Spectral line broadening by electron collisions in plasmas

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In this work we compute the broadening of the spectral line shape in a plasma. Precisely we focus our study on the broadening of the spectral line shape by the electrons collisions with the ions of the plasma. During the collision, the electron moves in the effective potential created by all the plasma components (ions and free electrons). Whereas the interaction between the colliding electron and the ion (assumed at the rest) is those of Deutsch. The latter takes into account the quantum effect at short distance. The corresponding broadening is computed for the case of the spectral line for Lyman-alpha of Li^{+2} (Hydrogen-like of Lithium) and compared with the case where the interaction is that of Coulomb.

Keywords: Stark broadening; broadening by electrons; deutsch potential; screened deutsch potential.

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1. Introduction

The analysis of the spectral line shapes allows us to discover the effects of particles surrounding the emitter (atom or ion) in a plasma. These effects are the result of the interactions between the charged particles of the plasma with the emitter. Qualifying the surrounding particles as the perturbers, the Stark broadening of the spectral line can be derived by the so-called dipolar approximation using the semi-classical treatment. In fact, the Stark effect occurs when an ion or an atom of the plasma, is perturbed by its interaction with the local electric field prevaling at all points of the plasma.

The theoretical studies of the broadening of the spectral line allow us to compare the predicted theoretical results with the experimental results, like the temperature, the density and the different species composing the plasma. In the present work, we are concerned by the broadening of the spectral line by the electrons by using the following approximations: a) the classic path [1], which requires that the perturber (electron) movement must be described by a classical mechanics, the perturber path was a straight line both for a neutral and charged emitter; however, it has been observed that the electron perturber has a hyperbola path when it collides with an ion in the plasma in the binary interaction model. b) the impact approximation [2] that means that the interactions are separated in time: in other words, the atom interacts with the perturber only at a given time: the mean duration, of an interaction must be much smaller than the mean time interval between two collisions, c) the semi-classical approximation [3–5] that consists to treat the emitter as a quantum object whereas the surrounding is a classic perturber.

The authors [6,7], taking in consideration the paper of [4], have developed the basic quantum formalism for the spectral line shape, and showed that the width and the shift of the spectral lines emitted from α level to a β level, can be expressed simply by computing the scattering matrix elements $S_{\alpha\alpha}$ and $S_{\beta\beta}$ between the emitter atom and the electron. For the isolated lines, Baranger has shown that the width is expressed as a sum of collision cross sections. So the calculation of the collision operator for the electrons, for these lines, has seen a great step while [7, 8], have used with success the theoretical results for [6] in the framework of the semi-classical approximation. However, in the above references, no discussion about the effect of energy interaction between the colliding electron and emitter in the formulation of the broadening by electron of the line shape, was evoked. In our work, we investigate the collision (electron with ion) by considering the interaction between the colliding electron and the emitter (a target), to be that of Deutsch potential energy [9]. Furthermore, the colliding electron moves in the effective potential created at any locality by all the plasma. We have computed, by using the mean field approximation, the effective potential of the plasma in Sec. 2.

This paper is organized in six sections: in the following section, we give the theoretical derivation of the effective potential of the plasma. We find that it obeys an integral equation. In Sec. 3 we derive the collision operator amplitude for any central potential. We show that the collision operator can be expressed simply by the scattering cross section. Section 4 is devoted to computing the cross section of scattering by using the integral equation for the effective potential stated in Sec. 2. Section 5 addresses to give results and contains a comparison between Lyman-α lines obtained from each potential: Coulomb and the effective one and some discussions. The comparison was made for astrophysical plasmas like the one we find in hot stars, in plasma fusion and in supernova explosions where the extreme conditions of high temperatures and densities are present [10]. We close this paper by a conclusion in Sec. 6.

2. The effective plasma potential

Consider a plasma at the thermodynamical equilibrium at a temperature T and electron density N_e. The velocities of the electrons are considered to obey the distribution function

$$f(\overrightarrow{r}, \overrightarrow{v}) = N_e \left(\frac{m}{2\pi k_B T}\right)^{3/2} \cdot \exp\left(-\frac{m\overrightarrow{v}^2}{2k_B T} - \frac{\Phi(r)}{k_B T}\right) \tag{1}$$

where k_B is the Boltzmann constant.

The effective potential energy of one electron located at the distance r is equal to a sum of three contributions

$$\Phi(r) = U_{\text{Deut}}(r) + \Phi(r)_{e-e} + \Phi(r)_{e-f} \tag{2}$$

where: - $U_{\text{Deut}}(r)$ is interaction energy of the pair (ion emitter-electron).

- $\Phi(r)_{e-e}$ is interaction energy of the pair (electron-electron).
- $\Phi(r)_{e-f}$ is interaction energy of the electron with a continuous background of positive charge representing the ions.

The interaction electron-electron is assumed to be the Coulomb potential, whereas the interaction ion emitter with an electron is that of the Deutsch interaction that takes into account the quantum effect at short distances:

$$U_{\text{Deut}}(r) = -\frac{Ze^2}{r}\left(1 - e^{-r/\lambda_T}\right) \tag{3}$$

where $\lambda_T = \hbar/\sqrt{(2\pi m k_B T)}$ is the thermal wavelength. Then the effective potential energy of the electron located at r from the frame origin is

$$\Phi(r) = U_{\text{Deut}}(r) + \int \frac{f(\overrightarrow{r}, \overrightarrow{v})\, e^2}{\left|\overrightarrow{r} - \overrightarrow{r'}\right|}\, d\overrightarrow{p}^3 d\overrightarrow{r'}^3 - \int \frac{N_e e^2}{\left|\overrightarrow{r} - \overrightarrow{r'}\right|}\, d\overrightarrow{r'} \tag{4}$$

When we integrate over the velocity in the above equation we find the integral equation that governs the effective potential energy as

$$\Phi(r) = -\frac{Ze^2}{r}(1 - e^{-r/\lambda_T}) + N_e e^2 \int \frac{(e^{-\beta\Phi(r')} - 1)}{\left|\overrightarrow{r} - \overrightarrow{r'}\right|} d\overrightarrow{r}' \tag{5}$$

where $\beta = (k_B T)^{-1}$. We easily read from the last integral equation that, if we put $\lambda_T = 0$ (neglecting the quantum effects at short distance) and put $\beta = 0$ (neglecting the collective interaction and keeping the binary interaction during the collision), we get the purely Coulomb potential ($-Ze^2/r$). In section four, we shall see how to solve this integral equation to get the effective potential $\Phi(r)$.

3. Collision operator of the electrons W_e

In the case of the impact approximation, and we do not take into account the fine structure of the radiator [11], the collision operator is given by

$$\begin{aligned} W_e = -N_e e^2 \left(\frac{\hbar}{m}\right)^2 \int\int 2\pi\rho v f(v) d\overrightarrow{v} d\rho \\ \times \Bigg\{ \int_{-\infty}^{+\infty} dt_1 \vec{R}_b.\vec{E}(t_1) \int_{-\infty}^{t_1} dt_2 \vec{R}_b.\vec{E}(t_2) \\ + \int_{-\infty}^{+\infty} dt_1 \vec{R}_a.\vec{E}(t_1) \int_{-\infty}^{t_1} dt_2 \vec{R}_a.\vec{E}(t_2) \\ - \int_{-\infty}^{+\infty} dt_1 \vec{R}_a.\vec{E}(t_1) \int_{-\infty}^{+\infty} dt_1 \vec{R}_b.\vec{E}(t_1) + \cdots \Bigg\} \end{aligned} \tag{6}$$

where v is the colliding electron velocity, N_e the electron density, ρ is the impact parameter and $\vec{R}_{a,b}$ is the position operator of the bounded electron for the lower state b and the upper state a corresponding to the line under consideration in our study. We note here that $\vec{E}(t)$ is the electric field due to the scattered electron at the emitter ion. It is natural that this electric field depends on the trajectory of the scattered electron that itself depends on the potential in which this electron moves. We have to choose four potentials as indicated above in the introduction. By using Newton's equation describing the electron movement around the ion emitter (located at the origin of the spatial frame of coordinates), and submitted only to the electric micro-field due to the local potential, we have

$$m\ddot{\vec{r}}(t) = -e\vec{E}(t) \tag{7}$$

If we substitute $-e\vec{E}(t)$ by $m\ddot{\vec{r}}(t)$ in the formula (6), we can integrate it by part on t_1 and t_2 and we immediately find

$$\begin{aligned} W_e = -N_e (\frac{\hbar}{e^2})^2 \int\int 2\pi\rho v f(v) d\overrightarrow{v} d\rho \\ \Bigg\{ \frac{1}{2}\left[\overrightarrow{R}_a \left(\frac{d\overrightarrow{r}}{dt}(+\infty) - \frac{d\overrightarrow{r}}{dt}(-\infty)\right)\right]^2 \\ + \frac{1}{2}\left[\overrightarrow{R}_b \left(\frac{d\overrightarrow{r}}{dt}(+\infty) - \frac{d\overrightarrow{r}}{dt}(-\infty)\right)\right]^2 \\ - \overrightarrow{R}_b \left[\frac{d\overrightarrow{r}}{dt}(+\infty) - \frac{d\overrightarrow{r}}{dt}(-\infty)\right] \\ \overrightarrow{R}_a \left[\frac{d\overrightarrow{r}}{dt}(+\infty) - \frac{d\overrightarrow{r}}{dt}(-\infty)\right] + \cdots \Bigg\} \end{aligned} \tag{8}$$

The in-velocity and the out-velocity (before and after the electron-ion collision) are related to the scattering angle θ and the impact parameter ρ by:

$$\frac{d\overrightarrow{r}}{dt}(+\infty) = \overrightarrow{v}(+\infty) = \overrightarrow{v}(-\infty) + \left(\frac{v}{\rho}\right)\overrightarrow{\rho}\sin\theta \quad (9)$$

where

$$v(-\infty) = v(+\infty) = v \quad (10)$$

The formula (9-10) are due to the fact that we deal in our investigation with conservative potentials. Then the collision operator W_e becomes as

$$W_e = ((\overrightarrow{R}_a)^2 + (\overrightarrow{R}_b)^2 - 2\overrightarrow{R}_a\overrightarrow{R}_b)\phi, \quad (11)$$

where

$$\phi = -\frac{4}{3}\pi N_e\left(\frac{\hbar}{e^2}\right)^2\int\int v^3 g(v)dv\rho d\rho\sin^2\left(\frac{\theta}{2}\right), \quad (12)$$

is the amplitude of the collision operator and $g(v)$ is Maxwell equilibrium velocities distribution given by

$$g(v) = 4\pi\left(\frac{m}{2\pi k_B T}\right)^{\frac{3}{2}} v^2\exp\left(\frac{-mv^2}{2k_B T}\right), \quad (13)$$

then the amplitude of the collision operator becomes

$$\phi = \frac{-16}{3}\pi^2 N_e\left(\frac{\hbar}{e^2}\right)^2\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} \times\int dv v^5\exp\left(\frac{-mv^2}{2kT}\right)\int\sin^2\left(\frac{\theta}{2}\right)\rho d\rho \quad (14)$$

Rather than to compute the trajectory of the scattered electron (ρ,θ), we shall use the scattering amplitude and the associated cross section for each potential. To do this, we replace θ appearing in the last integral by its expression as function of the impact parameter ρ. The expression can be found easily by using the amplitude diffusion and the scattering cross section formula. In one hand the differential cross section is defined as:

$$d\sigma = 2\pi\rho\frac{d\rho}{d\theta}d\theta. \quad (15)$$

In the other hand, it is related to the scattering amplitude $F(\theta)$, in the Born approximation, by

$$d\sigma = |F(\theta)|^2 d\omega \quad (16)$$

where $d\omega = 2\pi\sin\theta d\theta$ is the element of solid angle, and

$$F(q) = \frac{-m}{2\pi\hbar^2}\int\Phi(r)\exp(-i\overrightarrow{q}.\overrightarrow{r})d\overrightarrow{r} \quad (17)$$

where $(q = 2k\sin(\theta/2))$.

4. Calculation of the the effective potential

This concerns the collective effect, which consists of taking into account the interaction of a free electron of the plasma with all other particles (ions and electrons). The effective potential energy at the electron is given by [12]:

$$\Phi(r) = U_{\text{Deut}}(r) + N_e e^2\int\frac{\exp(-\beta\Phi(r')) - 1}{|\mathbf{r}-\mathbf{r}'|}d\overrightarrow{r'} \quad (18)$$

where $U_{\text{Deut}}(r)$ is the Deutsch potential described above. In the case of weakly coupled plasma, or in the case of high temperature, it is a good approximation if we replace $\exp(x)$ by $(1+x)$ as $x << 1$. Then the integral equation becomes

$$\Phi(r) = U_{\text{Deut}}(r) - N_e\beta e^2\int\frac{\Phi(r')}{|\mathbf{r}-\mathbf{r}'|}d\mathbf{r}' \quad (19)$$

Now we need the Fourier transform of the effective potential energy $\Phi(r)$ since it is related to the scattering amplitude $F(\theta)$ via Eq. (17). It suffices then to take the Fourier transform of Eq. (19) in both sides.

$$F(q) = F_{\text{Deut}}(q) - 4\pi(N_e\beta e^2/q^2)F(q), \quad (20)$$

or

$$F(q) = \frac{q^2 F_{\text{Deut}}(q)}{(q^2+\lambda''^2)}, \quad (21)$$

where

$$\begin{aligned} F_{\text{Deut}}(q) &= \frac{-m}{2\pi\hbar^2}\int U_{\text{Deut}}(r)\exp(-i\overrightarrow{q}.\overrightarrow{r})d\overrightarrow{r} \\ &= \frac{2m\alpha}{\hbar^2}\left(\frac{1}{q^2}+\frac{1}{\lambda'^2+q^2}\right) \end{aligned} \quad (22)$$

and

$$\lambda'^2 = 2\pi m k_b T/h^2 \sim \frac{1}{\lambda_T^2};$$
$$\lambda''^2 = 4\pi N_e e^2/(k_B T) \sim \frac{1}{\lambda_D^2} \quad (23)$$

It is worth to mention here if β=0 and λ_T=0 in Eq. (20), we recover the scattering amplitude of the Coulomb case $(F(q) \simeq 1/q^2)$. Using formulae (15-16)

$$\rho d\rho = 2\sin\left(\frac{\theta}{2}\right)\cdot\cos\left(\frac{\theta}{2}\right)|F(\theta)|^2 d\theta \quad (24)$$

or

$$\rho d\rho = \frac{q}{k^2}\left[\frac{q^2 F_{\text{Deut}}(q)}{(q^2+\lambda''^2)}\right]^2 dq \quad (25)$$

we find the amplitude of the collision operator (14) as the following expression

$$\phi = \frac{-16}{3}\pi^2 N_e\left(\frac{\hbar}{e^2}\right)^2\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}}\int_0^{+\infty} dv v^5 \times\exp\left(\frac{-mv^2}{2kT}\right)\int_{q_{\min}}^{q_{\max}}\left(\frac{q^3}{4k^4}\right)\left[\frac{q^2 F_{\text{Deut}}(q)}{(q^2+\lambda''^2)}\right]^2 dq. \quad (26)$$

By replacing $k = mv/\hbar$ and $F_{\text{Deut}}(q)$ by its expression (22) in the last formula, we find

$$\phi = \frac{-16}{3}\pi^2 N_e \left(\frac{Z\hbar}{m}\right)^2 \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} \times \int_0^{+\infty} v dv \exp\left(\frac{-mv^2}{2kT}\right)(G(q_{\max}) - G(q_{\min})), \quad (27)$$

where

$$G(x) = \int \left[\frac{\frac{1}{x^2} + \frac{1}{\lambda'^2 + x^2}}{\lambda''^2 + x^2}\right]^2 x^7 dx. \quad (28)$$

The last integral is easy to perform and it is equal to

$$G(x) = x^2 + \frac{1}{4}x^4 - \frac{a^6 - 2a^8 - 3a^4b^2 - 2a^4b^4 + 4a^6b^2}{2b^6 - 2a^6 - 6a^2b^4 + 6a^4b^2}\ln\left(a^2 + x^2\right) - \frac{2b^8 - b^6 + 3a^2b^4 - 4a^2b^6 + 2a^4b^4}{2b^6 - 2a^6 - 6a^2b^4 + 6a^4b^2}\ln\left(b^2 + x^2\right) + \frac{a^2b^6 + a^6b^2 + a^6x^2 + b^6x^2}{D}, \quad (29)$$

where we have considered $\lambda' = a$, $\lambda'' = b$ and

$$D = 2a^2b^6 - 4a^4b^4 + 2a^6b^2 + 2a^4x^4 + 2a^6x^2 + 2b^4x^4 + 2b^6x^2 - 4a^2b^2x^4 - 2a^2b^4x^2 - 2a^4b^2x^2. \quad (30)$$

So we need the limits $q_{\min}$ and $q_{\max}$ in the Eq. (27). From Eq. (25) we can extract $q_{\min}$ and $q_{\max}$ from $\rho_{\min}$ and $\rho_{\max}$ in the following way

$$\frac{\lambda_D^2}{2} = \rho_{\max}^2/2 = \int_{q_{\max}}^{2k} \frac{q}{k^2}\left[\frac{q^2 F_{\text{Deut}}(q)}{(q^2 + \lambda''^2)}\right]^2 dq = \Lambda(2k) - \Lambda(q_{\max}) \quad (31)$$

and

$$\frac{\lambda_T^2}{2} = \rho_{\min}^2/2 = \int_{q_{\min}}^{2k} \frac{q}{k^2}\left[\frac{q^2 F_{\text{Deut}}(q)}{(q^2 + \lambda''^2)}\right]^2 dq = \Lambda(2k) - \Lambda(q_{\min}) \quad (32)$$

where $\Lambda(x)$ is given by ($\lambda' = a, \lambda'' = b$)

$$\Lambda(x) = \left(\frac{2m\alpha}{k\hbar^2}\right)^2 \left[\frac{1}{2}x^2 - \frac{a^6 + a^2b^2 + a^2b^4 - 2a^4b^2}{b^6 - a^6 - 3a^2b^4 + 3a^4b^2}\ln\left(a^2 + x^2\right) - \frac{2a^2b^4 - a^2b^2 - b^6 - a^4b^2}{b^6 - a^6 - 3a^2b^4 + 3a^4b^2}\ln\left(b^2 + x^2\right) + \frac{-a^2b^4 - a^4b^2 - a^4x^2 - b^4x^2}{D}\right] \quad (33)$$

When we insert the expression of $\Lambda(x)$ in formula (31-32), we get numerically $q_{\min,\max}$ as function of the velocity v. When we replace them in the expression of $G(q_{\min})$ and $G(q_{\max})$ given by (30), we integrate over v in formula (27) to have the amplitude of the collision operator. The result of this task, is illustrated in the Figs. 1 and 2 for various densities and temperatures.

5. Results and discussion

It is easy to read from the Eqs. (31-32) that $q_{\min}$ and $q_{\max}$ depend on the velocity v because the presence of $k = mv/\hbar$ in those equations. This means that the integral over v in formula (27) must take this fact into account. Then, for a given $\rho_{\max}$, the Eq. (31) must be solved numerically for all velocity values. We get therefore $q_{\max}$ as a function of v. The same thing must be done with the Eq. (32) to get $q_{\min}$ as a function

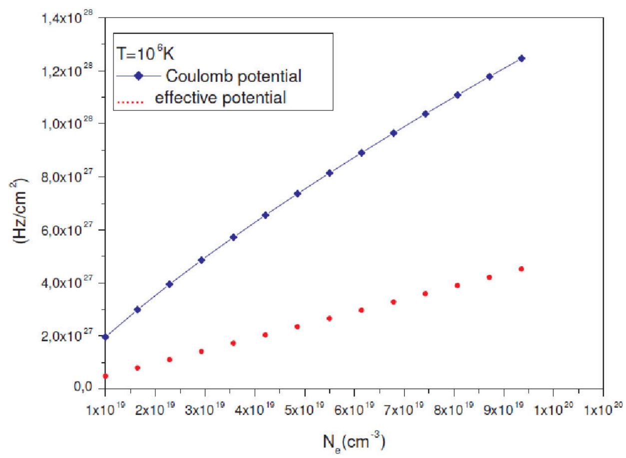

FIGURE 1. Amplitude of collision operator versus the density.

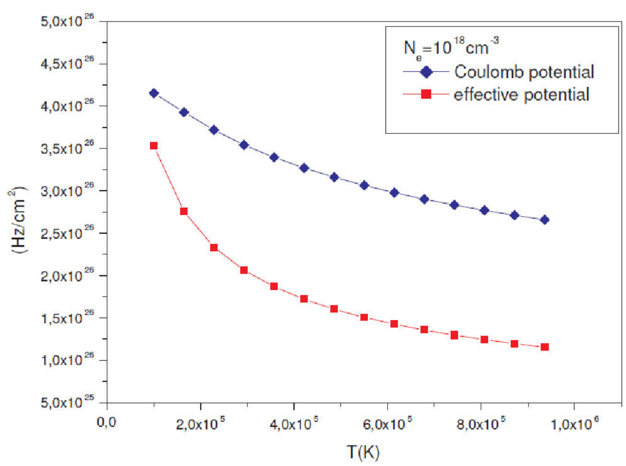

FIGURE 2. Amplitude of the collision operator versus the temperature.

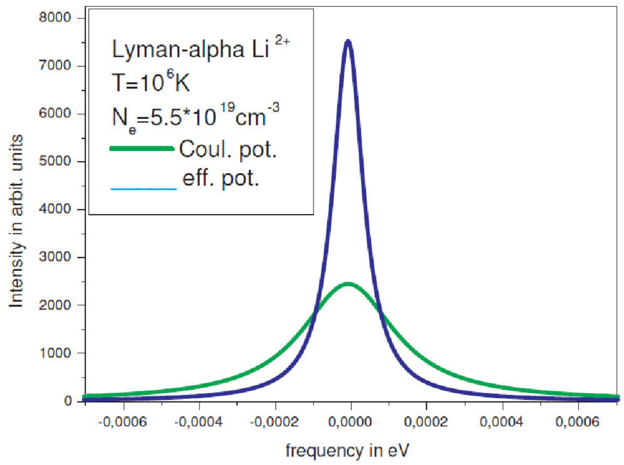

FIGURE 3. Spectral line shape Lyman alpha for different collision operators.

of the velocity v. The next step is to insert the obtained result of $q_{\max}$ and $q_{\min}$ in the Eqs. (29) and (27) and integrate over v to have the final result of the amplitude of the collision operator ϕ in (27) for the effective interaction potential.

Figure 1 shows the behaviour of the electronic collision operator versus the electron density in the case of the Coulomb and the effective potentials of the ion Li^{2+} (hydrogenlike) at a fixed temperature $T = 10^6 K$. We note that the variation of the electronic collision operator is proportional to the electron density for all potentials. This is because the number of collisions per second is proportional to the electron density. We note that the electronic collision operator relative to the effective potential is smaller than relative to Coulomb. Figure 2 shows the variation of the electronic collision operator for the different potentials versus the temperature at a fixed density $N_e = 10^{18}$ cm^{-3}. We note that the collision operator decreases slowly for the Coulomb potential according the temperature, whereas, it decreases exponentially for the case of the effective potential. In Figure 3, we present the spectral line for Lyman-α line, for Li^{+2} as hydrogen-like emitter for electron density equal to $(5.5)10^{19}$ cm^{-3} and a temperature equal to 10^6K. To do this, we have plotted two lines, each of them corresponds to Coulomb and the effective potential. We mention here, that only electron broadening are taken into account to plot the lines (Ionic Stark, Doppler and natural broadening are discarded). Each line corresponds then, to what force guides the electron during its collision with the emitter ion Li^{+2}. Then we can say that the line gives also, in addition to the diagnosis of density and temperature, an idea about the kind of the interaction between the unbounded electrons and the plasma. This figure (Fig. 3) illustrates the potential effect on the spectral line shape of Lyman-α without fine structure of hydrogen-like Li^{+2} at the temperature $T = 10^6$K and the density $N_e = (5.5)10^{19}$ cm^{-3}. We observe that when we use the effective potential, the width of the line decreases by 67.2 percent, and the intensity increases about 200 percent compared to the Coulomb case.

6. Conclusion

Assuming the interaction of the pair ion-electron in plasma to be generated by the Deutsch potential, that takes into account the quantum effects at short distance, and the effective interaction between the electron and all the plasma, we have presented in this paper a new expression of the collision operator in the framework of the impact approximation. The collision operator obtained is valid when the fine structure effect is neglected. We have applied it to compute and plot the Lyman-α lines for Li^{+2}. The result shows a difference between the two lines corresponding to the use of Coulomb and the effective potentials.

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