Decoherence from qubit-qubit dipolar interaction in an all silicon quantum computer

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The qubit-qubit dipolar interaction arises with the very presence of the nuclear qubits in an all–silicon quantum computer (ASQC). Since such interaction depends on the qubit spatial separation noise is induced on entanglement through a damping contribution to the density matrix. A closed condition of "robustness" against decoherence is found. The term "robustness" is define unambiguosly. The existence of at least two exceptional coherent states is infered. The preparation of such states is within the range of present technological capabilities. Thus, the harmful effects of decoherence coming from the dipolar interaction can be diminished considerably. An approximate condition against decoherence, to leading order in 1/(qubit separation)[3], is derived. By expanding the time evolution operator in a series of Chebyshev polynomials an approximated expression for the density matrix, whose precision is of order 10^{-4}, is also found. From this expression, a next-to-leading order approximation for the condition of robustness against decoherence is derived.

Keywords: All-silicon quantum computer; decoherence; robust.

La interacción dipolar qubit-qubit surge con la sola presencia de los qubits nucleares en una computadora cuántica de silicio (ASQC). Como dicha interacción depende de la separación espacial de los qubits, se induce cierto ruido en entrelazamiento a través de una contribución amortiguante a la matriz de densidad. Hallamos una condición exacta de "robustez" contra decoherencia. Eso nos permite defini claramente el término "robustez". De dicha condición, inferimos la existencia de al menos dos estados coherentes exepcionales. La preparación de tales estados es posible usando las capacidades tecnológicas actuales. Así, los efectos dañinos de la decoherencia provenientes de la interacción dipolar pueden ser disminuidos considerablemente. Por otra parte, derivamos una condición contra la decoherencia a primer orden en 1/(separación entre qubits)[3]. Mediante una expansión del operador de evolución temporal en una serie de polinomios de Chebyshev, se halla también una expresión aproximada para la matrix de densidad cuya precisión es del orden 10^{-4}. De tal expresión derivamos una aproximación para la condición exacta de "robustez" contra decoherencia.

Descriptores: Computadora cuántica hecha completamente de silicio; decoherencia; robusta.

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1. Introduction

To build an operative solid state quantum computer one needs to preserve the fragile states of qubits and, at the same time, allow for initialization, control and measurement. The environment disturbs the qubits causing them to decohere. The more intense the interaction with the enviroment is the less the nuclear qubits remain coherent. The longer a qubit remains coherent, the more operations a quantum computer is able to carry out. So far one of the most successful experimental applications of a multi-qubit, many-gate NMR quantum computer, is that discussed in Ref. 1. The main advantages of such computers would be the following:

(i) they remain isolated from control and measurement circuitry by employing weak measurements on a large (10^{18}) arrays of uncoupled identical molecules;

(ii) they behave non-classically [2]. However, as it already has been remarked in Ref. 3, their principal limitation is that they have a very small initial nuclear polarization.

As a consequence, the method cannot exceed the 10-qubit level without substantial modification [3,4]. There is still another NMR quantum computer which was proposed by Kane and others [5]. Through the use of single nuclear spins in a low-temperature solid, the so-called Kane quantum computer solves the scalability problem. The reason is that the quantum computer of Ref. 5 is highly susceptible to measurements with which the respective states have relatively short decoherence times.

The all silicon solid-state NMR quantum computer proposed in Ref. 3 is a good attempt to solve the difficultie mentioned. In Ref. 3 electron-mediated cooling is introduced. At the same time the weak ensemble measurements proper to a successful NMR quantum computer are maintained. With this the authors of this type of work intend to have long enough decoherence times. However, in the approach of Ref. 3 the way in which large decoherence times can be obtained is not clear. Besides, the all silicon qubits are very sensitive to thermal motion. The purpose of the present paper is to study decoherence in an all silicon quantum computer free of the difficultie mentioned above. In order to do this an all silicon quantum computer different from the one employed in Ref. 3 is proposed. The qubits of the present device overcome the decoherence. Furthermore, the constant magnetic fiel gradient in the active region is more reliable. Another of its advantages is that the system of qubits remains isolated from thermal motion. Finally, from the device pre-

sented here it is possible to implement coherent states. For the ASQC introduced here, there are several sources for decoherence, namely: magnetic fluctuation in the active region, fluctuation of paramagnetic impurities in the silicon, thermal motion on the active region, and uncontrolled dipolar couplings between nuclei. We consider that the last one as the most important. For this reason a closed expression for the density matrix elements corresponding to the uncontrolled dipolar interactions between qubits in the proposed ASQC is derived. Through the use of such a expression an exact general "robustness" condition against decoherence is derived. We prove that this condition. is always satisfie by at least two states. A definitio of the term "robustness", in terms of the size of interval of times where unitarity (coherence) prevails, will be given. From the leading order contribution in $1/r^3_{aa'}$ ($r_{aa'}$ is the distance between the active qubits a and a') to the density matrix, a condition against decoherence is found. In order to fin higher order corrections to the density matrix, the time evolution operator in a series of Chebyshev polynomials is decomposed. The series is truncated up to the third term; for this reason the next to leading order approximation for the density matrix has a precision of order 10^{-4}. This assures a high accuracy in the associated condition against decoherence.

Our paper is organized as follows. In Sec. 2 a short account of the main characteristics of the all silicon quantum computer proposed here is given and the conditions against decoherence, coming from the dipolar interactions, are found. In Sec. 3 we comment our proposal, specially, the improvements introduced. We conclude the paper with a discussion of our results.

2. Decoherence in an ASQC

In Fig. 1 a typical chain of silicon spin-1/2 nuclei contained in the active region is shown. The distance in the $z-$direction between two consecutive ^{29}Si nuclei in an atomic chain is $d \sim 1.9$ Å. The lateral sizes of the active region are 100 T/μm and 0.2 T/μm respectively. On the other hand, Fig. 2 illustrates two parallel planes in the active region containing each one, sets of quasi isolated silicon atomic chains. In Fig. 3 the quantum computer proposed in this work for the creation of coherent states is depicted. This device is completely made of silicon, without electrical gates or impurities. The respective qubits are spin-1/2 nuclei of silicon arranged in such a way that they defin quasi isolated atomic chains. The solenoid carrying a current I produces a large magnetic fiel gradient on each of the nuclei in the chains. Each nucleus has about 10^5 ensemble copies in a plane orthogonal to its chain. In Ref. 3 a detailed account of the advantages of using ^{29}Si as a material for a quantum computer is given.

The magnetic fiel gradient due to the solenoid is

$$\delta B/\delta\rho = \frac{96\pi n I b^2}{cL^4} = 1.4T/\mu\text{m}$$

where ρ is the radial coordinate, L is its length, n the linear density of spires, I the carried current, and b the radius. This gradient acts over all of the thickness of the active region and is superposed with a large homogeneous fiel

$$B_o = \frac{\mu_0 I_0}{2a} \sim 3T$$

pointing in the $z-$direction which is produced at the center by a circular spire of current I_0 and radius a. The gradient leads to a qubit-qubit frecuency difference of

$$\Delta\omega = d\gamma\delta B/\delta\rho = 2\pi \times 2 \text{ kHz},$$

being γ the corresponding ^{29}Si gyromagnetic ratio. The active region is 100 μm by 0.2 μm in area, containing $\mathcal{N} = 10^5$ chains persisting over the thickness. This active region is sufficientl small and the magnetic fiel sufficientl homogeneous that all equivalent qubits in an atomic plane lie within a band width of 0.6 kHz.

The secular component of the dipolar Hamiltonian coupling the ath to the a'th spin within one chain is written as [6]

$$\begin{aligned}\hat{\mathcal{H}}_{aa'} =& \frac{\mu_o}{4\pi}\gamma^2\hbar^2\frac{1-3\cos\theta_{aa'}}{r^3_{aa'}}\sigma_z^{(a)}\sigma_z^{(a')} \\ \equiv& -\hbar\delta\omega_{aa'}\sigma_z^{(a)}\sigma_z^{(a')},\end{aligned} \tag{1}$$

where $r_{aa'}$ is the relative distance between the spins, $\theta_{aa'}$ is the angle with the applied field For a nearest neighbor qubits $\cos^2\theta_{a,a+1} = 2/3$ which leads to

$$\delta\omega \equiv \delta\omega_{a,a+1} = 2\pi \times 0.4 \text{ kHz}.$$

According to Eq. (1), the total hamiltonian for a given chain can be decomposed as a sum of the consecutive neighbors, that is

$$\begin{aligned}H_{dip} =& -i\sum_{a\neq a'}\hat{\mathcal{H}}_{aa'} = -i\Big\{\sum_{a=1}^{N-1}\hat{\mathcal{H}}_{a,a+1} + \sum_{a=1}^{N-2}\hat{\mathcal{H}}_{a,a+2} \\ &+ \sum_{a=1}^{N-3}\hat{\mathcal{H}}_{a,a+3} + \cdots + \sum_{a=1}^{2}\hat{\mathcal{H}}_{a,a+N-2}\Big\} \\ =& H^{(1)} + H^{(2)} + H^{(3)} + \cdots + H^{(N-1)}.\end{aligned} \tag{2}$$

In order to study the density matrix elements we will consider only the leading order contribution in $1/r^3_{aa'}$ to Eq. (2), coming from $H^{(1)}$. Let us note that for the device in Fig. 1, the distance between pair of each consecutive neighbors is the same (*e.g.* $r_{a,a+1} = d$ for each $a = 0, 1, 2, \cdots N-1$). Consequently,

$$H^{(1)} = -i\hbar\delta\omega\sum_{a=1}^{N-1}\sigma_z^{(a)}\sigma_z^{(a+1)}. \tag{3}$$

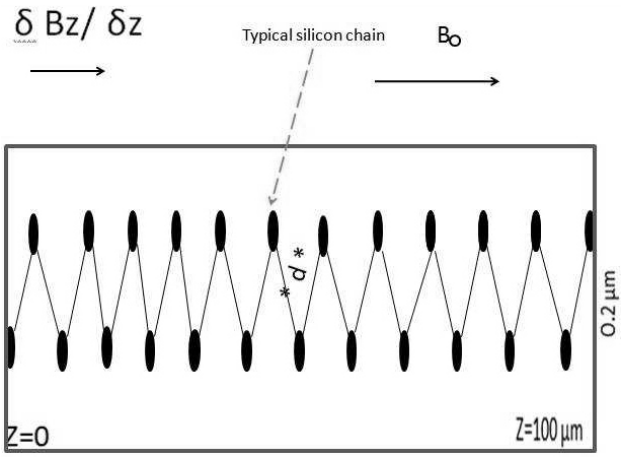

FIGURE 1. View from the top of a typical silicon chain of nuclear spins inside the active region. The values of the background magnetic fiel and the magnetic fiel gradient are: $B_0 = 3T/\mu$m and $\delta B_z/\delta z = 1.4T/\mu$m respectively.

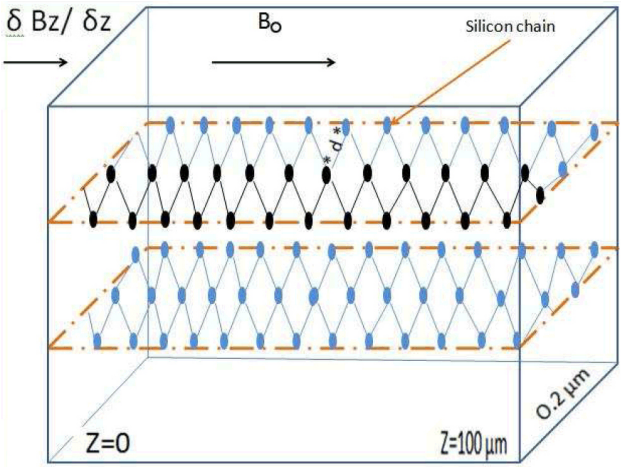

FIGURE 2. Two different planes of the silicon active region containing quasi isolated chains of nuclear spins. The values of the background magnetic fiel B_0 and the magnetic fiel gradient $\delta B_z/\delta z$ are the same as in Fig. 1.

The respective density matrix associated with the dipolar qubit-qubit coupling to leading order in $1/r^3_{a,a'}$, is derived from the Liouville-von Neumann equation in the interaction picture: $i\hbar \mathrm{d}\tilde{R}/\mathrm{d}t = [\tilde{H}^{(1)}(t), \tilde{R}]$ whose respective solution is

$$\tilde{R} = e^{-iH^{(1)}(R(0))t/\hbar} R(0) e^{iH^{(1)}(R(0))t/\hbar}.$$

By using the notation $|\mathbf{n}\rangle = |n_0, n_1, n_2, \cdots, n_N\rangle$ with $\sigma_z^{(a)}|\mathbf{n}\rangle = (-1)^{n_a}|\mathbf{n}\rangle$ being $n_a = 0, 1$, the following is obtained:

$$\rho^{(1)}_{\mathbf{mn}}(t) = \mathrm{e}^{-\{\delta\omega t \sum_{a=0}^{N-1}[(-1)^{m_a+m_{a+1}} - (-1)^{n_a+n_{a+1}}]\}} \rho_{\mathbf{mn}}(0), \quad (4)$$

where $\rho =$tr$\tilde{R}$ and $\delta\omega \equiv \delta\omega_{a,a+1} = 2\pi \times 0.4$ kHz.

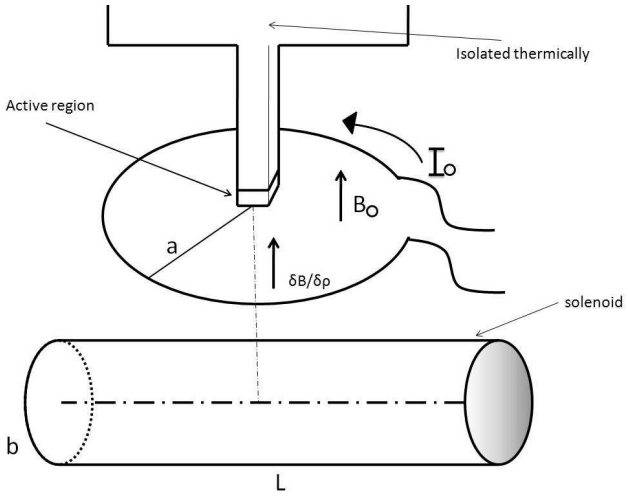

FIGURE 3. The ASQC proposed here. The background magnetic field acting on the active region at the center of the circular spire of current, is $B_0 = \mu_0 I_0/2a = 3$ T, where i is the current in the spire and a is its radius. The magnetic fiel gradient acting on the active region is produced by the radial magnetic fiel of a solenoid at $z = L/2$. The value of this gradient is $\delta B/\delta\rho = 96\pi n I b^2/cL = 1.4\ T/\mu$m , where L is the length of the solenoid, b its radius, n the linear density of spires, and I the carried current. The active region remains isolated from thermal field which might introduce harmful fluctuation in the operating magnetic fields

The interesting feature which emerges from the leading order result of Eq. (4) is that the qubit-qubit dipolar coupling means that at finit times, the states $\mathbf{m}$ and $\mathbf{n}$ are decoherent if

$$\sum_{a=0}^{N-1} \left[(-1)^{m_a+m_{a+1}} - (-1)^{n_a+n_{a+1}}\right] > 0.$$

By decoherence time is understood the value of t for which the off diagonal elements of the density matrix vanish [7]. With the above condition the decoherence times are very small due that $\rho^{(1)}_{\mathbf{mn}}(t) \sim e^{-1000t}$. Another notable, leading order result, which can be extracted from Eq. (4) is that there exists a condition of coherence for the qubit states, namely

$$\sum_{a=0}^{N-1} \left[(-1)^{m_a+m_{a+1}} - (-1)^{n_a+n_{a+1}}\right] \leq 0. \quad (5)$$

Indeed, the above condition together with Eq. (4) implies that $\rho^{(1)}_{\mathbf{mn}}(t)/\rho_{\mathbf{mn}}(0) > 0$ for $0 < t < \infty$ which means that to leading order the ASQC might have very large decoherence times even in the presence of dipolar noise.

The closed expression for the density matrix associated with the hamiltonian H_{dip} of Eq. (2) can be written as

$$\rho_{\mathbf{mn}}(t) = \mathrm{e}^{-\Omega t} \rho_{\mathbf{mn}}(0), \quad (6)$$

where

$$\Omega = \Bigg\{ \delta\omega_{a,a+1} \sum_{a=0}^{N-1} \left[(-1)^{m_a+m_{a+1}} - (-1)^{n_a+n_{a+1}} \right] + \delta\omega_{a,a+2} \sum_{a=0}^{N-2} \left[(-1)^{m_a+m_{a+2}} - (-1)^{n_a+n_{a+2}} \right] + \cdots + \delta\omega_{a,a+N} \left[(-1)^{m_0+m_N} - (-1)^{n_0+n_N} \right] \Bigg\}, \quad (7)$$

From Eqs. (6) and (7), it is easily seen that it is not a simple task to evaluate the density matrix associated with the exact qubit-qubit dipolar interaction. However, from these equations the following general condition of "robustness" against decoherence is extracted:

$$\Omega \leq 0. \quad (8)$$

With Eqs. (6) and (8) we obtain $\rho_{\mathbf{mn}}(t)/\rho_{\mathbf{mn}}(0) > 0$ for $0 < t < \infty$ which means very large decoherence times (*i.e.* large roots of $\rho_{\mathbf{mn}}(t)$ [7]). The term "robustness" employed above, means resistence to loose coherence or unitarity. That is, the length of the longest possible quantum computation is roughly given by the ratio of τ_Q, the time for which the system remains quantum mechanically coherent, to τ_{op}, the time it takes to perform elementary unitary operation [8]. The quantity τ_Q, is called decoherence time and it is the time for which the off diagonal elements of the density matrix become zero: $\rho_{\mathbf{mn}}(t = \tau_Q) = 0$ [7]. The condition (8) together with (6) implies that $\tau_Q/\tau_{op} \to \infty$, providing $\rho_{\mathbf{mn}}(0) \neq 0$. Consequently, for states $|\mathbf{m}\rangle$, $|\mathbf{n}\rangle$ satisfying condition (8), the system remains coherent all the time, allowing for the execution of a huge number of elementary unitary operations. This is the reason why we call (8) the condition of "robustness" against decoherence. On the other hand, by following Ref. 9, it is also possible to argue for Eq. (8) as a condition of "robustness" against decoherence, in an alternative and completely equivalent way. In Ref. 9, the coherence parameter is define as

$$\mathcal{C}(t) = \mathrm{e}^{-\{\delta\omega t \sum_{a=0}^{N-1} [(-1)^{m_a+m_{a+1}} - (-1)^{n_a+n_{a+1}}]\}}$$

being the decoherence time, the firs root of $\mathcal{C}(t)$. With Eq. (8) this is $\tau_Q \to \infty$.

Let us observe that there exist at least a couple of privileged, coherent states, which satisfy the most general condition of "robustness" against decoherence of (8), Indeed, consider for instance the states $|\ \mathbf{1}\rangle = |\ 1, 1, 1, \cdots, 1\rangle$ and $|\ \mathbf{0}\rangle = |\ 0, 0, 0, \cdots, 0\rangle$ which make $\Omega = 0$. It is worth noting here that these states are within present technological capabilities. In fact, the preparation of these ferromagnetic-like states can be engineered through gradient-diffusion methods [10-12] applied to the experimental device of Fig. 3 as follows. A weak initial magnetic fiel gradient $\delta B/\delta\rho \simeq 1.4$ T/μm parallel to the background static quantizing fiel $B_0 \simeq 3$T causes all of the spins in the active region of the silicon sample to precess around positive z-axis with a z-dependent Zeeman rate, thereby acquiring a phase in the silicon nuclei that varies linearly with z [10]. Due to the fact that the size of the active region is very small ($z \ll 100\,\mu$m), all of the phases are almost negligible leading to a common phase factor whose value is approximately one. In this way, the state $|\mathbf{0}\rangle$ where all of the spins are approximately pointing out in the direction $+z$ is obtained. By suddenly inverting the direction of the weak magnetic field to the opposite direction[i], the process is inverted and the precession of the silicon nuclei will be instead around the negative z-axis obtaining the transition to the fina state $|\mathbf{1}\rangle$. In this way, it is concluded that it is possible to neutralize the destructive effects of dipolar decoherence experimentally. Refering to the small errors in the Zeeman phases, it is worth remarking that if the initial magnetic fiel gradient is weak $\delta B/\delta\rho \simeq 1.4$ T/μm, then to a good approximation, a restricted set of errors dominates the information loss. Therefore, an active quantum error correction of the type developed in Ref. 13 can be successfully implemented. This is a task for subsequent research.

In order to gain a deeper insight into decoherence in the presence of the qubit-qubit dipolar interaction, we will make realistic estimations for higher order corrections to $\rho_{\mathbf{mn}}(t)$ which are based on an observation made in Ref. 14 refering to an expansion of the time evolution operator, $e^{-iHt/\hbar}$, in terms of a series of Chebyshev polynomials. The Chebyshev polynomials $T_k(x) = \cos(k arc\cos x)$ are define for $-1 < x < 1$. Consequently, our relevant hamiltonian H_{dip} of Eq. (2), should be rescaled by the factor E_o (the range of the values of the system's energy) and shifted by E_c (mean value of the system's energy):

$$\begin{aligned} E_c &= \frac{1}{2}(E_{max} + E_{min}), \\ E_o &= E_{max} - E_{min}, \\ E_{min} &= \min\langle H_{dip}\rangle = \min_{\langle\Phi|\Phi\rangle=1} \langle \Phi \mid H_{dip} \mid \Phi\rangle, \\ E_{max} &= \max\langle H_{dip}\rangle = \max_{\langle\Phi|\Phi\rangle=1} \langle \Phi \mid H_{dip} \mid \Phi\rangle. \end{aligned} \quad (9)$$

With Eq. (9) it is possible then to defin the rescaled operator $\mathcal{G} = 2(H_{dip} - E_c)/E_o$ which is bounded by -1 and 1: $-1 \leq \langle\Phi \mid \mathcal{G} \mid \Phi\rangle \leq 1$. Here, we take $E_c = 0$ and $E_o = 2\max(|\ E_{min}\ |, |\ E_{max}\ |)$, which implies $-E_o/2 \leq \langle H_{dip}\rangle \leq E_o/2$ and

$$\begin{aligned} E_o/2 \leq E_{max} &= || \ H_{dip} \ || \leq \sum_{a \neq a'} | \ \mathcal{H}_{aa'} \ | \\ &= \sum_{a \neq a'} \hbar \mid \delta\omega_{aa'} \mid || \ \sigma_z^{(a)} \ || \ || \ \sigma_z^{(a')} \ || \\ &= \sum_{a \neq a'} \frac{1}{4}\hbar \mid \delta\omega_{aa'} \mid \end{aligned} \quad (10)$$

From the above equation it follows that

$$E_o \leq E_1 = \sum_{a \neq a'} \frac{1}{2}\hbar \mid \delta\omega_{aa'} \mid, \tag{11}$$

so that the operator $\mathcal{G}$ can be define as $\mathcal{G} = 2\mathcal{H}/E_1$, which satisfie the inequality $-1 \leq \langle \mathcal{G} \rangle \leq 1$.

The Chebyshev's expansion of the evolution operator is now

$$\exp(-iH_{dip}t/\hbar) = \sum_{k=0}^{\infty} c_k T_k(\mathcal{G}). \tag{12}$$

The expansion coefficient c_k can be calculated using the orthogonal property of the polynomials $T_k(x)$,

$$c_k = \frac{a_k}{\pi} \int_{-1}^{1} \frac{T_k(x)\exp(-ix\tau)}{\sqrt{1-x^2}} dx = a_k(-i)^k J_k(\tau), \tag{13}$$

where $\tau = E_1 t/(2\hbar)$, $J_k(\tau)$ is the Bessel function of $k-$th order, $a_0 = 2$ and $a_k = 1$ for $k \geq 1$. The successive terms in the Chebyshev's series can be determined through the recursion formula

$$T_{k+1}(\mathcal{G}) = 2\mathcal{G}T_k(\mathcal{G}) + T_{k-1}(\mathcal{G}), \tag{14}$$

with the conditions $T_0(\mathcal{G}) = 1$ and $T_1(\mathcal{G}) = \mathcal{G}$.

As has already been pointed out in Ref. 14, the high precision of this approach comes from the fact that, for $k \gg \tau$, the value of the Bessel function decreases super-exponentially $J_k(\tau) \sim (\tau/k)^k$, so that termination of the series at $k = K$ leads to an error which decreases super-exponentially with K. In our case, the series is cut off at $K = 2$ so that the approximated expression for the respective density matrix should be

$$\rho_{\mathbf{nm}}(t) \simeq \Big[2J_0(\tau) - J_2(\tau)\big(2\mathcal{G}_n^2 + 1\big) - iJ_1(\tau)\mathcal{G}_n\Big]$$
$$\Big[2J_0(\tau) - J_2(\tau)\big(2\mathcal{G}_m^2 + 1\big) + iJ_1(\tau)\mathcal{G}_m\Big]\rho_{\mathbf{nm}}(0), \tag{15}$$

where $\mathcal{G}_x = (2\hbar/E_1)\sum_{a \neq a'}\sum_{a'} \delta\omega_{aa'}(-1)^{x_a + x_{a'}}$ with $x = m, n$. From the above equation there arises a next to leading order condition of coherence, namely $\mathcal{G}_m - \mathcal{G}_n = 0$ which is read as

$$\sum_{a \neq a'}^{N} \sum_{a'=0} \delta\omega_{aa'}\left[(-1)^{n_a + n_{a'}} - (-1)^{m_a + m_{a'}}\right] = 0. \tag{16}$$

Eqs. (15) and (16) imply that $\rho_{\mathbf{nm}}(t)/\rho_{\mathbf{nm}}(0) > 0$ for each $t \geq 0$. The latter means that the off diagonal elements of the density matrix never disappear, consequently at next to leading order, the quantum system remains coherent indefinitel .

3. Discussion

The ASQC model introduced here improves previous models in two basic aspects:

(i) a more reliable magnetic fiel gradient is considered, and

(ii) a better isolation of thermal currents is used.

As Eq. (1) shows, the dipolar qubit-qubit interaction depends on the qubit spatial separation; due to this, its influenc in the computing capabilities of the ASQC is definit ve. For instance, for a zero qubit-qubit spatial separation ($r_{aa'} \to 0$) in Eq. (1), the frequencies of the qubits grow dramatically, *i.e.* $\delta\omega_{aa'} \to \infty$; consequently, from (6) and (7), $\rho_{\mathbf{nm}}(t) = 0$. That is, the divergences make the decoherence more extreme; therefore, quantum computation does not make any sense. On the other hand, the larger the qubit spatial separation ($r_{aa'} \to \infty$), the smaller the values of the frequencies $\omega_{aa'} \to 0$ and then the lower the intensity of the dipolar interaction ($\hat{\mathcal{H}}_{aa'} \to 0$); thus, the more propicious the preparation of coherence. The latter follows from Eqs. (1), (6), (7), and (8). The leading order approximation in Eq. (4) gives a qualitative understanding of decoherence. The reason for this is that the contribution to the total hamiltonian, H_{dip}, of the nearest neighbor hamiltonian, $H^{(1)}$, is one order of magnitude greater than the contribution to H_{dip} coming from the next to leading order Hamiltonian, $H^{(2)}$. The fact that $\mid\mid H^{(2)} \mid\mid \sim \frac{1}{10} \mid\mid H^{(1)} \mid\mid$ follows from Eq. (1) with $r_{a,a+2} \sim 2r_{a,a+1}$ and $\cos_{a,a+2} \sim 2/3$. The theoretical value of the leading order condition of coherence, given in Eq. (5), lies in the fact that this is a qualitative criterion which suggests the way the states must be prepared for diminishing the undesired effects. An important result of our work is the closed expressions for the density matrix elements due to the uncontrolled qubit-qubit dipolar interaction in the ASQC of Fig. 3 given in Eq. (6). In fact,we can observe from (6) and (7) that if the states $\mid \mathbf{m}\rangle$ and $\mid \mathbf{n}\rangle$ are such that $\Omega > 0$, a finit qubit spatial separation will introduce noise through a —damping— exponential contribution. This leads to the strong decoherence regime ($\tau_Q \sim 0$). On the other hand, when $\Omega \leq 0$, the states are coherent. In this regime this condition define a robust protocol for the ASQC because $\tau_Q/\tau_{op} \to \infty$. By protocol we mean transitions among the qubit states $\mid \mathbf{n}\rangle$ and $\mid \mathbf{m}\rangle$ of Eqs. (6)-(7). A particular and suggestive physical realization where the general condition of "robustness" against decoherence of Eq. (8) is satisfied is the completely polarized states $|\mathbf{1}\rangle = \mid 1, 1, 1, \cdots, 1\rangle$ and $|\mathbf{0}\rangle = \mid 0, 0, 0, \cdots, 0\rangle$. As was explained above, with the ASCQ proposed in this work recurrent transitions between the two highly coherent states can be experimentally implemented. It is worth observing that for weak magnetic field and a very small active region in Fig. 3, the errors in the qubit Zeeman phases are negligible. This would make it possible to implement quantum error correction codes. In fact, a novel and interesting new direction of research is searching for quantum error correction codes in the presence of the dipolar decoherence studied here. This will be investigated in a future work.

We conclude this paper by stating that it is possible to neutralize the harmful effects of decoherence from a dipolar interaction among silicon nuclei. This can be achieved if the system satisfie condition (8), since in such a case $\tau_Q/\tau_{op} \gg 1$. The above condition implies a higher capacity of computing. Clearly, $\mid \mathbf{0}\rangle$ and $\mid \mathbf{1}\rangle$ are not the unique coherent states that satisfy the condition given in Eq. (8), altough they are very suggestive. The states that satisfy the coherence condition of Eq. (16), still process a high number of unitary operations per unit time. The power of the method (*i.e.* the fast convergence) is due to the expansion of the evolution operator in a series of Chebyshev polynomials leading to the coherence condition of Eq. (16). On truncating the series (11) at K=2 order, the precision in the density matrix, Eq. (14), would be of the order 10^{-4}. The condition of coherence given by Eq. (16) is a natural generalization of the weaker, leading order condition, of Eq. (5) although this last result is technologically easier to implement.

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i This can be accomplished by applying pulses of approximated frequencies $\delta\omega = 2\pi\times$ kHz during an interval of time equivalent to two times the Rabi period [8].

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