Engng. Thermophys.* (1993) **14** 13. (in Chinese).
42. J.M. Gordon, *Am. J. Phys.* **59** (1991) 551.
43. C. Wu, *Energy Convers. Mgmt.* (1993) **34** 1239.
44. D.C. Agrawal and V.J. Menon, *J. Phys. D: Appl. Phys.* (1997) **30** 357.
45. J. Chen, *J. Appl. Phys.*, 1996, **79** 2717.
46. J. Chen and B. Andresen, *Int. J. Pow. Energy System* **17** (1997) 23.
47. J. Chen and C. Wu, *J. Energy Res. Tech.* **122** (2000) 61.
48. R.Y Nuwayhid and F. Monkalled, *Energy Convers. Mgmt.* **45** (2003) 647.
49. L. Chen, J. Gong, F. Sun, and C. Wu, *Int. J. Therm. Sci.* (2002) **41** 95.
50. J. Chen, B. Lin, H. Wang, and G. Lin, *Semicond. Sci. Thenol.* **15** (2000) 184.
51. D.T. Crane and G.S. Jackson, *Energy Convers. Mgnt.* **45** (2004) 1565.
52. L. Chen, F. Sun, and C. Wu, *Appl. Energy* **81** (2005) 358.
53. M. Chen, L. Rosendahl, I. Bach, T. Condra, and J. Pedersen, *Am. J. Phys.* **75** (2007) 815.
54. L. Chen, F. Sun, and C. Wu *Int. J. Ambient Energy* **28** (2007) 135.
55. L. Chen, J. Li, F. Sun, and C. Wu, *Appl. Energy* **82** (2005) 300.
56. C. Wu and W. Schulden, *Energy Convers. Mgmt.* **35** (1994) 459.
57. L. Chen, C. Wu, and F. Sun, *Appl. Thermal Engng. 17* (1997) 103.
58. Y. Bi, L. Chen, C. Wu, and F. Sun, *J. Non-Equilib. Thermodyn.* **26** (2001) 41.
59. L. Chen, J. Li, F. Sun, and C. Wu, *Int. J. Ambient Energy* **28** (2007) 189.
60. L. Chen, J. Li, F. Sun, and C. Wu, *Appl. Energy* **85** (2008) 641.
61. D. Xu, *Application technology of semiconductor refrigerating* (2nd Edition). (Shanghai: Shanghai Jiaotong University Press. 309, 1999).